Improved Multistage Parallel Interference Cancellation Using Limit Cycle Mitigation

D. Richard Brown III
Electrical and Comp. Engr. Dept.
Worcester Polytechnic Institute
Worcester, MA 01609
e-mail: drb@wpi.edu

Abstract —

This paper describes a new technique for improving the performance of multistage parallel interference cancellation (PIC) multiuser detection. The focus of the paper is on the hard-decision multistage PIC detector due to the fact that it possesses two desirable properties: (a) very low computational complexity in binary communication systems and (b) the optimum (joint maximum likelihood) bit estimates are a fixed point of the hard-decision PIC iteration. Unfortunately, the hard-decision PIC iteration is also known to sometimes demonstrate two modes of undesirable convergence behavior: convergence to non-optimum fixed points and limit cycles. This paper demonstrates that limit cycles tend to occur with much greater probability than convergence to non-optimum fixed points. To improve the performance of the hard-decision multistage PIC detector, we propose a reactive limit cycle mitigation algorithm. The results suggest that significant performance improvements may be possible in some cases with only modest increases in computational complexity.

I. Introduction

One promising technique for mitigating multiple access interference in CDMA communication systems is parallel interference cancellation (PIC) multiuser detection. PIC multiuser detection was first introduced for CDMA communication systems in [1] and [2] as the multistage detector and was shown to have low computational complexity, good performance, and close connections to the optimum joint maximum likelihood detector. PIC multiuser detection has also been the subject of extensive research more recently due to its applicability to 3G cellular standards [3].

Since Varanasi and Aazhang's pioneering work in the early 1990's, there has been an increasing interest in understanding the performance of hard-decision multistage PIC detection, e.g. [4]–[8]. The results of these papers suggest that the hard-decision multistage PIC detector may offer significant performance gains with respect to the matched filter detector and even outperform the linear MMSE and decorrelation detectors in some cases. An intuitive explanation for this good overall performance is that the optimum (joint maximum likelihood [9]) decisions are a fixed point of the hard-decision PIC iteration. Near-optimum performance is achieved when the hard-decision PIC iteration converges to the optimum fixed point with high probability.

Simulations and analysis have shown, however, that the bit error rate performance of the hard-decision multistage PIC detector is not equivalent to that of the optimum detector, even for an infinite number of PIC stages. This suboptimum performance is due to two factors, both demonstrated in this paper:

1. The hard-decision PIC iteration may possess one or more non-optimum fixed points in $\{-1, +1\}^K$.
2. The hard-decision PIC iteration may not converge to any fixed point but enter a limit cycle instead.

Evidence of limit cycles in the hard-decision PIC iteration can be seen in the simulation results of [6].

With the suboptimum performance of the hard-decision multistage PIC detector well documented, recent attention has turned towards methods to improve the performance of PIC detection. The research to date generally falls into one of two approaches. The first approach is to modify the initialization of the multistage PIC detector such that the probability of "good" convergence is improved. This may be achieved, for instance, by using a linear transformation on the bank of matched filter outputs prior to multistage PIC detection. Decorrelation of the matched filter outputs was explored in [2] and was shown in [9] to reduce the likelihood of limit cycle behavior with respect to using a conventional matched filter multistage PIC initialization. The tradeoff of this approach is that the linear transformation results in increased computational complexity which may be prohibitive in cases when the spreading codes are pseudo-random.

The second approach to improving the performance of the multistage hard-decision PIC detector is to modify the interference cancellation nonlinearity of the detector such that the effects of incorrect interference estimates are minimized. As an example of this approach, the linear PIC detector (first described in [10]) replaces the nonlinear $\mathrm{sgn}(\cdot)$ function of the hard-decision PIC detector with a linear mapping. The performance of the linear PIC detector has been extensively investigated, e.g. [11]–[15]. Other examples of this approach are partial interference cancellation [16], weighted linear/nonlinear cancellation [17], linear clipping and deadzone nonlinearities [18]–[19] and sigmoidal interference cancellation nonlinearities [20]–[22]. The tradeoff of these approaches is that the optimum fixed point of the original hard-decision PIC iteration is lost, in general, and that the implementation complexity of these improved PIC detectors tends to be higher than that of the hard-decision multistage PIC detector.

