ECE504 Homework Assignment Number 7
Due by 8:45pm on 10-Nov-2009

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 3 pts. Chen 5.5.

2. 3 pts. For what values of parameter α is the state equation below BIBO stable? Explain.

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} 0 & \alpha \\ 2 & -1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \boldsymbol{x}(t) \end{aligned}$$

3. 3 pts. Is the state equation below BIBO stable? Explain.

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} 1/2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \boldsymbol{x}(t) \end{aligned}$$

4. 3 pts. Suppose you are given a DT-LTI system with

$$\begin{aligned} \boldsymbol{x}[k+1] &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \boldsymbol{x}[k] + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u[k] \\ y[k] &= \begin{bmatrix} c_1 & c_2 \end{bmatrix} \boldsymbol{x}[k] \end{aligned}$$

   Find the set of reachable states, parameterized by b_1 and b_2. Under what conditions on b_1 and b_2 is this a reachable system? Are there any choices for b_1 and b_2 so that the set of reachable states is not equivalent to the set of controllable states in this system?

5. 3 pts. Using the same system as problem 4, find a basis for the subspace of unobservable states, parameterized by c_1 and c_2. Under what conditions on c_1 and c_2 is this an observable system?