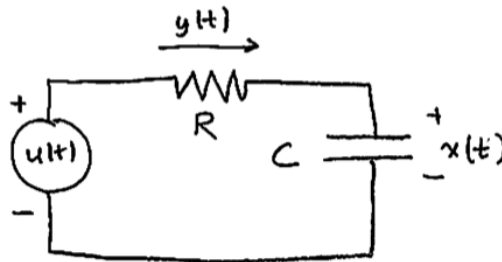


ECE504 Midterm Exam

20-Oct-2009

Notes:

- This exam is worth 350 points and is to be completed in 90 minutes.
 - Look over all the questions before starting.
 - Budget your time to allow enough time to work on each question.
 - To receive maximum credit, you must show your reasoning and/or work.
 - Points may be deducted for a disorderly presentation of your solution.
1. 60 points. Compare and contrast **transfer function** and **state space** mathematical descriptions of systems. What types of systems can be represented by both transfer functions and state-space mathematical descriptions? What types of systems can only be represented with transfer functions? What types of systems can only be represented with state-space descriptions? Be explicit.
 2. 120 points total. Given the circuit



- (a) 60 points. Let the state $x(t)$ be voltage across the capacitor and let the output $y(t)$ be the current. Find a state-space representation of the system such that

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)u(t) \\ y(t) &= \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)u(t).\end{aligned}$$

- (b) 60 points. Find an expression for the output of this system to a unit-step input

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

with initial state $x(0) = 2$.

3. 80 points. Given the system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \begin{bmatrix} -1 & e^{-2t} \\ 0 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [1 \quad 1] \mathbf{x}(t)\end{aligned}$$

find the state transition matrix $\Phi(t, s)$. Hint: the chain rule $(fg)' = f'g + fg'$ might be useful here.

4. 90 points total. Suppose $\theta \in [0, 2\pi)$ is a constant and that

$$\mathbf{A} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

- (a) 30 points. Under what conditions on θ is \mathbf{A} invertible? Hint: recall that $\sin^2 x + \cos^2 x = 1$.
- (b) 60 points. Under what conditions on θ is \mathbf{A} diagonalizable?