ECE531 Homework Assignment Number 7

Due by 8:50pm on Thursday 26-Mar-2009

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. Poor textbook Chapter IV, Problem 12.

2. 4 points. Suppose $Y_k \overset{\text{i.i.d.}}{\sim} p_Y(y; \theta)$ for $k = 0, \ldots, n - 1$ where

$$p_Y(y; \theta) = \begin{cases} \frac{y}{\sigma^2} \exp\left(\frac{-y^2}{2\sigma^2}\right) & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find a non-trivial sufficient statistic for estimating $\theta = \sigma^2$.
(b) Is your sufficient statistic complete?

3. 4 points. Repeat the previous problem for the case $Y_k \overset{\text{i.i.d.}}{\sim} \mathcal{U}[-\theta, \theta]$ for $k = 0, \ldots, n - 1$.

4. 4 points. Recall the linear model $Y = H\theta + W$ discussed in lecture with observations $Y \in \mathbb{R}^p$. Assume that $W \sim \mathcal{N}(0, \sigma^2 I)$ with $\sigma^2$ known. Find the MVU estimator of $\theta \in \mathbb{R}^m$ by using the Neyman-Fisher factorization and the Rao-Blackwell-Lehmann-Sheffe theorems. Hint: This is a non-random parameter estimation problem. You do not have a prior distribution on $\theta$.
   Another hint: If you are having trouble with the vector case, try $m = 1$ and $n = 1$ first.

5. 4 points. Poor textbook Chapter IV, Problem 13(a).