ECE531 Screencast 13.2: Constant False Alarm Rate (CFAR) Detectors

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Estimate and Plug (part 1 of 2)

Recall the problem of detecting the presence/absence of a known signal $s = a\mathbf{1}$ in AWGN. If the noise variance is known, we have the N-P rule

$$\rho(y) = \begin{cases} 1 & \bar{y} \geq \sqrt{\frac{\sigma^2}{N}} Q^{-1}(\alpha) \\ 0 & \bar{y} < \sqrt{\frac{\sigma^2}{N}} Q^{-1}(\alpha) \end{cases}$$

If σ^2 is not known, this detector is unrealizable since the decision threshold depends on σ^2. Idea: Can we estimate σ^2 and plug the estimate in?

The problem here is that we don't know if we are in $\mathcal{H}_0$ or $\mathcal{H}_1$. Under $\mathcal{H}_0$, the MLE of σ^2 is

$$\hat{\sigma_0^2}(y) = \frac{1}{N} y^\top y$$

but under $\mathcal{H}_1$ the MLE of σ^2 is

$$\hat{\sigma_1^2}(y) = \frac{1}{N} (y - \mathbf{1}a)^\top (y - \mathbf{1}a)$$

Estimate and Plug (part 2 of 2)

To determine the decision threshold satisfying the false positive probability constraint, we always assume we are in $\mathcal{H}_0$. So we should use the MLE $\hat{\sigma_0^2}(y)$. But if we use this estimator when we are actually in $\mathcal{H}_1$, then

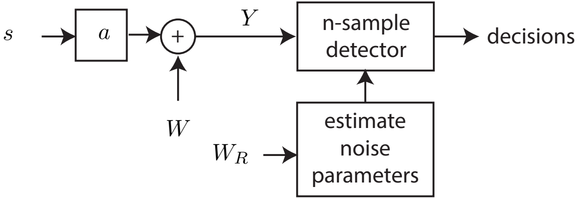
$$\begin{aligned} \mathrm{E}[\hat{\sigma_0^2}(Y)] &= \frac{1}{N}\mathrm{E}\left[Y^\top Y \,|\, \mathcal{H}_1\right] \\ &= \frac{1}{N}\left(a^2 1^\top 1 + \mathrm{E}[W^\top W]\right) \\ &= a^2 + \sigma^2 \end{aligned}$$

which is clearly biased since $a > 0$.

The bias when the signal is present causes the decision threshold to increase and, consequently, P_D to decrease. What we need are some "reference" noise samples (in which we know the signal is not present) to avoid this problem.

Constant False Alarm Rate Detection

The main idea of a CFAR detector is that we estimate the statistics of reference noise samples to normalize the decision statistic so that it no longer depends on the unknown characteristics of the noise.



The reference noise W_R is assumed to have the same distribution as W but to be independent of W.

CFAR detectors are sometimes used in radar problems where reference noise can be gathered from directions other than the look direction of the radar system.

CFAR Example (part 1 of 2)

Recall the problem of detecting the presence/absence of a known signal $s = a1$ in AWGN with unknown variance σ^2. Our decision statistic is $T(y) = \bar{y}$ which we know is Gaussian distributed with variance σ^2/N.

From the reference noise samples, we can estimate σ^2 as

$$\hat{\sigma^2}(w_R) = \frac{1}{N} w_R^\top w_R$$

Suppose we use this estimate to normalize our decision statistic to try to make its variance no longer depend on σ^2. We can write

$$T_{\rm norm}(Y, \hat{\sigma^2}(W_R)) = \frac{T(Y)}{\sqrt{\hat{\sigma^2}(W_R)/N}} = \frac{\frac{\sqrt{N}}{\sigma}\bar{Y}}{\sqrt{\frac{\hat{\sigma^2}(W_R)}{\sigma^2}}} = \frac{\frac{\sqrt{N}}{\sigma}\bar{Y}}{\sqrt{\frac{1}{N}\frac{W_R^\top W_R}{\sigma^2}}}$$

What is the distribution of $T_{\rm norm}$?

CFAR Example (part 2 of 2)

We have

$$T_{\rm norm}(Y, \hat{\sigma^2}(W_R)) = \frac{\frac{\sqrt{N}}{\sigma}\bar{Y}}{\sqrt{\frac{1}{N}\frac{W_R^\top W_R}{\sigma^2}}} = \frac{U}{\sqrt{\frac{1}{N}V}}$$

Under $\mathcal{H}_0$, we can write

$$U \sim \mathcal{N}(0,1)$$
$$V \sim \chi^2_N$$

and, since W_R is independent of Y, we can say that $U = f(Y)$ and $V = g(W_R)$ are independent. The decision statistic $T_{\rm norm}$ is Student's-t distributed with N degrees of freedom.

Notice that the distribution of $T_{\rm norm}$ no longer depends on σ^2 since neither the distribution of U nor the distribution of V depend on σ^2. Hence we can find a decision threshold v such that

$$P_{\sf fp} = {\rm Prob}(T_{\rm norm} > v) = \alpha$$

such that v does not depend on the unknown noise variance. This is called a CFAR detector. CFAR detectors are realizable but not necessarily optimal.