The Effect of Channel State Information on Optimum Energy Allocation and Energy Efficiency of Cooperative Wireless Transmission Systems

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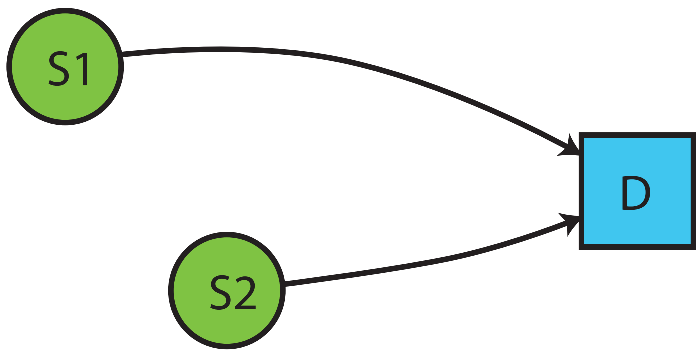
Scenario

- Two-source, one-destination communication system
- Sources cooperate via the amplify-and-forward protocol
- Both sources must satisfy outage probability constraints
- Fading channels



Problem Statement

- How should transmit energy be allocated to minimize total energy?
- How does CSIT affect energy allocation and overall energy efficiency?

Selected Related Papers

1. A. Host-Madsen *et al.*
   "Capacity bounds and power allocation for wireless relay channels ",
   IEEE Transaction on Information Theory 2005

2. J. Adeane *et al.*
   "Centralised and distributed power allocation algorithms in cooperative networks", *IEEE 6th workshop on Signal Processing Advances in Wireless Communications and Mobile Computing* 2005

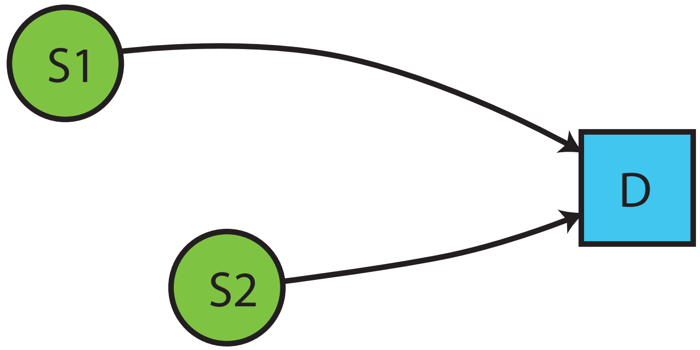
3. D. Gunduz and E. Erkip.
   "Outage minimization by opportunistic cooperation", *International Conference on Wireless Networks, Communications and Mobile Computing* 2005

Our Contribution

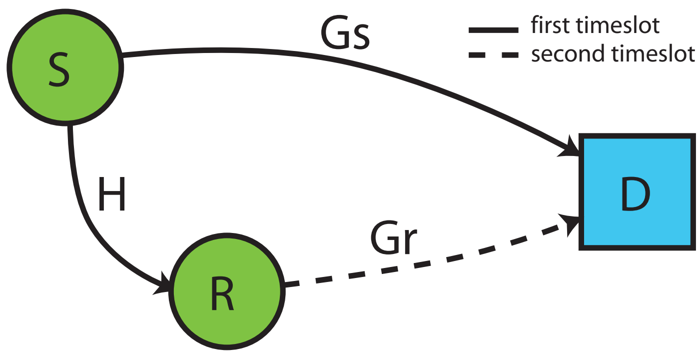
- Optimum energy allocation analysis for the *amplify-and-forward* protocol:
  - with partial CSIT
    (1) when to cooperate?
    (2) Optimum energy allocation strategy satisfying a fixed outage probability constraint
  - without CSIT
    (1) outage probability analysis and bounds
    (2) Optimum energy allocation strategy satisfying a fixed outage probability constraint
- Investigate the effect of CSIT on optimum energy allocation and energy efficiency

System Model

- Scenario



- Protocol: amplify-and-forward
- Two-source, one-destination cooperative transmission system model:



Destination Processing

Question: How should the destination combine the obervations?

Assumption: Destination has access to full CSI
$\Rightarrow$Maximal Ratio Combining (MRC) to maximize SNR

- With partial CSIT

$$\mathsf{SNR}_{\mathsf{partialcsit}} = G_s\mathcal{E}_s + \frac{H\mathcal{E}_s G_r \mathcal{E}_r}{1 + H\mathcal{E}_s + G_r\mathcal{E}_r}.$$

- Without CSIT

$$\mathsf{SNR}_{\mathsf{nocsit}} = G_s\mathcal{E}_s + \frac{H\mathcal{E}_s G_r \mathcal{E}_r}{1 + \mathsf{E}[H]\mathcal{E}_s + G_r\mathcal{E}_r}.$$

Part I: Energy Analysis for Partial CSIT

Assumption 1: Feedback channel provides perfect channel amplitude estimates to both sources

Assumption 2: Sources dynamically allocate their transmit energies based on the instantaneous channel state

Main Results

- When to cooperate?
- Optimum energy allocation strategy satisfying a fixed outage probability constraint

When to cooperate?

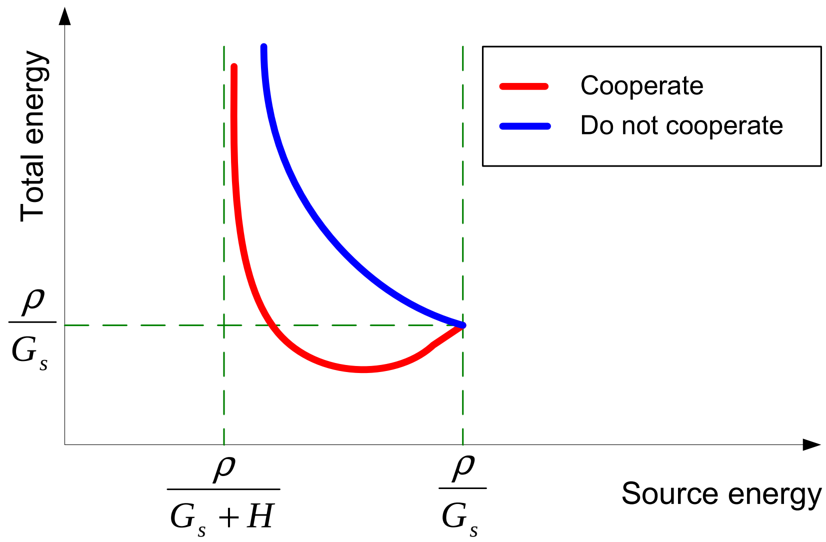
Notation: $\rho :=$ SNR target.

Notation: Energy required to satisfy $\text{SNR}_{\text{pcsit}} = \rho$ using direct transmission:

$$\mathcal{E} = \mathcal{E}_s + 0 = \frac{\rho}{G_s}$$

Proposition: There exists $\mathcal{E} < \frac{\rho}{G_s}$ if and only if

$$\frac{G_r}{G_s} > 1 + \frac{G_s}{H\rho}$$



What does it mean?

When the sources have partial CSIT:

- Cooperative transmission can achieve a transmit energy reduction *iff* the condition

$$\frac{G_r}{G_s} > 1 + \frac{G_s}{H\rho}$$

  is satisfied.

- At most, one source should cooperate in each interval.

- In some cases, total energy is minimized if neither source cooperates and both sources satisfy their SNR targets via direct transmission.

