ECE531 Screencast 1.1: Introduction to Estimation

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Introduction

- Detection: Make a choice between two (or several) discrete situations from observations.
- Estimation: Determine as accurately as possible the actual value of one or more parameter(s) from observations.

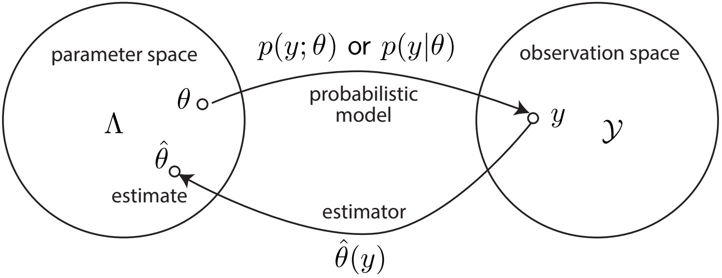
Our focus for the first half of the course is on parameter estimation.

Two basic approaches/philosophies for parameter estimation:

- **Bayesian**: The unknown parameter(s) are assumed to be random and have a known prior distribution.
- **Non-random (classical)**: The unknown parameter(s) are assumed to be deterministic and do not possess any known prior distribution.

Chapters 2-9 of Kay vI are about non-random parameter estimation.
Chapters 10-13 of Kay vI are about Bayesian estimation.

Mathematical Model for Estimation



Notation and terminology:

- The parameter space Λ is assumed to be a subset of $\mathbb{R}^m$.
- Y denotes the random observation with realizations $y \in \mathcal{Y} \subseteq \mathbb{R}^n$.
- An **estimator** is a function mapping $\mathcal{Y} \mapsto \Lambda$.
- An **estimate** is a realization of the estimator corresponding to a particular observation $Y = y$.
- The shorthand notation $\hat{\theta}$ can refer to the estimate or the estimator.

Example 1

Estimating the mean and variance of Gaussian distributed observations.
Parameters to estimate:

$$\begin{aligned} \theta &= \begin{bmatrix} \mu \\ \sigma^2 \end{bmatrix} \\ \mu &\in \mathbb{R} \\ \sigma^2 &\in [0, \infty) \end{aligned}$$

Observation model:

$$Y_k \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, \sigma^2) \text{ for } k = 0, \dots, n-1$$

Reasonable estimators? $\hat{\theta} = [\hat{\mu}, \hat{\sigma^2}]^\top$

$$\hat{\mu}(y) = \frac{1}{n}\sum_{k=0}^{n-1} y_k \qquad \hat{\sigma^2}(y) = \frac{1}{n}\sum_{k=0}^{n-1}(y_k - \hat{\mu}(y))^2$$

Example 2

Estimating the unknown frequency of a sinusoid in noise: $\theta \in (-\pi/2, \pi/2]$.

Observation model:

$$Y_k = \cos(\theta k) + W_k \text{ for } k = 0, \dots, n-1 \text{ with } W_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$$

A reasonable estimator?

$$\hat{\theta}(y) = \arg \max_{\omega \in (-\pi/2, \pi/2]} \left| \sum_{k=0}^{n-1} y_k e^{-jk\omega} \right|$$

Example 3

Estimating the unknown interval of uniformly distributed observations: $\theta \in (0, \infty)$.

Observation model:

$$Y_k \overset{\text{i.i.d.}}{\sim} \mathcal{U}(0, \theta) \text{ for } k = 0, \dots, n-1$$

A reasonable estimator?

$$\hat{\theta}(y) = \max\{y_0, \dots, y_{n-1}\}$$