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6.11 Exponential families

Many of the distribution families we've encountered so far are special cases of general families called *exponential*. This is of more than merely academic interest: When we establish a result for the general family, it applies automatically to all subfamilies.

The **one-parameter exponential family** is defined by the following:

$$f(x;\,\theta) = B(\theta)h(x)\exp[Q(\theta)R(x)], \tag{1}$$

where "f" can be either the p.d.f. of a continuous distribution or the p.f. of a discrete distribution. Table 6-1 lists some one-parameter families by name with corresponding identifications of the functions B, Q, R, and h.

Table 6-1

Name	p.f. or p.d.f.	$B(\theta)$	$Q(\theta)$	$R(x)$	$h(x)$
Ber(θ)	$\theta^x(1-\theta)^{1-x},\ x = 0, 1$	$1-\theta$	$\log\frac{\theta}{1-\theta}$	x	1
Bin(n, θ)	$\binom{n}{x}\ \theta^x(1-\theta)^{n-x},\ x = 0, \ldots, n$	$(1-\theta)^n$	$\log\frac{\theta}{1-\theta}$	x	$\binom{n}{x}$
Geo(θ)	$\theta(1-\theta)^{x-1},\ x = 1, 2, \ldots$	$\frac{\theta}{1-\theta}$	$\log(1-\theta)$	x	1
Negbin(r, θ)	$\binom{x-1}{r-1}\theta^r(1-\theta)^{(x-r)},\ x = r, r+1, \ldots$	$\frac{\theta^r}{(1-\theta)^r}$	$\log(1-\theta)$	x	$\binom{x-1}{r-1}$
Poi(θ)	$\frac{\theta^x}{x!}e^{-\theta},\ x = 0, 1, \ldots$	$e^{-\theta}$	$\log\theta$	x	$1/x!$
Exp(θ)	$\theta e^{-\theta x},\ x > 0$	θ	$-\theta$	x	1
$\mathcal{N}(0,\,\theta)$	$\frac{1}{\sqrt{2\pi\theta}}e^{-x^2/2\theta}$	$(2\pi\theta)^{-1/2}$	$-(2\theta)^{-1}$	x^2	1
$\mathcal{N}(\theta,\,1)$	$\frac{1}{\sqrt{2\pi}}e^{-(x-\theta)^2/2}$	$(2\pi)^{-1/2}\,e^{-\theta^2/2}$	θ	x	$e^{-x^2/2}$
Gam(α,θ)	$\frac{\theta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\theta x},\ x > 0$ (α given)	θ^α	$-\theta$	x	$\frac{x^{\alpha-1}}{\Gamma(\alpha)}$