Optimum energy allocation with partial CSIT (I)

Temporary assumption: $outage\ probability = 0$

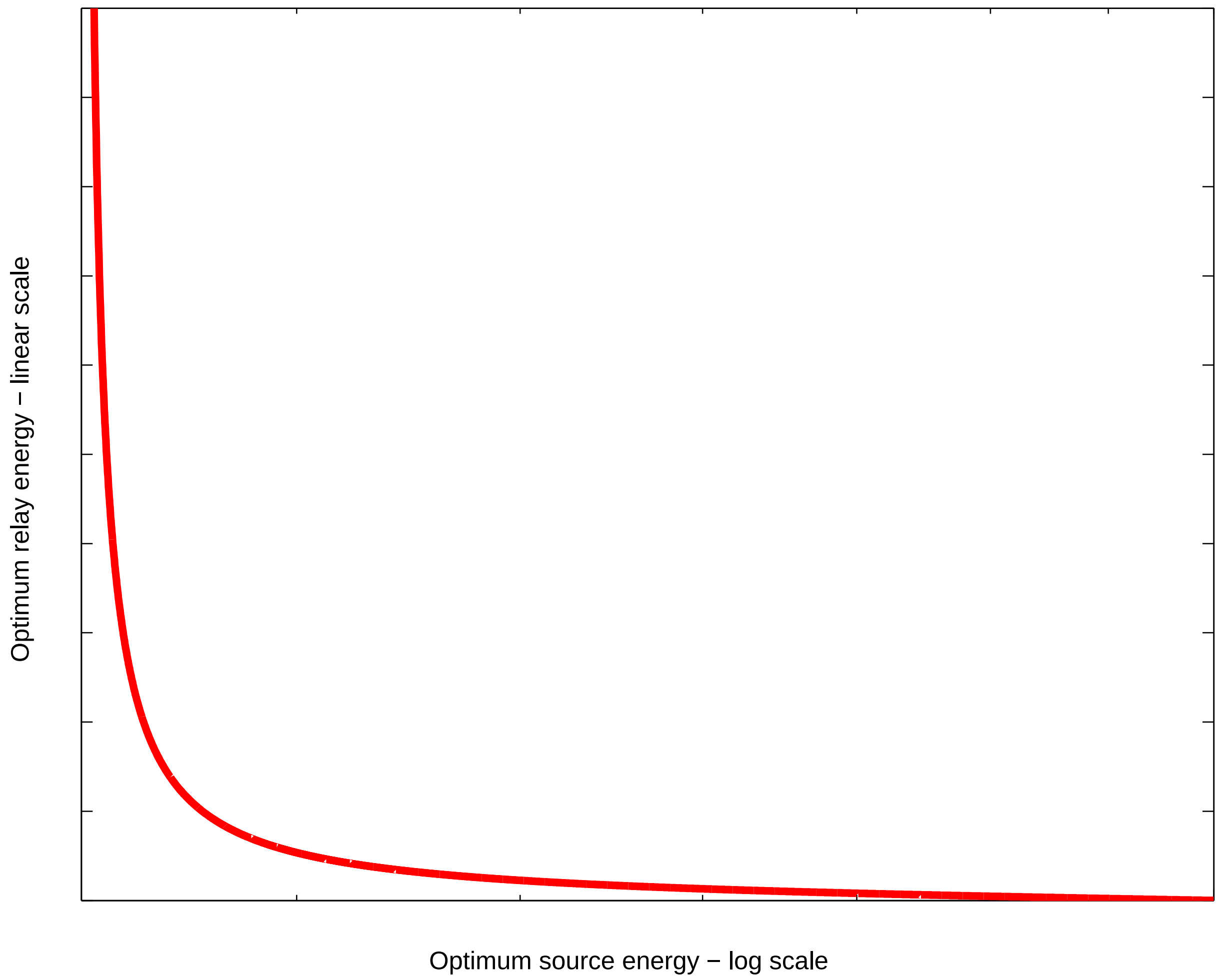
- Goal: Minimize $\mathcal{E} = \mathcal{E}_r + \mathcal{E}_s$ satisfying:
  - $\mathsf{SNR}_{\mathsf{pcsit}} = \rho$
  - $\mathcal{E}_s \in \left(\frac{\rho}{H+G_s}, \frac{\rho}{G_s}\right]$; $\mathcal{E}_r \in [0, \infty)$
- Optimum source energy allocation:

$$\mathcal{E}_s^* = \frac{\rho}{H + G_s} + \frac{(\rho H)^{1/2}(G_s + (1+\rho)H)^{1/2}}{(H + G_s)\left(H(G_r - G_s) + G_s G_r\right)^{1/2}}$$

- Optimum relay energy can be derived by the SNR contraint and optimum source energy.

Optimum energy allocation with partial CSIT (II)

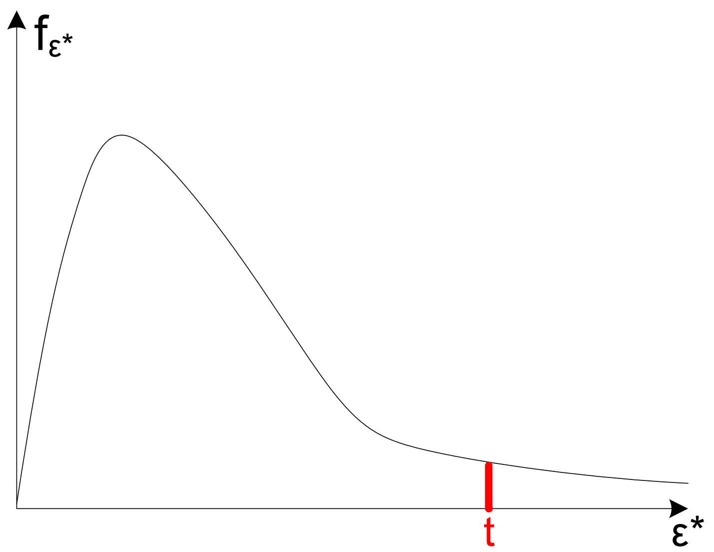
Optimum relay energy vs. optimum source energy satisfying $SNR_{\mathsf{pcsit}} = \rho$.



Optimum energy allocation with partial CSIT (III)

