ECE531: Principles of Detection and Estimation
Course Introduction

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First Lecture: Major Topics

1. Administrative details:
   - Course web page.
   - Syllabus and textbook.
   - Academic honesty policy.
   - Students with disabilities statement.
   - Piazza.
   - Weekly quizzes.
   - Kalman filter midterm project.
2. Course introduction.
3. Review of essential probability concepts.

Why No Homework?

I have found almost no value in collecting and grading homework.

- Homework grades are a poor assessment of student understanding. In my experience, homework grades are almost entirely uncorrelated to final course grades.
- Academic honesty problems.
- Student focus is often on handing something in rather than really learning the material.

You still need to do problems to understand this material. I will provide several suggested problems and solutions each week for you to use for practice. You are encouraged to collaborate!

Why Weekly Quizzes?

Traditional midterm/final format leads to poor long-term retention.

Immediate feedback for instructor.

We don't lose two class periods to exams.

Why No Lectures?

It is very difficult to pay attention to a three-hour evening lecture.

Recent trends “flip teaching” and “mobile learning”:

- Watch lecture materials outside of the classroom on your own schedule
- Do assigned reading
- Work on suggested problems
- Collaborate with other students (in person or via Piazza) and reinforce your understanding of the material
- Classroom meeting time can be used for more hands-on discussion

Bottom line: I can spend class time interacting with you and helping you understand the material, instead of lecturing.

Why Piazza?

I will answer questions on Piazza. Those questions/answers will be visible to everyone.

You can answer questions (or contribute to the discussion) on Piazza too. Helping others understand the material is an excellent way to reinforce your own understanding.

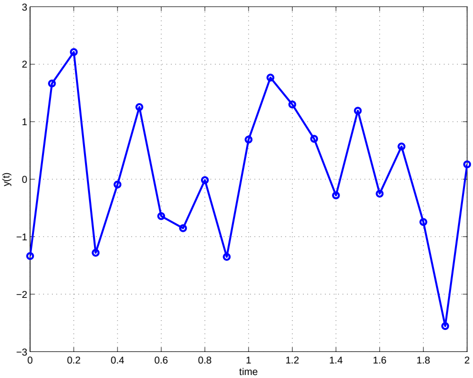
I can mark questions/answers as "good questions" and "good answers". I can also mark duplicate questions. Try not to post duplicate questions.

If you have a question, chances are someone else has that question too. In fact, your question might already be answered on Piazza.

Some Notation

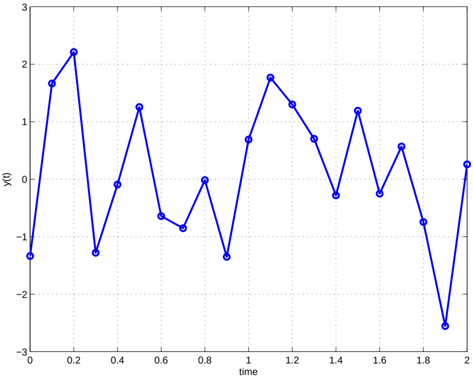
- A set with discrete elements: $\mathcal{S} = \{-1, \pi, 6\}$.
- The cardinality of a set: $|\mathcal{S}| = 3$.
- The set of all integers: $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$.
- The set of all real numbers: $\mathbb{R} = (-\infty, \infty)$.
- Intervals on the real line: $[-3, 1]$, $(0, 1]$, $(-1, 1)$, $[10, \infty)$.
- Multidimensional sets:
  - $\{a, b, c\}^2$ is shorthand for the set $\{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$.
  - $\mathbb{R}^2$ is the two-dimensional real plane.
  - $\mathbb{R}^3$ is the three-dimensional real volume.
- An element of a set: $s \in \mathcal{S}$.
- A subset: $\mathcal{W} \subseteq \mathcal{S}$.
- The probability of an event A: $\mathrm{Prob}[A] \in [0, 1]$.
- The joint probability of events A and B: $\mathrm{Prob}[A, B] \in [0, 1]$.
- The probability of event A conditioned on event B: $\mathrm{Prob}[A \,|\, B] \in [0, 1]$.

