ECE531 Homework Assignment Number 3

Due by 8:50pm on Wednesday 16-Feb-2011

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. Specify the N-P decision rule and probability of detection for the following hypothesis testing problem

$$\mathcal{H}_0 : p_Y(y\,;\mathcal{H}_0) = e^{-y}u(y)$$
$$\mathcal{H}_1 : p_Y(y\,;\mathcal{H}_1) = 2e^{-2y}u(y)$$

   as a function of the significance level α where $u(y)$ is the usual unit step function equal to one for $y \geq 0$ and equal to zero otherwise. Plot the probability of detection as a function of the significance level of the test.

2. 4 points. Specify the N-P decision rule and probability of detection for the following hypothesis testing problem

$$\mathcal{H}_0 : y \sim \mathcal{N}(0,1)$$
$$\mathcal{H}_1 : y \sim \mathcal{U}(-1,1)$$

   as a function of the significance level α. Plot the probability of detection as a function of the significance level of the test.

3. 4 points. Specify the Bayes decision rule for problem 1 as a function of the prior probability $\pi_0 = \text{Prob}\{\text{state is } x_0\}$ assuming the uniform cost assignment (UCA). Also plot the Bayes risk as a function of π_0.

4. 5 points. (Revisiting the quality checking problem from HW2): Suppose you work in a microprocessor manufacturing facility and that, before boxing and shipping, each microprocessor undergoes a quality check to avoid shipping defective units. The quality checking machine has the following characteristics:

   - It declares good microprocessors to be defective (D) with probability $p = 0.15$.
   - It declares defective microprocessors to be good (G) with probability $q = 0.03$.

   Suppose there are n such quality checking machines that give independent results with the same probabilities. Also let $\mathcal{H}_0$ be the hypothesis that the microprocessor is good and let $\mathcal{H}_1$ be the hypothesis that the microprocessor is defective. You receive some new information from manufacturing and customer service saying that

   - It costs \$200 to replace a defective microprocessor after it has been shipped to the customer.
   - It costs \$50 each time good microprocessors are discarded.
   - It costs nothing when good microprocessors are shipped and bad microprocessors are discarded.

   Assume $n = 2$. Determine the Bayes decision rule and the associated Bayes risk as a function of the prior probability $\pi_0 = \text{Prob}\{\text{microprocessor is good}\}$. Comment on your decision rule in the extreme cases when $\pi_0 \to 0$ and $\pi_0 \to 1$. Plot the risk of the Bayes detector as a function of π_0.

5. 4 points. Kay II: 3.18.

6. 4 points. Kay II: 3.21. Be explicit about how you derive your decision regions.