General case: $outage\ probability > 0$

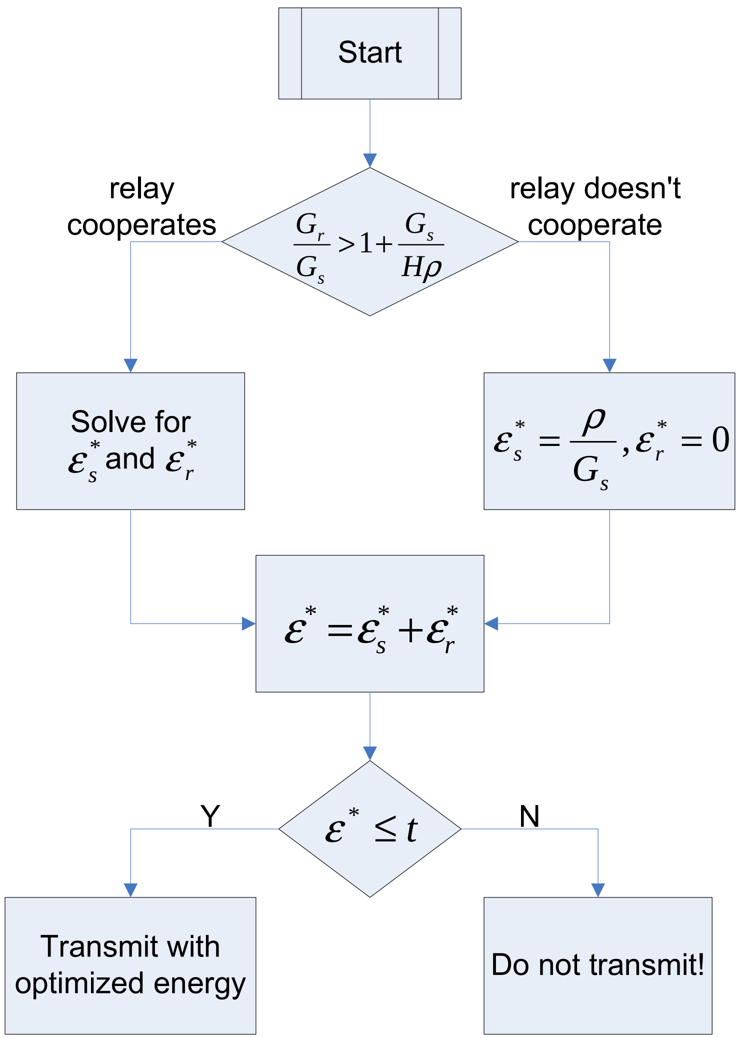
Preliminaries: Probability density function of minimum total energy $\mathcal{E}^* = \mathcal{E}_s^* + \mathcal{E}_r^*$ satisfying constraint $\text{SNR}_{\text{pcsit}} = \rho$ almost surely.



Threshold t selected such that $F_{\mathcal{E}^*}(t) = 1 - p$

$\Rightarrow$ Minimum total energy $\mathcal{E}^*$ exceeds threshold t with probability p

Optimum energy allocation strategy with Partial CSIT



Part II: Energy Analysis for no CSIT

Assumption 1: Sources do not have access to the instantaneous channel amplitudes

Assumption 2: Sources allocate fixed transmit energies based only on knowledge of the channel statistics

Main Results

- Outage probability analysis and bounds
- Optimal fixed energy allocation

No CSIT: Outage Probability Analysis & Bounds

Exact outage probability:

$$p = \int_{R(\rho)} f_{G_s,G_r,H}(\boldsymbol{x})\,d\boldsymbol{x}$$

Lower bound: Assume perfect source-relay channel

$$p \geq \frac{\mu_s\mathcal{E}_s\left(1-\exp\left(\frac{-\rho}{\mu_s\mathcal{E}_s}\right)\right)-\mu_r\mathcal{E}_r\left(1-\exp\left(\frac{-\rho}{\mu_r\mathcal{E}_r}\right)\right)}{\mu_s\mathcal{E}_s-\mu_r\mathcal{E}_r}.$$

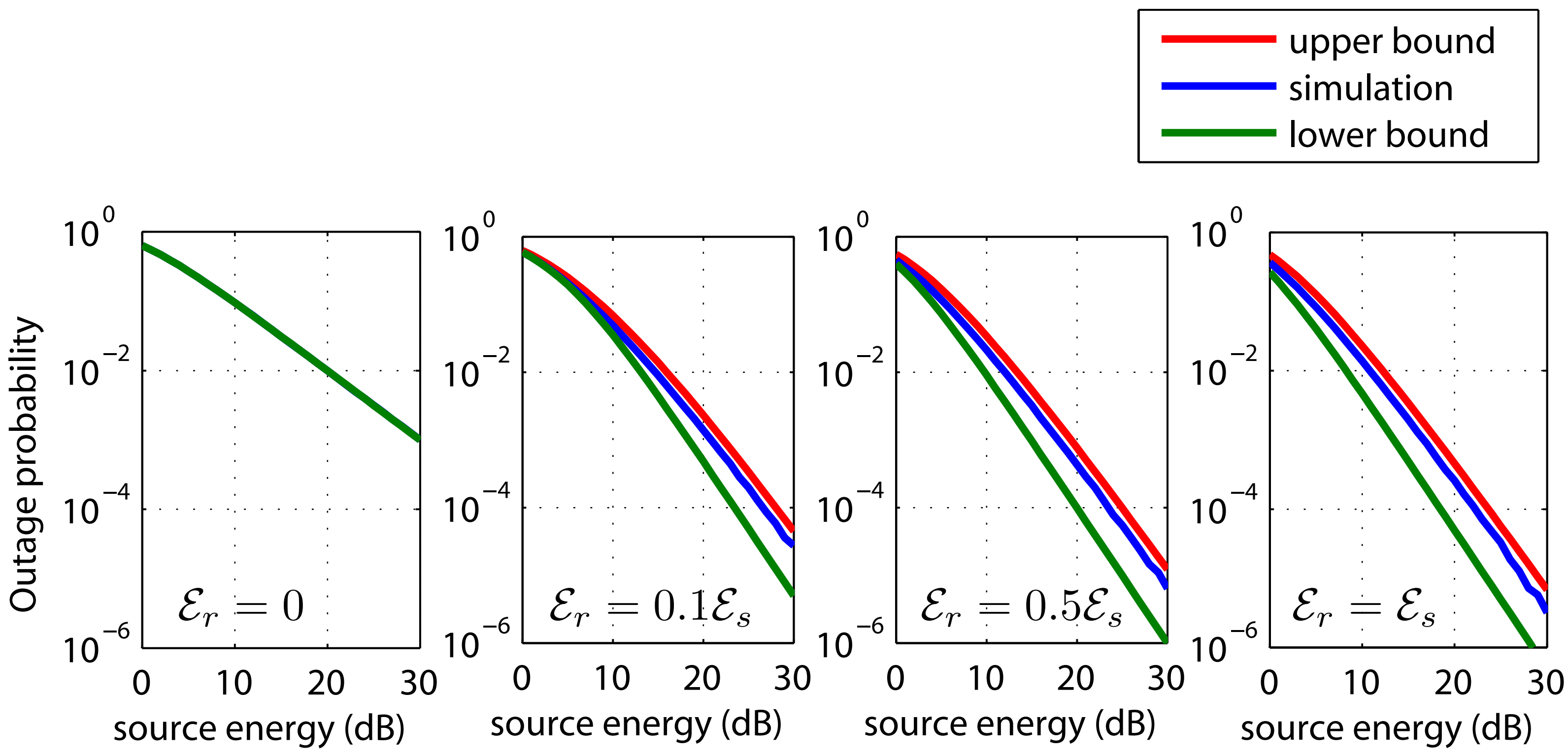
Upper bound: Overbound the integration region

$$p \leq \left(1-\exp\left(\frac{-\rho}{\mu_s\mathcal{E}_s}\right)\right)\cdot\left(1-\exp\left(\frac{-\rho}{\mu_H\mathcal{E}_s}\right)\psi K_1(\psi)\right)$$

where $K_1(\psi)$ is the modified Bessel function of the second kind and $\psi := 2\sqrt{\frac{\rho(1+\mu_H\mathcal{E}_s)}{\mathcal{E}_s\mathcal{E}_r\mu_r\mu_H}}$

