ECE531 Homework Assignment Number 8

Due by 8:50pm on Thursday 2-Apr-2009

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. The equality condition on the Cauchy-Schwarz inequality implies that the information bound is attained if and only if

$$\frac{\partial}{\partial\theta} \ln p_Y(y\,;\,\theta) \quad = \quad k(\theta)\left[\hat{\theta}(y) - \mathrm{E}_\theta\left\{\hat{\theta}(Y)\right\}\right]$$

almost surely for some $k(\theta)$. Assuming all of the regularity conditions hold, show that

$$k(\theta) \quad = \quad \frac{I(\theta)}{\frac{\partial}{\partial\theta}\mathrm{E}_\theta\left\{\hat{\theta}(Y)\right\}}.$$

What is $k(\theta)$ when we have an unbiased estimator?

2. 4 points. Poor textbook Chapter IV, Problem 13 (c).

3. 4 points. Poor textbook Chapter IV, Problem 20 (b). Hint: For a Gaussian random variable Y, we know $\frac{\mathrm{E}\{Y^4\}}{\mathrm{E}^2\{Y^2\}} = 3$.

4. 4 points. Poor textbook Chapter IV, Problem 21 (a), (b), (c), and (e).

5. 4 points. Suppose we have a polynomial fitting problem where the observed samples are given as

$$Y_k \quad = \quad \sum_{\ell=0}^{L-1} \theta_\ell k^\ell + W_k \text{ for } k = 0, 1, \ldots, n-1$$

where $W_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$. Find the Fisher information matrix for estimating $\{\theta_0, \ldots, \theta_{L-1}\}$. If you have difficulty with the general case, try $L = 2$ (fitting a straight line) first.