ECE531 Spring 2013 Quiz 8

Your Name: SOLUTION

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

A flu test was developed with the following characteristics:

- For patients that **do not have the flu**, the test falsely indicates the presence of the flu 10% of the time and correctly indicates the absence of the flu 90% of the time.
- For patients that **have the flu**, the test correctly indicates the presence of the flu 80% of the time and incorrectly indicates the absence of the flu 20% of the time.

Suppose a doctor orders two independent flu tests to be given to a patient. Given two outcomes of this flu test from an unknown test subject, we wish to design an optimal decision rule for deciding whether the test subject has the flu or not.

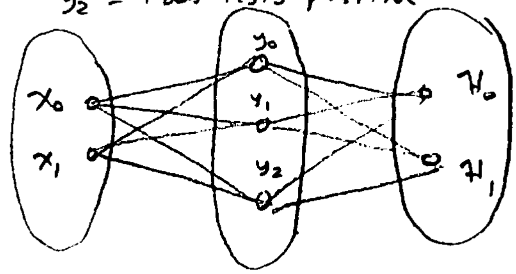
1. 25 points. Set up this hypothesis testing problem by explicitly defining the states, hypotheses, observations, and the conditional distributions on the observations for each state.
2. 25 points. Sketch the Pareto-optimal risk tradeoff surface as accurately as possible assuming the uniform cost assignment (UCA). Label all vertices. Hint: You do not need to find the whole set of achievable risk vectors here. The Pareto-optimal risk tradeoff surface can be found by only computing the conditional risks of "good" decision rules.
3. 25 points. Suppose you use the decision rule "decide the flu is present only if both tests indicate the presence of the flu". What is the false positive probability and the detection probability of this decision rule? Is this a Pareto-optimal decision rule?
4. 25 points. Find the Neyman-Pearson decision rule for $\mathcal{H}_0$:absence versus $\mathcal{H}_1$:presence that gives a false positive probability of $\alpha = 0.05$. Simplify your decision rule as much as possible. What is the probability of detection?

1. states x_0 = no flu
x_1 = has flu

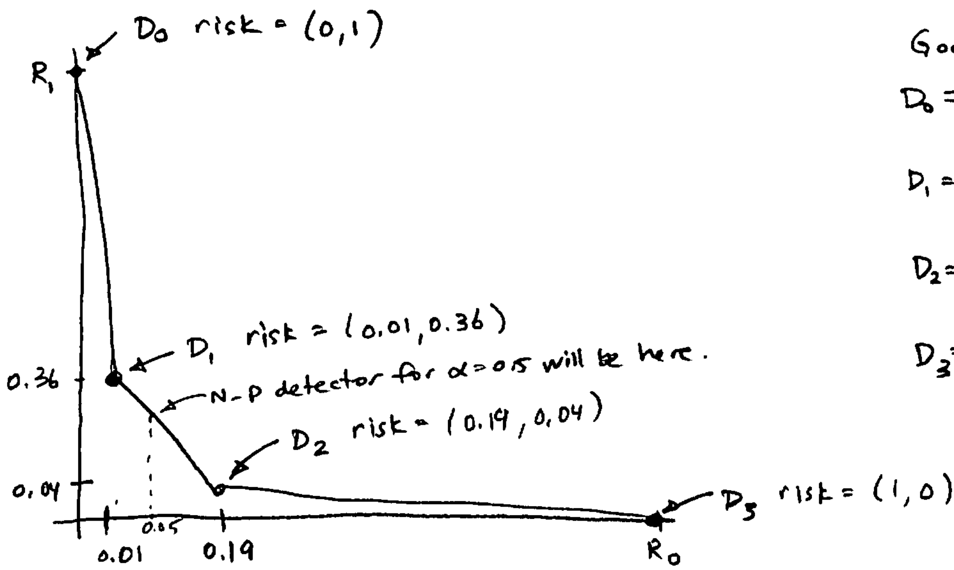
observations
y_0 = both tests negative
y_1 = one test positive
y_2 = two tests positive

Hypotheses
$\mathcal{H}_0 : x_0$
$\mathcal{H}_1 : x_1$

$$P = \begin{bmatrix} 0.81 & 0.04 \\ 0.18 & 0.32 \\ 0.01 & 0.64 \end{bmatrix}$$



2.



Good decision rules

$D_0 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ always $\mathcal{H}_0$

$D_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$D_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$D_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ always $\mathcal{H}_1$

3. This is D_1. The conditional risk of D_1 is $R_0 = 0.01$
$R_1 = 0.36$

$$\Rightarrow \boxed{\begin{aligned} P_{fp}(D_1) &= R_0 = 0.01 \\ P_D(D_1) &= 1 - R_1 = 0.64 \end{aligned}}$$

4. Now $\alpha = 0.05$. D_2 has too much false positive probability so we will need to randomize between D_1 and D_2.

$(1-\gamma)\,0.01 + \gamma \cdot 0.19 = 0.05$, solve for γ...

$0.18\gamma = 0.04 \Rightarrow \gamma = 0.2222$

$$\Rightarrow \boxed{\rho_{NP} = \begin{cases} 1 & y = y_2 \\ 0.2222 & y = y_1 \\ 0 & y = y_0 \end{cases}}$$

$$\boxed{\begin{aligned} P_D &= 0.64 + \gamma \cdot 0.32 \\ &= 0.7111 \end{aligned}}$$