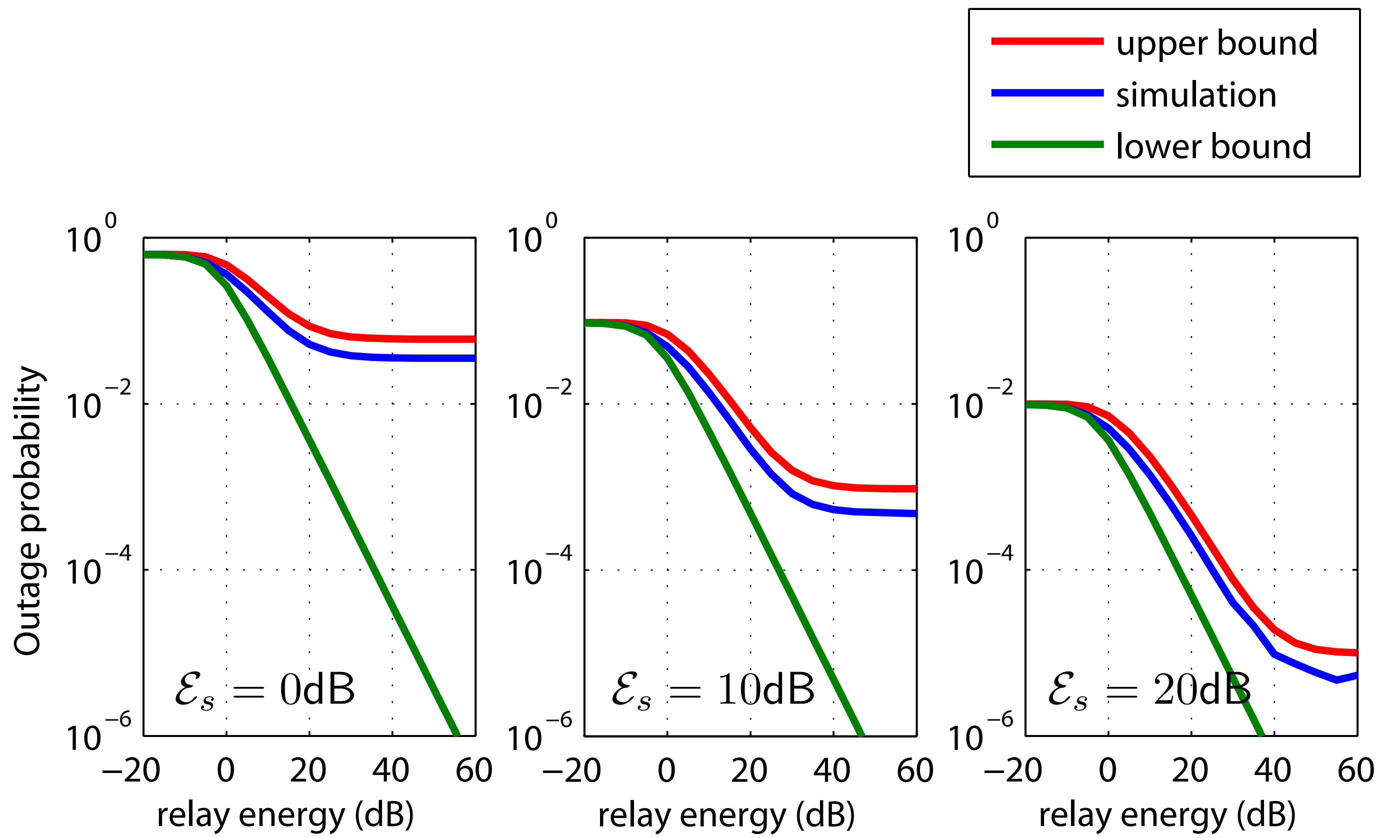
Evaluation of Outage Probability Bounds (I)

Variable source energy with proportional relay energy

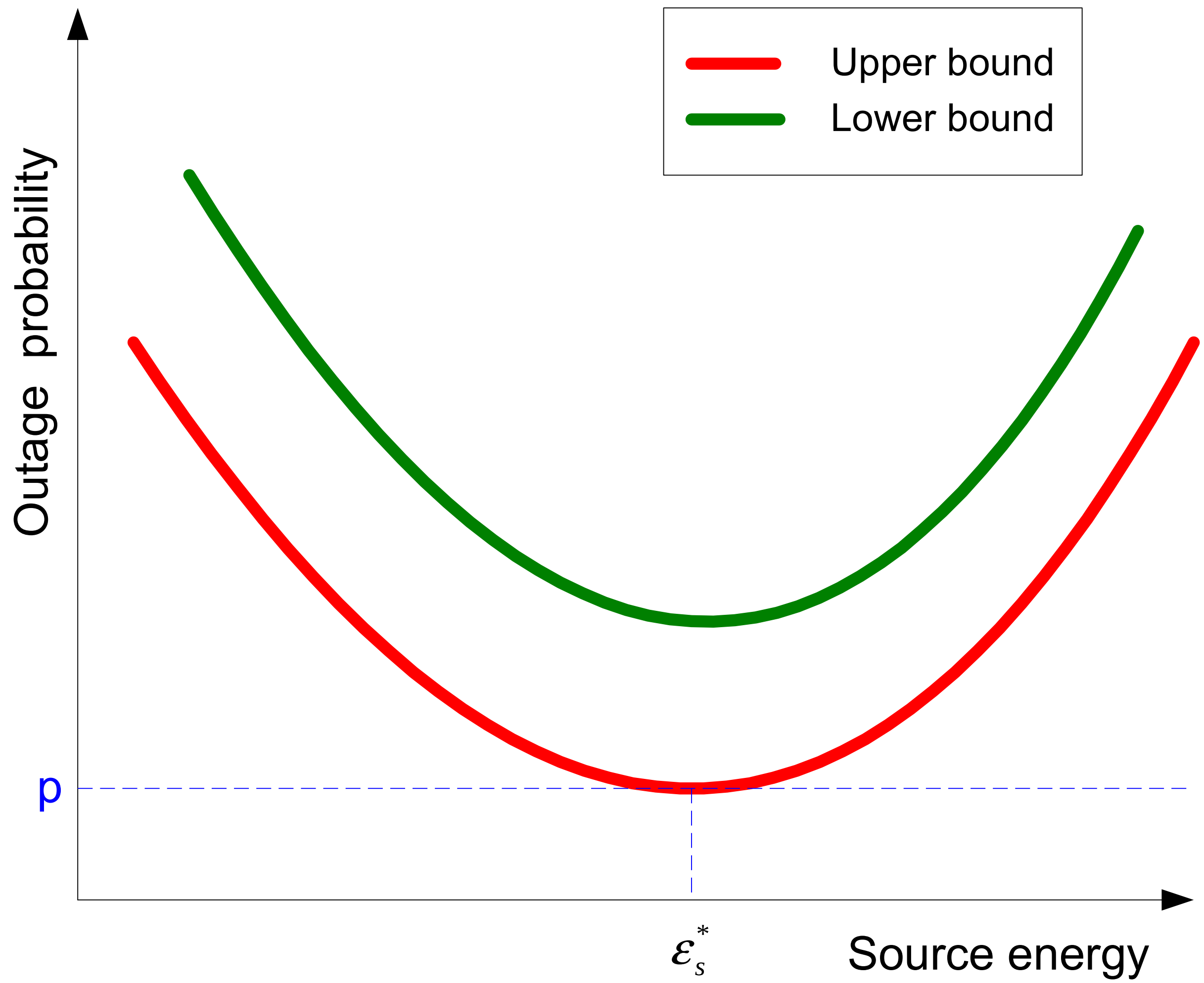


Evaluation of Outage Probability Bounds (II)

