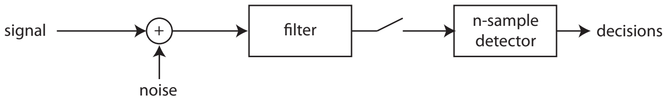
ECE531 Screencast 10.1: Introduction to Deterministic Signal Detection

D. Richard Brown III

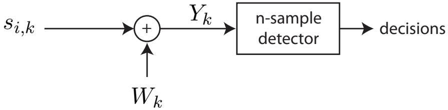
Worcester Polytechnic Institute

Detection of Deterministic Discrete-Time Signals in Noise

Continuous-Time Signal Model



Equivalent Discrete-Time Signal Model



In the **deterministic** model, we have a finite number of **known** signal vectors $s_i = [s_{i,0}, \dots, s_{i,n-1}]^\top$, $i = 0, \dots, M-1$, observed in additive noise $W = [W_0, \dots, W_{n-1}]^\top$.

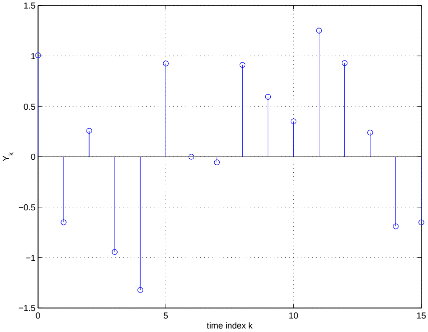
The problem is to use the n-sample observation $[Y_0, \dots, Y_{n-1}]^\top$ to determine which of the M signals was sent.

Example 1

Suppose

$$s_0 = [0, \ldots, 0]^\top$$
$$s_1 = [0, \sin(\pi/4), \ldots, \sin((n-1)\pi/4)]^\top$$

and that $W_k \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$. Example observation:



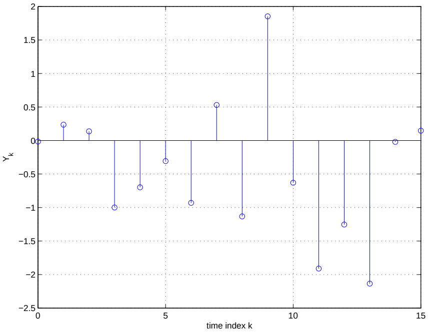
Which signal was sent, s_0 or s_1?

Example 2

Suppose

$$s_0 = [1, \cos(\pi/4), \ldots, \cos((n-1)\pi/4)]^\top$$
$$s_1 = [0, \sin(\pi/4), \ldots, \sin((n-1)\pi/4)]^\top$$

and that $W_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$. Example observation:



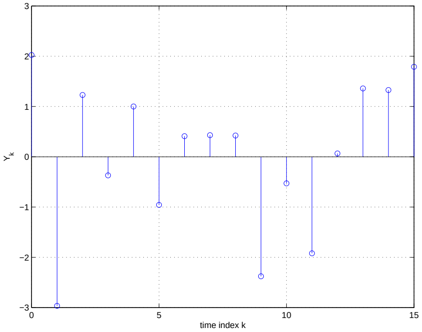
Which signal was sent, s_0 or s_1?

Example 3

Suppose

$$s_0 = [1, \cos(\pi/4), \dots, \cos((n-1)\pi/4)]^\top \quad s_2 = [-1, -\cos(\pi/4), \dots, -\cos((n-1)\pi/4)]^\top$$
$$s_1 = [0, \sin(\pi/4), \dots, \sin((n-1)\pi/4)]^\top \quad s_3 = [0, -\sin(\pi/4), \dots, -\sin((n-1)\pi/4)]^\top$$

and that $W_k \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$. Example observation:



Which signal was sent, s_0 or s_1?

Detection of Deterministic Signals in Noise

Detection of deterministic signals in noise is a simple hypothesis testing problem with $N = M$ states and hypotheses.

1. Example 1: $N = M = 2$.
2. Example 2: $N = M = 2$.
3. Example 3: $N = M = 4$.

The first two examples are **simple binary** hypothesis testing problems. Those examples are particularly easy to solve because we know that the optimum decision rule is going to be of the form

$$\rho(y) = \begin{cases} 1 & \text{if } L(y) > v \\ \gamma & \text{if } L(y) = v \\ 0 & \text{if } L(y) < v \end{cases}$$

where v and γ depend on the criterion (N-P or Bayes) and $L(y) = \frac{p_1(y)}{p_0(y)}$.