ECE531 Homework Assignment Number 5

Due by 8:50pm on Thursday 26-Feb-2009

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. Poor textbook Chapter II, Problem 13.
2. 6 points. Consider n uniform i.i.d. $Y_i \sim \mathcal{U}(0, x)$ random variables with $x > 0$ and hypotheses

$$\begin{aligned} \mathcal{H}_0 &: x \le \lambda \\ \mathcal{H}_1 &: x > \lambda \end{aligned}$$

   (a) Define $T(Y) = \max_i Y_i$ and find the significance level of the following decision rule

$$\rho(Y) = \begin{cases} 1 & \text{if } T(Y) > \lambda \\ \alpha & \text{if } T(Y) \le \lambda \end{cases}$$

   where $\rho(Y) = \alpha$ means that we decide $\mathcal{H}_1$ with probability α. Explain the intuition behind ρ. Derive the power function of ρ. Is ρ a UMP decision rule of size α?

   (b) Show that the vector observation $Y = (Y_0, \dots, Y_{n-1})$ has a monotone likelihood ratio in $T(Y)$.

   (c) Find a UMP decision rule of size α and compare its power function to that of ρ.

3. 4 points. Consider a Laplacian random variable Y with unknown mean $x \in [0, \infty)$ and conditional density $p_x(y) = \frac{b}{2} e^{-b|y-x|}$. Given one observation of Y, we want to decide $\mathcal{H}_0 \leftrightarrow x = 0$ versus $\mathcal{H}_1 \leftrightarrow x > 0$ subject to an upper bound α on the false positive probability. Find a UMP decision rule and write an expression for its power function. You can check your results against the lecture notes.
4. 6 points. Consider an observation of n i.i.d. Cauchy random variables

$$Y_i \sim \frac{1}{\nu(1 + (y - \lambda)^2)}$$

   and the one-sided test

$$\begin{aligned} \mathcal{H}_0 &: \lambda \le 0 \\ \mathcal{H}_1 &: \lambda > 0 \end{aligned}$$

   (a) Show that there is no UMP decision rule for this binary composite HT problem.

   (b) Find a LMP decision rule and its power function. Hint 1: You may need to numerically solve for the decision threshold. Hint 2: When n is large, you can use a Gaussian approximation on the test statistic to write the threshold as a Q function.