Fixed source energy with variable relay energy

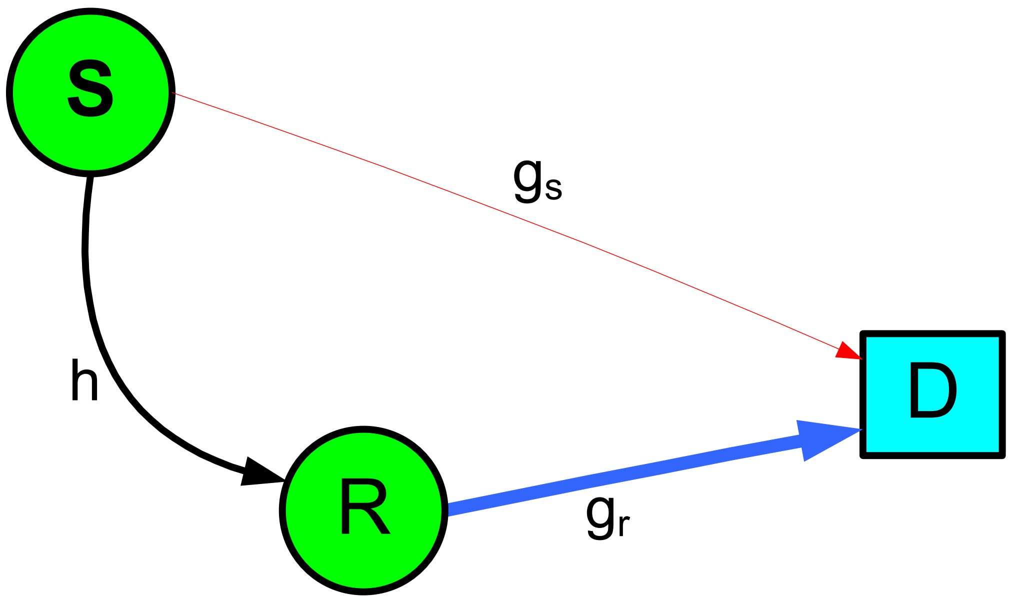


Numerical solution for optimum energy allocation



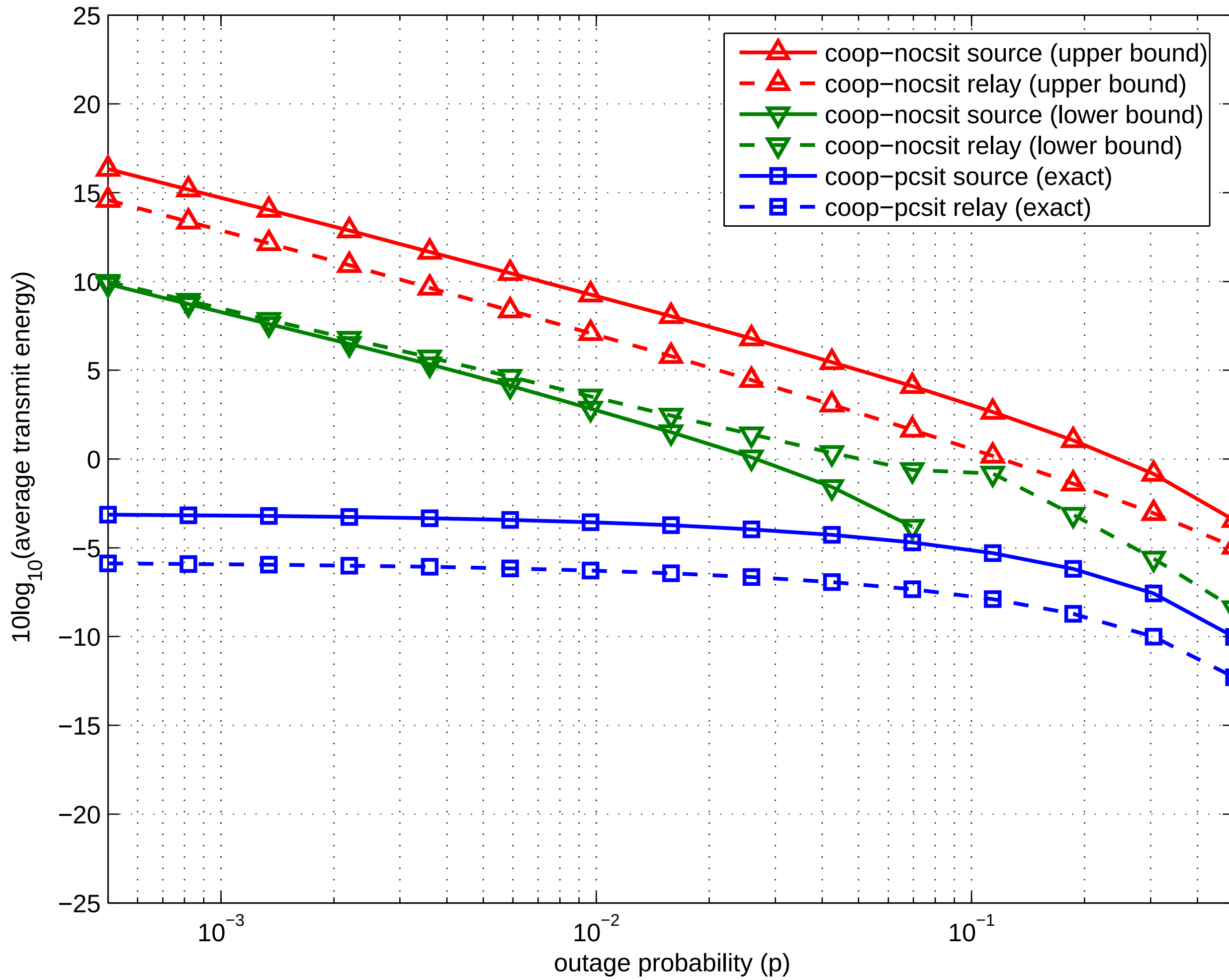
Numerical Results: relay advantaged (I)

System model:



