ECE531 Spring 2013 Quiz 10

Your Name: SOLUTION

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

Suppose you have a communication system with two signals given as

$$s_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$$
$$s_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

and these signals are observed as

$$Y = s_i + W$$

for $i \in \{0,1\}$, depending on which signal was transmitted, with $W \sim \mathcal{N}(0,\Sigma)$ and

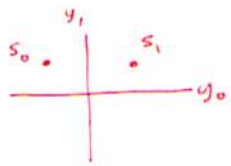
$$\Sigma = \begin{bmatrix}4 & 0\\0 & 1\end{bmatrix}.$$

Noise is uncorrelated but not i.i.d.

1. 50 points. Assuming equal prior probabilities, determine the minimum probability of error decision rule and the resulting error probability.

2. 50 points. Redesign the signals s_0 and s_1 so that the error probability is minimized subject to the constraint $\|s_0\|^2 = \|s_1\|^2 = 1$. Are s_0 and s_1 uniquely determined? Compute the resulting error probability.

1. Equal priors, UCA $\Rightarrow$ Minimum distance detector



clearly the decision depends only on y_0 and not y_1.

$$\delta^{BIT}(y) = \begin{cases}1 & y_0 > 0\\ 0/1 & y_0 = 0\\ 0 & y_0 < 0\end{cases}$$

$$P_e = \frac{1}{2}\,\text{Prob}(y_0 > 0 \mid s_0 \text{ sent}) + \frac{1}{2}\,\text{Prob}(y_0 < 0 \mid s_1 \text{ sent}) = Q\left(\frac{1/\sqrt{2}}{2}\right)$$
$$= Q\left(\frac{1}{2\sqrt{2}}\right)$$

2. We want s_0 and s_1 to be antipodal on the y_1 axis since the noise variance is minimal on y_1. We can use $s_0 = \begin{bmatrix}0\\1\end{bmatrix}$ and $s_1 = \begin{bmatrix}0\\-1\end{bmatrix}$

In this case, $P_e = \frac{1}{2}\,\text{Prob}(y_1 > 0 \mid s_1 \text{ sent}) + \frac{1}{2}\,\text{Prob}(y_1 < 0 \mid s_0 \text{ sent})$
$= Q(1)$

which is better than $Q\left(\frac{1}{2\sqrt{2}}\right)$ 1

These signals are uniquely determined, but can be flipped.