Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. Poor textbook Chapter IV, Problem 13 (b).

2. 4 points. Poor textbook Chapter IV, Problem 20 (c) and (d).

3. 4 points. Poor textbook Chapter IV, Problem 21 (d).

4. 4 points. Poor textbook Chapter IV, Problem 23.

5. 4 points. Write a Matlab program to simulate one dimensional motion as described on Slide 14 of Lecture 10b, with $T = 1$ and $X[0] = [0, 0]^\top$. Your input sequence $U[n]$ should be Gaussian with zero mean and unit variance and autocorrelation

$$
E[U[n]U[k]] = \begin{cases} 
1 & n = k \\
0 & \text{otherwise.}
\end{cases}
$$

Plot the position and velocity of the particle (see slide 6 of Lecture 10b) for several different realizations of the input sequence. Also, assuming that your observations are noiseless, write Matlab code to generate estimates of the state $X[n-1]$ for $n \geq 1$ given observations $Y[0], Y[1], \ldots, Y[n]$ (this is a “smoothing” problem). You don’t need to find an optimum estimator here; just use your intuition and confirm that your estimates are at least close to the true state.