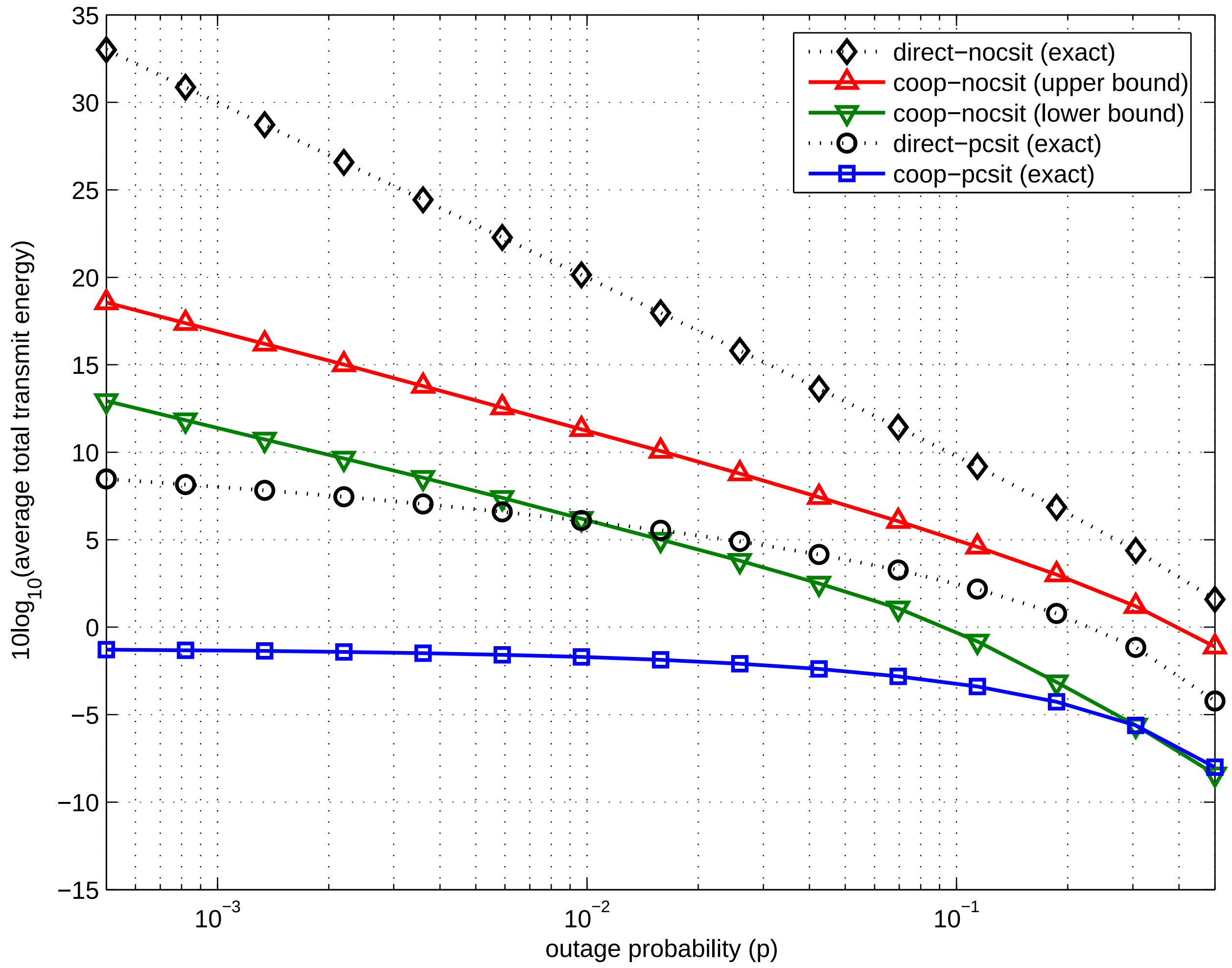
Numerical Results: relay advantaged (II)

Optimum energy allocation:



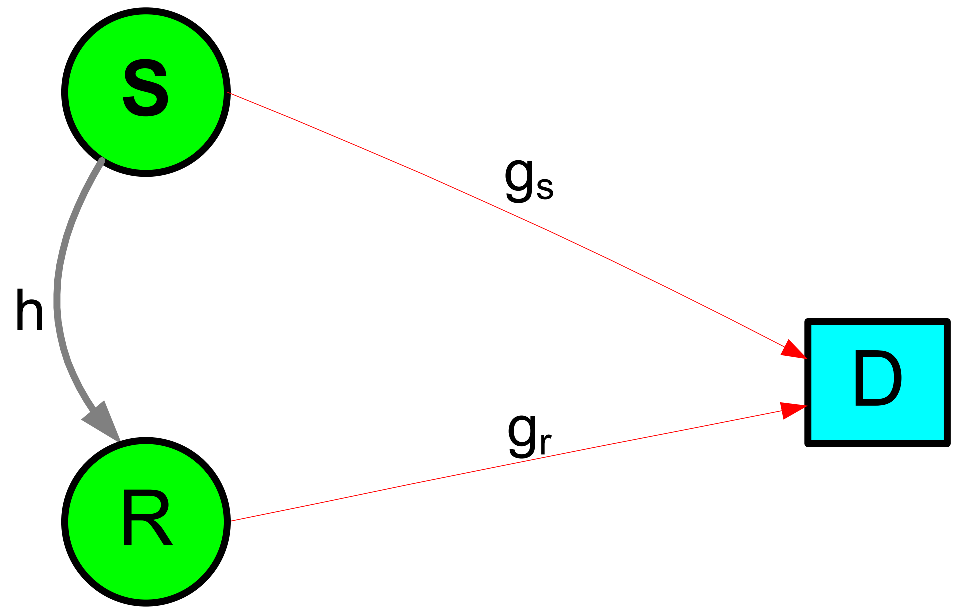
Numerical Results: relay advantaged (III)

Energy efficiency:



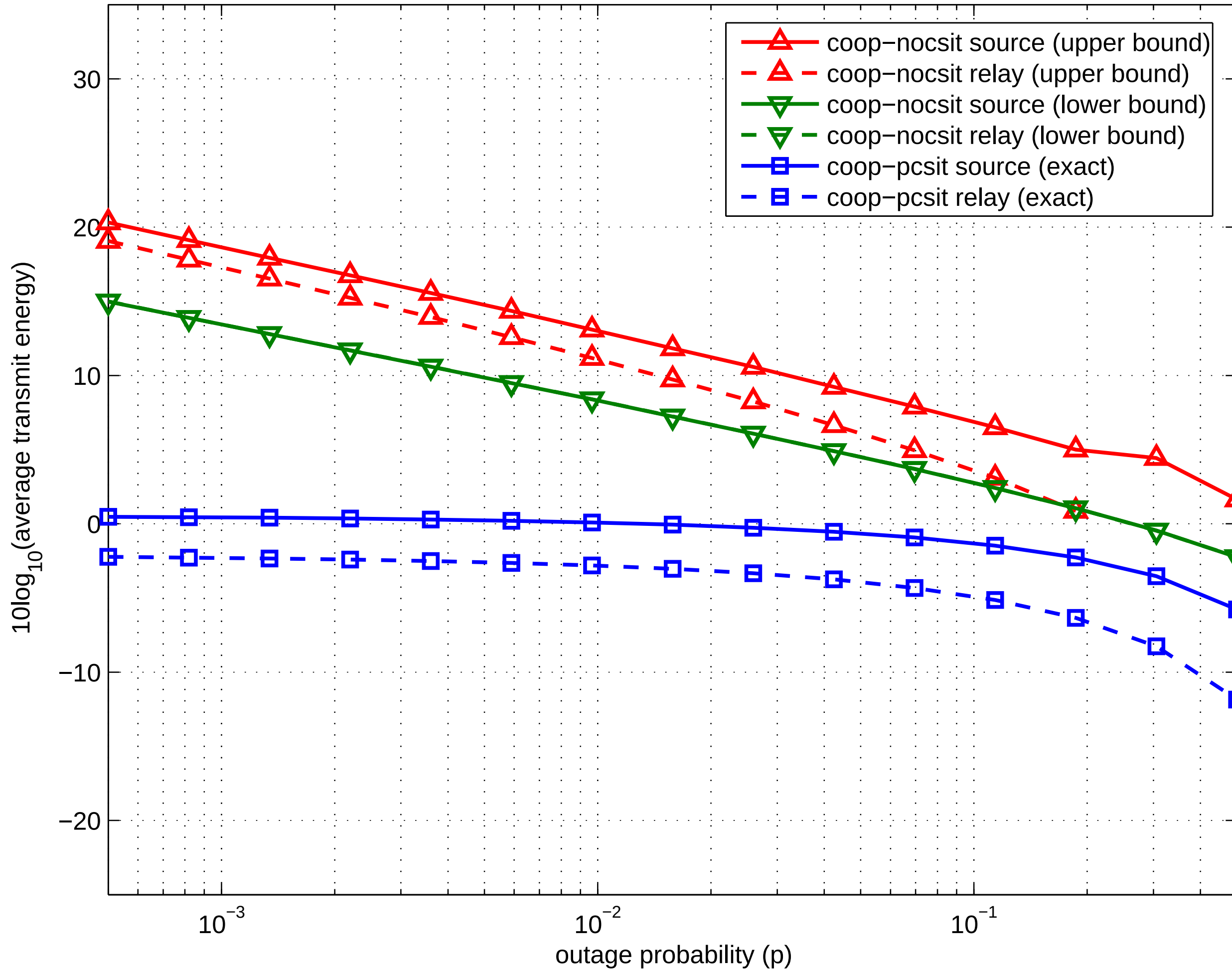
Numerical Results: relay symmetric (I)