Typical Detection Problems



- Is this a sine wave plus noise, or just noise?
- Is the frequency of the sine wave 1Hz or 2Hz?
- Detection is about making smart choices from a finite set of possibilities (and the consequences).

Typical Estimation Problems



- What is the frequency, phase, and/or amplitude of the sine wave?
- What is the mean and/or variance of the noise?
- Estimation is about "guessing" values from an infinite number of possibilities (and the consequences).

Joint Estimation and Detection

Suppose we have a binary communication system with an intersymbol interference channel. M symbols are sent through the channel and we observe

$$y_k = \sum_{\ell=0}^{L-1} h_\ell s_{k-\ell} + w_k$$

for $k \in \{0, \dots, L+M-2\}$ where

- Unknown binary symbols $[s_0, \dots, s_{M-1}] \in \{-1, +1\}^M$
- Unknown discrete-time impulse response of channel $[h_0, \dots, h_{L-1}] \in \mathbb{R}^L$
- Unknown noise $[w_0, \dots, w_{L+M-2}] \in \mathbb{R}^{L+M-1}$

In some scenarios, we may want know the bits that were sent and the channel coefficients. This is a joint estimation and detection problem. Why?

Consequences

To develop optimal decision rules or estimators, we need to quantify the **consequences** of incorrect decisions or inaccurate estimates.

Simple Example

It is not known if a coin is fair (F) or double headed (HH). We are given one observation of the coin flip. Based on this observation, how do you decide if the coin is F or HH?

Observation	Rule 1	Rule 2	Rule 3	Rule 4
H	HH	F	HH	F
T	HH	F	F	HH

Suppose you have to pay $100 if you are wrong. Which decision rule is "optimum"?

Rule 1: Always decide HH

Note that the observation is ignored here.

- If the coin is F (fair), the decision was wrong and you must pay \$100.
- If the coin is HH (double headed), the decision was right and you pay nothing.

The **maximum cost** (between HH or F) for Rule 1 is \$100.
The **average cost** for Rule 1 is

$$\bar{C}_1 = \mathrm{Prob}[\mathsf{F}] \cdot \$100 + \mathrm{Prob}[\mathsf{HH}] \cdot \$0$$

where $\mathrm{Prob}[\mathsf{F}]$ and $\mathrm{Prob}[\mathsf{HH}]$ are the prior probabilities (our belief before any observations) on the coin being fair or double headed, respectively. For purposes of illustration, lets assume $\mathrm{Prob}[\mathsf{F}] = \mathrm{Prob}[\mathsf{HH}] = 0.5$ so that $\bar{C}_1 = \$50$.

Rule 2: Always decide F

Again, the observation is being ignored. Same analysis as for Rule 1...

- If the coin is F (fair), the decision was right and you pay nothing.
- If the coin is HH (double headed), the decision was wrong and you must pay $100.

The **maximum cost** for Rule 2 is $100.
The **average cost** for Rule 2 is

$$\bar{C}_2 = \mathrm{Prob}[\mathsf{F}] \cdot \$0 + \mathrm{Prob}[\mathsf{HH}] \cdot \$100$$

If $\mathrm{Prob}[\mathsf{F}] = \mathrm{Prob}[\mathsf{HH}] = 0.5$, then $\bar{C}_2 = \$50$.

Rule 3: Decide HH if H observed, F if T observed

- If the coin is F (fair), there is a 50% chance the observation will be H and you will decide HH. This will cost you \$100. There is also a 50% chance that the observation will be T and you will decide F. In this case, you made the correct decision and pay nothing.

$$C_{\mathsf{F}} = \mathrm{Prob}[\mathsf{H}|\mathsf{F}] \cdot \$100 + \mathrm{Prob}[\mathsf{T}|\mathsf{F}] \cdot \$0 = \$50$$

- If the coin is HH (double headed), what is our cost? \$0

The **maximum cost** for Rule 3 is \$50.
The **average cost** for Rule 3 is

$$\bar{C}_3 = \mathrm{Prob}[\mathsf{F}] \cdot \$50 + \mathrm{Prob}[\mathsf{HH}] \cdot \$0$$

If $\mathrm{Prob}[\mathsf{F}] = \mathrm{Prob}[\mathsf{HH}] = 0.5$, then $\bar{C}_3 = \$25$.

