ECE531 Homework Assignment Number 2

Due by 8:50pm on Wednesday 2-Feb-2011

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 14 points. Suppose you work in a microprocessor manufacturing facility and that, before boxing and shipping, each microprocessor undergoes a quality check to avoid shipping defective units. The quality checking machine has the following characteristics:

   - It declares good microprocessors to be defective (D) with probability $p = 0.15$.
   - It declares defective microprocessors to be good (G) with probability $q = 0.03$.

   Suppose there are n such quality checking machines that give independent results with the same probabilities. Also let $\mathcal{H}_0$ be the hypothesis that the microprocessor is good and let $\mathcal{H}_1$ be the hypothesis that the microprocessor is defective.

   (a) 2 points. For the case $n = 1$ (each microprocessor is only checked by one machine), explicitly describe the states $\mathcal{X}$, possible observations $\mathcal{Y}$, the conditional probability matrix P, and the set of all possible deterministic decision rules. What type of hypothesis testing problem is this?

   (b) 2 points. Assume the uniform cost assignment (UCA) and $n = 1$. For each possible deterministic decision rule, plot the conditional risk vector $R(D) = [R_0(D), R_1(D)]^\top$ in the R_0, R_1 plane. Color in the set of all achievable conditional risk vectors in the R_0, R_1 plane and also identify the optimal tradeoff surface.

   (c) 2 points. Determine the Neyman-Pearson decision rule for significance levels $\alpha = 0.03$ and $\alpha = 0.01$ when $n = 1$. What is the probability of detection, i.e. β, for the N-P detector at each of these significance levels?

   (d) 6 points. Repeat (a)-(c) for the case $n = 2$ (each microprocessor is checked by two separate machines). Discuss on the how the set of achievable conditional risk vectors and the optimal tradeoff surface change when two quality checks are performed.

   (e) 2 points. How many quality checking machines would be needed to satisfy $P_{\mathsf{fp}} < 0.001$ and $P_D > 0.999$?

2. 5 points. Suppose you want to determine if a six-sided die is fair by rolling it and observing the outcomes of the rolls. There are two hypotheses

   - $\mathcal{H}_0$ the die is fair (all six numbers are equiprobable)
   - $\mathcal{H}_1$ the die in unfair with the following probabilities: $P(1) = 0.27$, $P(2) = 0.10$, $P(3) = 0.05$, $P(4) = 0.20$, $P(5) = 0.16$, and $P(6) = 0.22$.

   You observe only one roll of the die. Determine the Neyman-Pearson decision rule for significance level α and plot the probability of detection β as a function of α.

3. 3 points. Kay II:3.1.

4. 3 points. Kay II:3.4