ECE531 Homework Assignment Number 10

Due by 8:50pm on Thursday 20-Apr-2011

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 5 points. Prove that the innovation

$$\tilde{Y}[\ell+1\,|\,\ell] := Y[\ell+1] - H[\ell+1]\hat{X}[\ell+1\,|\,\ell]$$

is a zero-mean Gaussian random vector uncorrelated with $Y[0], \ldots, Y[\ell]$. Also prove that the innovation sequence is white.

2. 5 points. Suppose you have a scalar-state, scalar-observation linear time-invariant dynamic model given by

$$\begin{aligned} x[n+1] &= fx[n] + gu[n] \\ y[n] &= hx[n] + v[n] \end{aligned}$$

for $n = 0, 1, \ldots$ with f, g, h all non-zero scalars. The process noise $u[n]$ is a zero-mean Gaussian random process with $\mathrm{E}[u[m]u[n]] = q\delta_{m,n}$ with $q > 0$. The measurement noise $v[n]$ is a zero-mean Gaussian random process with $\mathrm{E}[v[m]v[n]] = r\delta_{m,n}$ with $r > 0$ and is assumed to be uncorrelated with $\{u[0], u[1], \ldots\}$. Write the Kalman filter recursion and determine $\lim_{\ell\to\infty} \Sigma[\ell+1\,|\,\ell]$ and $\lim_{\ell\to\infty} \Sigma[\ell\,|\,\ell]$, i.e. the steady-state prediction and estimation covariances. Interpret your results.

3. 5 points. Kay I: Problem 13.10

4. 10 points. Kay I: Problem 13.11. To clarify what I am expecting here, I would like you to write some Matlab/Octave code to run the KF over lots of realizations of the process noise and measurement noise, generate Monte-Carlo results for the MSE, and compare the Monte-Carlo MSE results to the error covariance matrices (actually scalars in this problem). You can also use your results from Problem 2 to confirm your KF is working correctly in part (b).