Rule 4: Decide F if H observed, HH if T observed

Obviously, this is a bad rule.

- If the coin is F (fair), there is a 50% chance the observation will be T and you will decide HH. This will cost you \$100. There is also a 50% chance that the observation will be H and you will decide F. In this case, you made the correct decision and pay nothing.

$$C_{\mathsf{F}} = \mathrm{Prob}[\mathsf{T}|\mathsf{F}] \cdot \$100 + \mathrm{Prob}[\mathsf{H}|\mathsf{F}] \cdot \$0 = \$50$$

- If the coin is HH (double headed), what is our cost? \$100

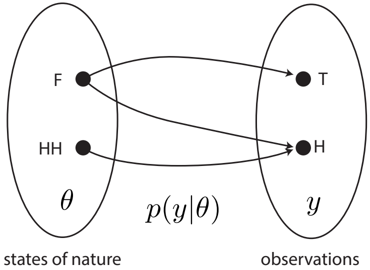
The **maximum cost** for Rule 4 is \$100.
The **average cost** for Rule 4 is

$$\bar{C}_3 = \mathrm{Prob}[\mathsf{F}] \cdot \$50 + \mathrm{Prob}[\mathsf{HH}] \cdot \$100$$

If $\mathrm{Prob}[\mathsf{F}] = \mathrm{Prob}[\mathsf{HH}] = 0.5$, then $\bar{C}_4 = \$75$.

Remarks

- The notion of **maximum cost** is the maximum over the possible "states of nature" (HH and F in our example), but averaged over the probabilities of the observation.



- In our example, we could always lose \$100, irrespective of the decision rule. But the maximum cost of Rule 3 was \$50.
- Is Rule 3 optimal?

Probability Basics: Events

Let A be a possible (or impossible) outcome of a random experiment. We call A an "event" and $\text{Prob}[A] \in [0,1]$ is the probability that A happens.

Examples:

- $A =$ tomorrow will be sunny in Worcester, $\text{Prob}[A] = 0.4$.
- $A =$ a 9 is rolled with two fair 6-sided dice, $\text{Prob}[A] = \frac{4}{36}$.
- $A =$ a 13 is rolled with two fair 6-sided dice, $\text{Prob}[A] = 0$.
- $A =$ an odd number is rolled with two fair 6-sided dice,

$$\text{Prob}[A] = \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = \frac{1}{2}$$

- $A =$ any number but 9 is rolled with two fair 6-sided dice, $\text{Prob}[A] = 1 - \frac{4}{36} = \frac{32}{36}$

The last result used the fact that $\text{Prob}[A] + \text{Prob}[\bar{A}] = 1$, where $\bar{A}$ means "not event A" and $\text{Prob}[\bar{A}]$ is the probability that A doesn't happen.

Probability Basics: Random Variables

Definition

A random variable is a mapping from random experiments to real numbers.

Example: Let X be the Dow Jones average at the close on Friday.

We can easily relate events and random variables.

Example: What is the probability that $X \geq 13500$?

- X is the random variable. It can be anything on the interval $[0, \infty)$.
- The event is $A =$ "X is no less than 13500".

To answer these types of questions, we need to know the probabilistic distribution of the random variable X. Every random variable has a **cumulative distribution function** (CDF) defined as

$$F_X(x) := \mathrm{Prob}[X \leq x]$$

for all $x \in \mathbb{R}$.

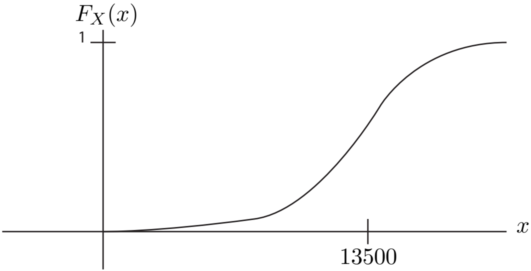
Probability Basics: Properties of the CDF

$$F_X(x) := \text{Prob}[X \le x]$$

The following properties are true for any random variable X:

- $F_X(-\infty) = 0$.
- $F_X(\infty) = 1$.
- If $y > x$ then $F_X(y) \ge F_X(x)$.

Example: Let X be the Dow Jones average at the close on Friday.