System model:



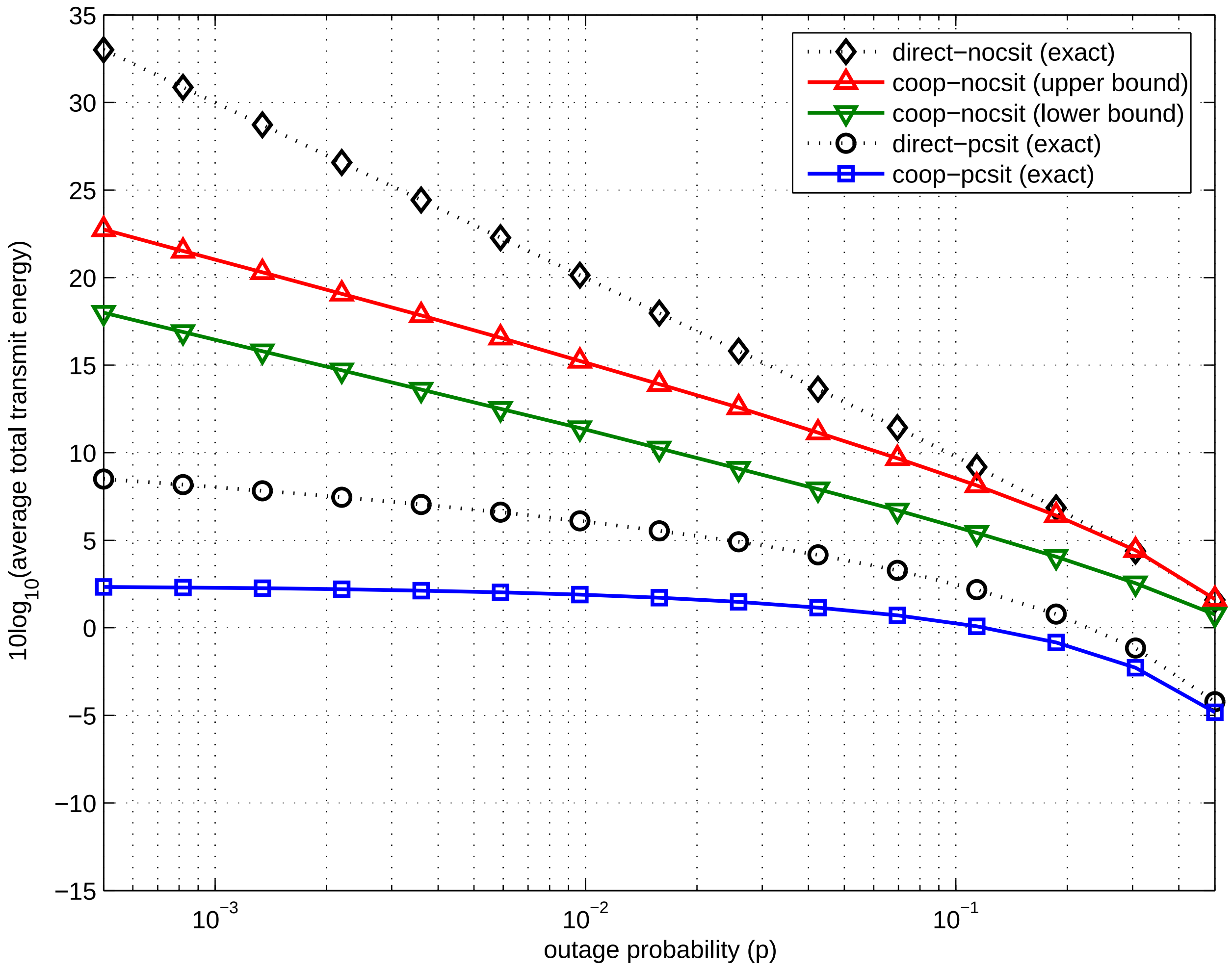
Numerical Results: relay symmetric (II)

Optimum energy allocation:



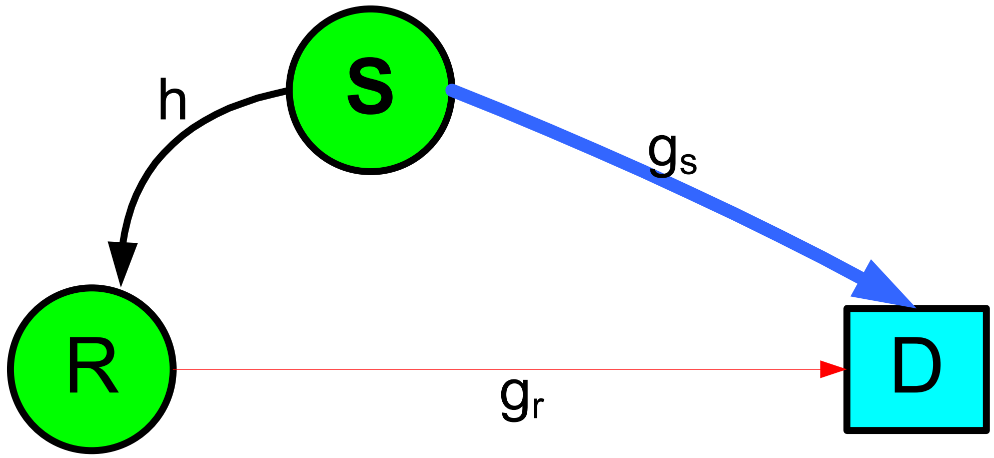
Numerical Results: relay symmetric (III)

Energy efficiency:



