ECE531 Homework Assignment Number 4

Due by 8:50pm on Wednesday 23-Feb-2011

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 5 points total. Suppose you have a communication system in which two signals are transmitted to convey one bit of information. The signals are

$$x_0 : \cos\left(\frac{\pi}{4}n\right)$$
$$x_1 : \cos\left(\frac{\pi}{4}n + \phi\right)$$

for $n = 0, 1, \ldots, 3$ where $0 < \phi \leq \pi$ is known. The observation at the detector is corrupted by stationary zero-mean additive Gaussian noise $w[n]$ with covariance

$$\mathrm{E}[w[n]w[m]] = \begin{cases} 0.50 & n = m \\ 0.25 & |n-m| = 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) 3 points. Suppose $\phi = \pi/2$. Determine the minimum probability of error decision rule and its probability of error as a function of the prior probability π_0.

   (b) 2 points. Suppose $\pi_0 = 0.5$ and you are allowed to design the signal in state 1 to minimize the probability of error. Determine the value of ϕ that minimizes the probability of error.

2. 4 points. Kay II: 4.6

3. 4 points. Kay II: 4.7

4. 4 points. Suppose the state $x \sim \mathcal{U}(0, 2)$ and we receive one observation $Y = x + W$ where $p_W(t) = e^{-t}u(t)$. Determine the Bayes decision rule to decide between the composite hypotheses

$$\mathcal{H}_0 : 0 \leq x < 1$$
$$\mathcal{H}_1 : 1 \leq x \leq 2.$$

5. 4 points. Suppose you have n independent and identically distributed (i.i.d.) observations, each taking on the values 1 and 0 with probabilities p and $1-p$, respectively. Find a uniformly most powerful decision rule with false positive probability $\alpha = 2^{-n}$ for the hypothesis pair

$$\mathcal{H}_0 : p = \frac{1}{2}$$
$$\mathcal{H}_1 : p > \frac{1}{2}.$$

6. 4 points. Kay II: 6.21