Probability Basics: The PDF

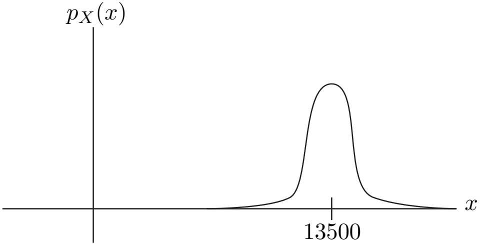
The **probability density function** (PDF) of the random variable X is

$$p_X(x) := \frac{d}{dx} F_X(x)$$

The following properties are true for any random variable X:

- $p_X(x) \geq 0$ for all x.
- $\int_{-\infty}^{\infty} p_X(x)\,dx = 1$.
- $\text{Prob}[a < X \leq b] = \int_a^b p_X(x)\,dx = F_X(b) - F_X(a)$.

Example: Let X be the Dow Jones average at the close on Friday.



Probability Basics: Mean and Variance

Definition

The **mean** of the random variable X is defined as

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x p_X(x)\, dx.$$

The mean is also called the **expectation**.

Definition

The **variance** of the random variable X is defined as

$$\mathrm{var}[X] = \int_{-\infty}^{\infty} (x - \mathrm{E}[X])^2 p_X(x)\, dx.$$

Remark: The standard deviation of X is equal to $\mathrm{std}[X] = \sqrt{\mathrm{var}[X]}$.

Probability Basics: Properties of Mean and Variance

Assuming c is a known constant, it is not difficult to show the following properties of the mean:

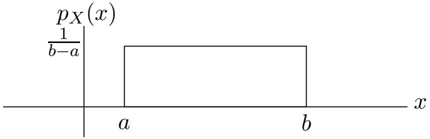
1. $\mathrm{E}[cX] = c\mathrm{E}[X]$ (by linearity)
2. $\mathrm{E}[X + c] = \mathrm{E}[X] + c$ (by linearity)
3. $\mathrm{E}[c] = c$

Assuming c is a known constant, it is not difficult to show the following properties of the variance:

1. $\mathrm{var}[cX] = c^2\mathrm{var}[X]$
2. $\mathrm{var}[X + c] = \mathrm{var}[X]$
3. $\mathrm{var}[c] = 0$

Uniform Random Variables

Uniform distribution: $X \sim \mathcal{U}(a,b)$ for $a \leq b$.



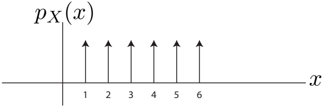
$$p_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Suppose $X \sim \mathcal{U}(1,5)$.

- Sketch the CDF.
- What is $\mathrm{Prob}[X = 3]$?
- What is $\mathrm{Prob}[X < 2]$?
- What is $\mathrm{Prob}[X > 1]$?
- What is $\mathrm{E}[X]$?
- What is $\mathrm{var}[X]$?

Discrete Uniform Random Variables

Uniform distribution: $X \sim \mathcal{U}(\mathcal{S})$ where $\mathcal{S}$ is a finite set of discrete points on the real line. Each element in the set is equally likely. Example:



Given $\mathcal{S} = \{s_1, \dots, s_n\}$, then $\mathrm{Prob}[X = s_1] = \dots \mathrm{Prob}[X = s_n] = \frac{1}{n}$ and

$$p_X(x) = \frac{1}{n}\left(\delta(x - s_1) + \dots + \delta(x - s_n)\right)$$

Suppose $X \sim \mathcal{U}(\{1, 2, 3\})$.

- Sketch the CDF.
- What is $\mathrm{Prob}[X = 3]$?
- What is $\mathrm{Prob}[X < 2]$?
- What is $\mathrm{Prob}[X \leq 2]$?
- What is $\mathrm{E}[X]$?
- What is $\mathrm{var}[X]$?