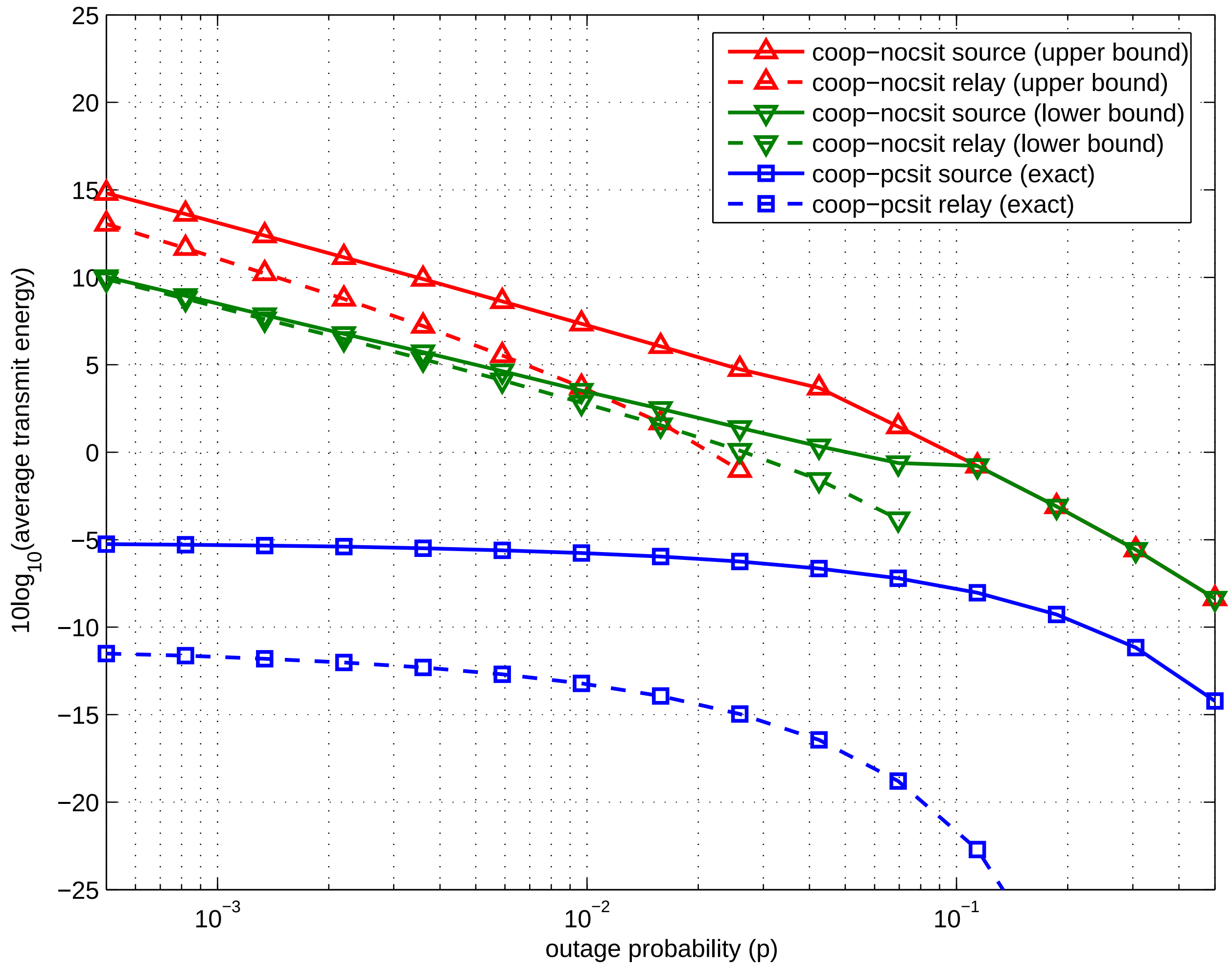
Numerical Results: relay disadvantaged (I)

System model:



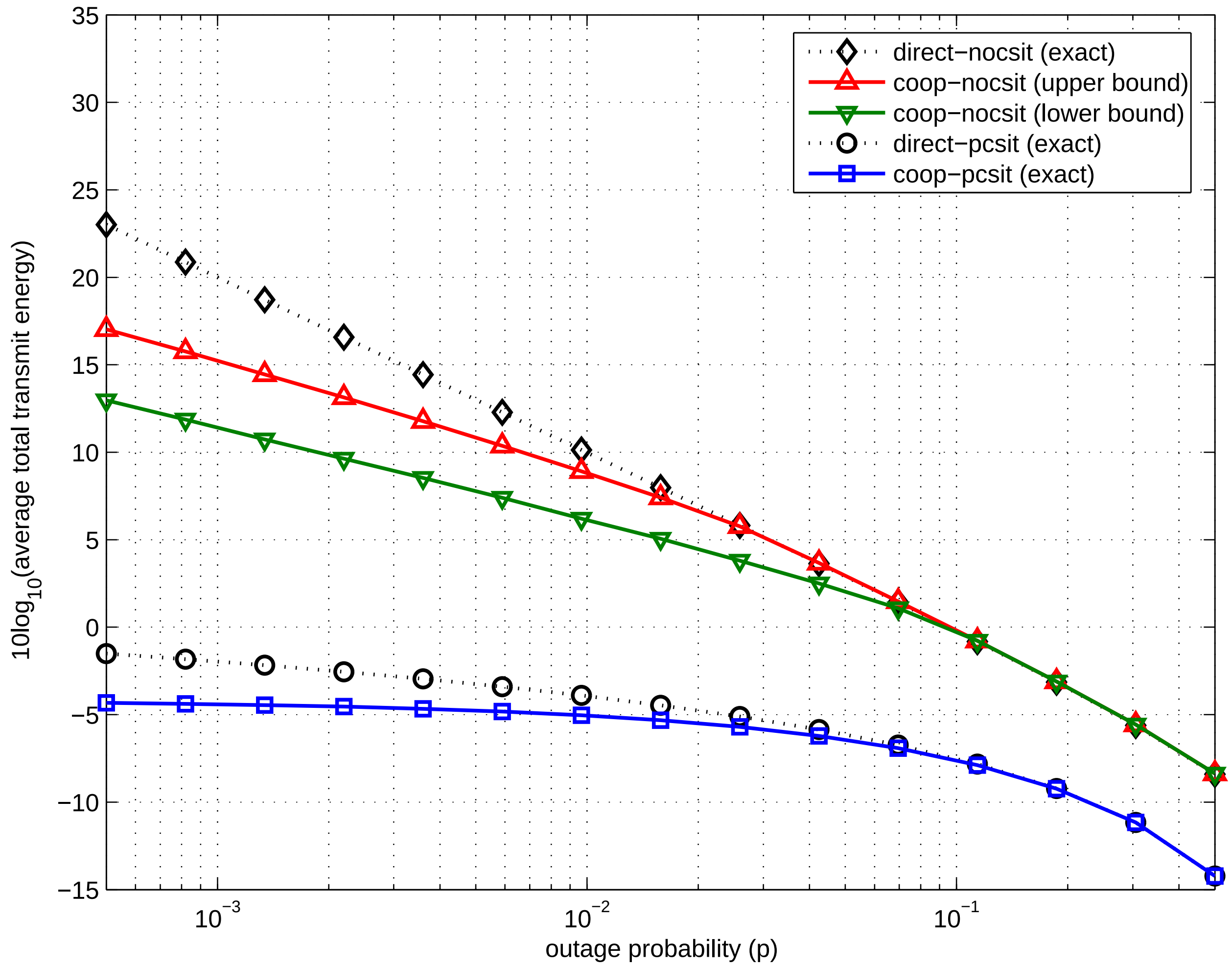
Numerical Results: relay disadvantaged (II)

Optimum energy allocation:



Numerical Results: relay disadvantaged (III)

Energy efficiency:



Conclusions

- When both sources have access to partial CSIT, cooperative transmission is more efficient than direct transmission *iff*

$$\frac{G_r}{G_s} > 1 + \frac{G_s}{H\rho}$$

- Partial CSIT facilitates **opportunistic** transmission

  ⇒ significantly improves the energy efficiency of both cooperative and direct transmission

- New outage probability bounds derived for the case without CSIT

  ⇒ used to determine optimum energy allocations

- Opportunistic direct transmission with partial CSIT is often more energy efficient than cooperative transmission without CSIT