HW5 corrected solution

2. From problem 1, part a, we have

$$V = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad V^{-1} = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad V^{-1}AV = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{\Lambda}$$

So $e^{tA} = \boxed{V^{-1} e^{t\Lambda} V}$ ← this is incorrect. $e^{tA} = V e^{t\Lambda} V^{-1}$

$$= \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$e^{tA} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

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replace this with V

replace this with V^{-1}

$$= \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e^t & 0 & 4e^t \\ 0 & 0 & e^{2t} \\ 0 & e^{2t} & 0 \end{bmatrix} = \boxed{\begin{bmatrix} e^t & 0 & 4e^t - 4e^{2t} \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} = e^{tA}}$$

similarly $A^{100} = \boxed{V^{-1} \Lambda^{100} V}$ ← this is incorrect $A^{100} = V \Lambda^{100} V^{-1}$

$$= \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{100} & 0 \\ 0 & 0 & 2^{100} \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 2^{100} \\ 0 & 2^{100} & 0 \end{bmatrix}$$

hence $\boxed{A^{100} = \begin{bmatrix} 1 & 0 & 4 - 4 \cdot 2^{100} \\ 0 & 2^{100} & 0 \\ 0 & 0 & 2^{100} \end{bmatrix}}$ (check this in Matlab)

For $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (part b) we can just use the definition of $e^{tA} = \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \cdots$

hence $\boxed{e^{tA} = \begin{bmatrix} 1 & t \\ 0 & 0 \end{bmatrix}}$

this is incorrect (matrix addition mistake) this should be a 1. $e^{tA} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$

Also $\boxed{A^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$

$$e^{tA} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e^t & -4e^t & 0 \\ 0 & 0 & e^{2t} \\ 0 & e^{2t} & 0 \end{bmatrix} = \begin{bmatrix} e^t & 4e^{2t}-4e^t & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{2t} \end{bmatrix}$$

$$A^{100} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{100} & 0 \\ 0 & 0 & 2^{100} \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^{100} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 2^{100} \\ 0 & 2^{100} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4\cdot 2^{100}-4 & 0 \\ 0 & 2^{100} & 0 \\ 0 & 0 & 2^{100} \end{bmatrix}$$

these are the correct answers to problem 2a.