Gaussian Random Variables

Gaussian distribution: $X \sim \mathcal{N}(\mu, \sigma^2)$ for any μ and σ.

$$p_X(x) \quad = \quad \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

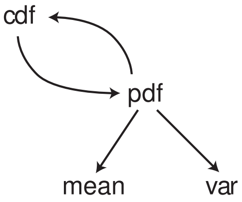
Remarks:

1. $\mathrm{E}[X] = \mu$.
2. $\mathrm{var}[X] = \sigma^2$.
3. Gaussian random variables are completely specified by their mean and variance.
4. Lots of things in the real world are Gaussian or approximately Gaussian distributed, e.g. exam scores, etc. The Central Limit Theorem explains why this is so.
5. Probability calculations for Gaussian random variables often require the use of erf and/or erfc functions (or the Q-function).

Final Remarks on Scalar Random Variables

1. The PDF and CDF completely describe a random variable.
   - If X and Y have the same PDF, then they have the same CDF, the same mean, and the same variance.
2. The mean and variance are only partial statistical descriptions of a random variable.
   - If X and Y have the same mean and/or variance, they might have the same PDF/CDF but not necessarily.

One Random Variable



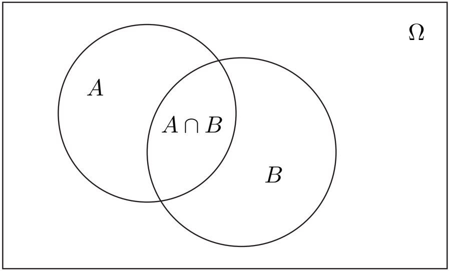
Joint Events

Suppose you have two events A and B. We can define a new event

$$C = \text{both } A \text{ and } B \text{ occur}$$

and we can write

$$\text{Prob}[C] = \text{Prob}[A \cap B] = \text{Prob}[A, B]$$



Conditional Probability of Events

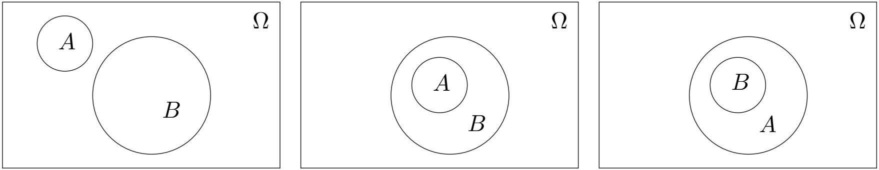
Suppose you have two events A and B. We can condition on the event B to write the probability

$$\text{Prob}[A \,|\, B] = \text{the probability of event } A \text{ given event } B \text{ happened}$$

When $\text{Prob}[B] > 0$, this conditional probability is defined as

$$\text{Prob}[A \,|\, B] = \frac{\text{Prob}[A \cap B]}{\text{Prob}[B]} = \frac{\text{Prob}[A, B]}{\text{Prob}[B]}$$

Three special cases:



Conditional Probabilities: Our Earlier Example

It is not known if a coin is fair (F) or double headed (HH). We can write the conditional probabilities of a one-flip observation as

$$\begin{aligned}
\mathrm{Prob}[\text{observe H} \,|\, \text{coin is F}] &= 0.5 \\
\mathrm{Prob}[\text{observe T} \,|\, \text{coin is F}] &= 0.5 \\
\mathrm{Prob}[\text{observe H} \,|\, \text{coin is HH}] &= 1 \\
\mathrm{Prob}[\text{observe T} \,|\, \text{coin is HH}] &= 0
\end{aligned}$$

Can you compute $\mathrm{Prob}[\text{coin is HH} \,|\, \text{observe H}]$?

We can write

$$\begin{aligned}
\mathrm{Prob}[\text{coin is HH} \,|\, \text{observe H}] &= \frac{\mathrm{Prob}[\text{coin is HH}, \text{observe H}]}{\mathrm{Prob}[\text{observe H}]} \\
&= \frac{\mathrm{Prob}[\text{observe H} \,|\, \text{coin is HH}]\mathrm{Prob}[\text{coin is HH}]}{\mathrm{Prob}[\text{observe H}]}
\end{aligned}$$

We are missing two things: $\mathrm{Prob}[\text{coin is HH}]$ and $\mathrm{Prob}[\text{observe H}]$....

Conditional Probabilities: Our Earlier Example

The term $\mathrm{Prob}[\text{coin is HH}]$ is called the **prior** probability, i.e. it is our belief that the coin is unfair **before we have any observations**. This is assumed to be given in some of the problems that we will be considering, so let's say for now that $\mathrm{Prob}[\text{coin is HH}] = 0.5$.

