ECE531 Homework Assignment Number 1

Due by 8:50pm on Thursday 29-Jan-2009

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. You are given an urn containing one ball, known to be either black or white with equal probability. You drop in a white ball, shake the urn, and remove a white ball. What is the probability that the ball remaining in the urn is white?

2. 4 points. A passenger next to you on an airplane (whom you never previously met) tells you she has two children. What is the probability that they are both girls if she says "yes" to

   (a) Is at least one of them a girl?

   (b) Is the older one a girl?

3. 4 points. Suppose you have a wired communication system where the transmitter puts +1Vdc on the communication circuit if a binary one is transmitted and puts -1Vdc on the circuit if a binary zero is transmitted. Binary ones are transmitted with probability p and binary zeros are transmitted with probability $1-p$. The receiver forms its scalar observation by sampling the voltage in the circuit.

   The signal at the receiver is corrupted by noise. The sampled voltage at the receiver can be modeled as a random variable $Y = V + W$ where $V \in \{-1, +1\}$ is the voltage applied to the circuit by the transmitter and $W \in \mathbb{R}$ is a Gaussian random variable with zero mean and a standard deviation of 0.3 volts.

   (a) Plot the *posterior* probability that the transmitter sent +1Vdc as a function of the prior probability p given an observation of $Y = 0.3$Vdc.

   (b) Plot the *posterior* probability that the transmitter sent -1Vdc as a function of the prior probability p given an observation of $Y = 0.3$Vdc.

4. 8 points. Two random variables X and Y have the joint probability density function

$$p_{X,Y}(x,y) = \begin{cases} c(x-y)^2 & -1 \le x \le 1 \text{ and } -1 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the appropriate value for c.

   (b) Compute the joint probability $\mathrm{Prob}[X > 0, Y > 0]$.

   (c) Compute the conditional distribution $p_X(x \mid y)$.

   (d) Compute the conditional expectation $\mathrm{E}[X \mid y]$.

   (e) Compute the conditional probability $\mathrm{Prob}[X > 0 \mid Y \le 0]$.

   (f) Compute the conditional probability $\mathrm{Prob}[X > 0 \mid Y > 0]$.

   (g) What do parts (e) and (f) imply about the unconditional probability $\mathrm{Prob}[X > 0]$? Explain.