ECE531 Screencast 12.1: Introduction to Deterministic Signal Detection with Unknown Parameters

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Introduction

We now consider **parametric detection** problems for known/deterministic signals with one or more unknown parameters.

- We focus on the case of an N-sample discrete time observation $Y \in \mathcal{Y}$ with binary hypotheses

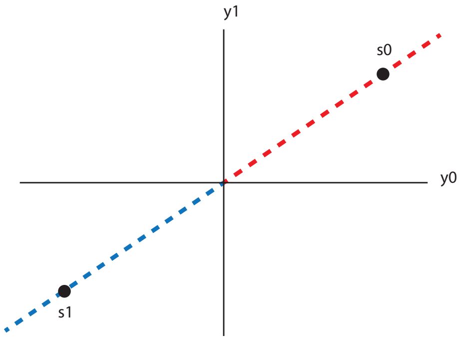
$$\begin{aligned} \mathcal{H}_0 &: Y \sim p_x(y) \text{ for } x \in \mathcal{X}_0 \\ \mathcal{H}_1 &: Y \sim p_x(y) \text{ for } x \in \mathcal{X}_1 \end{aligned}$$

  with $\mathcal{X}_0 \bigcup \mathcal{X}_1 = \mathcal{X}$ and $\mathcal{X}_0 \bigcap \mathcal{X}_1 = \emptyset$
- Our approach: Since we will have more than one state associated with one or both hypotheses, we will use our tools from **composite hypothesis testing** to develop good detectors.

Example 1

Suppose we have a known signal $s \in \mathbb{R}^n$ and we have a communication system that transmits either $s_0 = -s$ or $s_1 = s$. The signal arrives with unknown amplitude $a > 0$ and is corrupted by zero-mean AWGN with known variance σ^2.

For $n = 2$, we can illustrate this situation:



Example 1 (continued)

A key step in these kinds of problems is to set them up correctly. If possible, we want to absorb the unknown parameter(s) into the state.

In this example, since $s_1 = s = -s_0$, we have

$$Y \sim \mathcal{N}(as, \sigma^2 I)$$

from any state $a \in \mathcal{X}$ with $a < 0$ when s_0 is sent and $a > 0$ when s_1 is sent. If we let the state $x = a$, then the hypotheses can be written as

$$\begin{aligned} \mathcal{H}_0 &: \quad x < 0 \\ \mathcal{H}_1 &: \quad x > 0 \end{aligned}$$

with the state space $\mathcal{X} = \mathbb{R}\backslash 0$.

This is a composite binary hypothesis testing problem with composite null and composite alternative hypotheses.

Example 2

Suppose now we have a known signal $s \in \mathbb{R}^n$ and we want to detect its presence or absence. The signal arrives with unknown amplitude $a > 0$ and is corrupted by zero-mean AWGN with known variance σ^2.

As before, we have

$$Y \sim \mathcal{N}(as, \sigma^2 I)$$

from any state $a \in \mathcal{X}$. In this case, we can say $a = 0$ when the signal is absent and $a > 0$ when the signal is present. If we let the state $x = a$, then the hypotheses can be written as

$$\begin{aligned} \mathcal{H}_0 &: \quad x = 0 \\ \mathcal{H}_1 &: \quad x > 0 \end{aligned}$$

with the state space $\mathcal{X} = [0, \infty)$.

This is a composite binary hypothesis testing problem with a simple null and a composite alternative hypotheses.

Strategies for Finding Good Detectors

For these types of detection problems, we have three approaches:

1. N-P criterion with significance level α:
   - Our strategy: **reduce the composite HT problem to a simple one**.
   - Check for the existence of a UMP decision rule (check the critical region and/or monotone likelihood ratio) that maximizes $P_{D,x}$ for all $x \in \mathcal{X}_1$ subject to $P_{\mathsf{fp},x} \leq \alpha$ for all $x \in \mathcal{X}_0$.
   - If a UMP rule doesn't exist, we can try to find an LMP decision rule.
2. Bayesian decision rule:
   - Need a prior on all of the states.
   - Need a cost assignment (typically the UCA).
   - Compute commodity costs $g_i(y, \pi)$ and pick smallest.
3. GLRT decision rule:
   - Need to find MLE of the unknown parameter(s).
   - Plug in MLE estimates to make a simple hypothesis testing problem.
   - Suboptimal, but usually easier to compute than a Bayesian detector and usually gives good performance.