The term $\mathrm{Prob}[\text{observe H}]$ is the **unconditional probability** that we observe heads. Can we calculate this?

Theorem (Total Probability Theorem)

If the events $B_1, \dots, B_n$ are mutually exclusive, i.e. $\mathrm{Prob}[B_i, B_j] = 0$ for all $i \neq j$, and exhaustive, i.e. $\sum_{i=1}^n \mathrm{Prob}[B_i] = 1$, then

$$\mathrm{Prob}[A] = \sum_{i=1}^{n} \mathrm{Prob}[A \,|\, B_i] P[B_i].$$

So how can we use this result to compute $\mathrm{Prob}[\text{observe H}]$?

Independence of Events

Two events are independent if and only if their joint probability is equal to the product of their individual probabilities, i.e

$$\text{Prob}[A, B] = \text{Prob}[A]\text{Prob}[B]$$

Lots of events can be assumed to be independent. For example, suppose you flip a fair coin twice with $A =$"the first coin flip is heads", $B =$"the second coin flip is heads", and $C =$"both coin flips are heads".

- Are A and B independent?
- Are A and C independent?

Note that when events A and B are independent,

$$\text{Prob}[A \mid B] = \frac{\text{Prob}[A, B]}{\text{Prob}[B]} = \frac{\text{Prob}[A]\text{Prob}[B]}{\text{Prob}[B]} = \text{Prob}[A].$$

This should be intuitively satisfying since knowing B happened doesn't give you any useful information about A.

Joint Random Variables

When we have two random variables, we require a joint distribution. The joint CDF is defined as

$$F_{X,Y}(x,y) = \mathrm{Prob}[X \leq x \cap Y \leq y] = \mathrm{Prob}[X \leq x, Y \leq y]$$

and the joint PDF is defined as

$$p_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y)$$

If you know the joint distribution, you can get the marginal and conditional distributions:

$$F_X(x) = F_{X,Y}(x,\infty) \qquad F_Y(y) = F_{X,Y}(\infty,y)$$

$$p_X(x) = \int_{-\infty}^{\infty} p_{X,Y}(x,y)\,dy \qquad p_Y(y) = \int_{-\infty}^{\infty} p_{X,Y}(x,y)\,dx$$

$$p_X(x|Y=y) = \frac{p_{X,Y}(x,y)}{p_Y(y)} \qquad p_Y(y|X=x) = \frac{p_{X,Y}(x,y)}{p_X(x)}$$

Marginals are not enough to specify the joint distribution, except in special cases.

Conditional Distributions

If X and Y are both continuous random variables, then the conditional PDF of Y given $X = x$ is

$$p_Y(y \,|\, X = x) = p_Y(y \,|\, x) = \frac{p_{X,Y}(x, y)}{p_X(x)}.$$

with $p_{X,Y}(x, y)$ as the usual joint distribution of X and Y.
If X and Y are both discrete random variables (both are drawn from finite sets) with $\mathrm{Prob}[X = x] > 0$, then

$$\mathrm{Prob}[Y = y \,|\, X = x] \quad = \quad \frac{\mathrm{Prob}[X = x, Y = y]}{\mathrm{Prob}[X = x]}$$

If Y is a discrete random variable and X is a continuous random variable, then the conditional probability that $Y = y$ given $X = x$ is

$$\mathrm{Prob}[Y = y \,|\, X = x] = \frac{p_{X,Y}(x, y)}{p_X(x)}$$

where $p_{X,Y}(x, y) := \lim_{h \to 0} \frac{\mathrm{Prob}[x - h < X \leq x, Y = y]}{h}$ is the joint PDF-probability of the random variable X and the event $Y = y$.

Joint, Marginal, and Conditional Probabilities

Suppose you have two random variables X and Y that can take on discrete values $\{1, 2\}$. We can specify the joint distribution in a table, e.g.

	X=1	X=2
Y=1	0.1	0.3
Y=2	0.2	0.4

where the value in each element of the table corresponds to the probability of the joint event $X = a$ and $Y = b$, denoted as $p_{X,Y}(a, b)$.

Note the elements in the table sum to one.

- What is the most likely event?
- What is the marginal distribution $p_X(x)$?
- What is the conditional distribution on Y given $X = 1$?

Joint Statistics

Note that the means and variances are defined as usual for X and Y.
When we have a joint distribution, we have two new statistical quantities:

Definition

The **correlation** of the random variables X and Y is defined as

$$\mathrm{E}[XY] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy p_{X,Y}(x,y)\, dx\, dy.$$

Definition

The **covariance** of the random variables X and Y is defined as

$$\mathrm{cov}[X,Y] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \mathrm{E}[X])(y - \mathrm{E}[Y]) p_{X,Y}(x,y)\, dx\, dy.$$

Conditional Statistics

Definition

The **conditional mean** of a random variable Y given $X = x$ is defined as

$$\mathrm{E}[Y \,|\, x] \;=\; \int_{-\infty}^{\infty} y p_Y(y \,|\, x)\, dy.$$

The definition is identical to the regular mean except that we use the conditional PDF.

Definition

The **conditional variance** of a random variable Y given $X = x$ is defined as

$$\mathrm{var}[Y \,|\, x] \;=\; \int_{-\infty}^{\infty} (y - \mathrm{E}[Y \,|\, x])^2 p_Y(y \,|\, x)\, dx.$$

Independence of Random Variables

Two random variables are independent if and only if their joint distribution is equal to a product of their marginal distributions, i.e.

$$p_{X,Y}(x,y) = p_X(x)p_Y(y).$$

If X and Y are independent, the conditional PDFs can be written as

$$p_Y(y\,|\,x) = \frac{p_{X,Y}(x,y)}{p_X(x)} = \frac{p_X(x)p_Y(y)}{p_X(x)} = p_Y(y)$$

and

$$p_X(x\,|\,y) = \frac{p_{X,Y}(x,y)}{p_Y(y)} = \frac{p_X(x)p_Y(y)}{p_Y(y)} = p_X(x).$$

These results should be intuitively satisfying since knowing $X = x$ (or $Y = y$) doesn't tell you anything about Y (or X).

Jointly Gausian Random Variables

Definition: The random variables $\boldsymbol{X} = [X_1, \dots, X_k]^\top$ are jointly Gaussian if their joint density is given as

$$p_{\boldsymbol{X}}(\boldsymbol{x}) = |2\pi \boldsymbol{P}|^{-1/2} \exp\left(\frac{(\boldsymbol{x}-\boldsymbol{\mu}_X)^\top \boldsymbol{P}^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_X)}{2}\right)$$

where $\boldsymbol{\mu}_X = \mathrm{E}[\boldsymbol{X}]$ and $\boldsymbol{P} = \mathrm{E}[(\boldsymbol{X}-\boldsymbol{\mu}_X)(\boldsymbol{X}-\boldsymbol{\mu}_X)^\top]$.

Remarks:

1. $\boldsymbol{\mu}_X = [\mathrm{E}[X_1], \dots, \mathrm{E}[X_k]]^\top$ is a k-dimensional vector of means
2. $\boldsymbol{P}$ is a $k \times k$ matrix of covariances, i.e.,

$$\boldsymbol{P} = \begin{bmatrix} \mathrm{E}[(X_1-\mu_{X_1})(X_1-\mu_{X_1})] & \dots & \mathrm{E}[(X_1-\mu_{X_1})(X_k-\mu_{X_k})] \\ \vdots & \ddots & \vdots \\ \mathrm{E}[(X_k-\mu_{X_k})(X_1-\mu_{X_1})] & \dots & \mathrm{E}[(X_k-\mu_{X_k})(X_k-\mu_{X_k})] \end{bmatrix}$$

What's Next?

This is the only "lecture" in ECE531. The remaining class meetings will be structured as

- First half: Discussion/examples related to reading assignment, screencasts, suggested problems.
- Second half: 60-minute quiz.

Next week's meeting will be focused on topics from Kay vI:1-2. These chapters provide an introduction to the problem of parameter estimation and develop the concept minimum-variance unbiased (MVU) estimation.

We will be focused on MVU estimation for the first 3-4 weeks of the course.

Any questions/discussion during the week should be posted to Piazza. Students are encouraged to collaborate on everything except the quizzes.