In this paper, we consider a new approach towards improving the performance of multistage PIC detection. Our results suggest that limit cycles are the largest cause of poor performance in the hard-decision multistage PIC detector and we propose an algorithm to mitigate limit cycle behavior. The proposed algorithm does not modify the initialization or interference cancellation nonlinearity of the hard-decision PIC detector but rather observes the output of the hard-decision PIC iteration and reactively corrects for poor convergence behavior. The advantages of this approach are that the correction only needs to be applied when needed, the desirable properties of the original PIC iteration are retained while the undesirable properties are mitigated, and the computational complexity remains low.

We assume a synchronous CDMA multiuser communication scenario with binary signaling, nonorthogonal transmissions, and an additive white Gaussian noise channel. The communication system model is identical to the basic synchronous CDMA model described in [9]. The number of users in the system is denoted by K and all multiuser detectors considered in this paper operate on the K-dimensional matched filter bank output given by the expression

$$\boldsymbol{y} = \boldsymbol{R}\boldsymbol{A}\boldsymbol{b} + \sigma \boldsymbol{n} \tag{1}$$

where $\boldsymbol{R} \in \mathbb{R}^{K\times K}$ is a symmetric matrix of normalized user signature crosscorrelations such that $\boldsymbol{R}_{kk} = 1$ for $m = 1,\dots,K$ and $|\boldsymbol{R}_{k\ell}| \le 1$ for all $k \neq \ell$, $\boldsymbol{A} \in \mathbb{R}^{K\times K}$ is a diagonal matrix of positive real amplitudes, $\boldsymbol{b} \in \mathbb{B}^{K\times 1}$ is the vector of i.i.d. equiprobable binary user symbols where $\mathbb{B} = \{\pm 1\}$, σ is the standard deviation of the additive channel noise, and $\boldsymbol{n} \in \mathbb{R}^{K\times 1}$ represents a matched filtered, unit variance AWGN process where $\mathrm{E}[\boldsymbol{n}] = \boldsymbol{0}$ and $\mathrm{E}[\boldsymbol{n}\boldsymbol{n}^\top] = \boldsymbol{R}$. The channel noise and user symbols are assumed to be independent.

II. Hard-Decision Multistage PIC Detection

Under the assumption that the receiver has perfect knowledge of the amplitudes and signature crosscorrelations of all the users in the system, the hard-decision multistage PIC detector's output after iteration $m+1$ is given as

$$\boldsymbol{d}(m+1) \quad = \quad \mathrm{sgn}\left(\boldsymbol{y} - (\boldsymbol{R}-\boldsymbol{I})\boldsymbol{A}\boldsymbol{d}(m)\right) \tag{2}$$

where $\boldsymbol{d}(m)$ is the K-vector of tentative binary decisions at the output of the m^{th} stage. Typically, the PIC iteration is initialized by setting $\boldsymbol{d}(0) = \mathrm{sgn}(\boldsymbol{y})$. The multistage PIC detector's final decisions may occur at some pre-set final stage M or, as is the case in this paper, the iteration may be monitored such that final decisions are generated upon convergence of the iteration.

Since $\boldsymbol{R}$, $\boldsymbol{I}$, and $\boldsymbol{A}$ are all symmetric matrices, the hard-decision multistage PIC detector can be shown to be a specific case of a symmetrically connected iterated map neural network [23] or a symmetrically connected binary Hopfield neural network operating in fully parallel update mode [24]. Both of these papers prove that neural networks belonging to this class can only demonstrate two types of convergence behavior: convergence to a fixed point or period-2 limit cycles. The following four-user example demonstrates both of these types of convergence behavior for the hard-decision multistage PIC detector.

<u>Example 1:</u>
Suppose that $\boldsymbol{A} = \boldsymbol{I}$,

$$\boldsymbol{R} \quad = \quad \frac{1}{4}\begin{bmatrix} 4 & 1 & 1 & -3 \\ 1 & 4 & -2 & -2 \\ 1 & -2 & 4 & 0 \\ -3 & -2 & 0 & 4 \end{bmatrix},$$

and that $\boldsymbol{y} = [-0.25, 1.75, -1.75, -0.75]^\top$. It can be shown that the hard-decision PIC iteration in (2) has two fixed points: $\boldsymbol{d} = [-1, 1, -1, -1]^\top$ (the optimum fixed point) and $\boldsymbol{d} = [1, 1, -1, 1]^\top$. The hard-decision PIC iteration also has a period-2 limit cycle between the points $\boldsymbol{d} = [-1, 1, -1, 1]^\top$ and $\boldsymbol{d} = [1, 1, -1, -1]^\top$.

The following lemmas provide additional intuition on the attractors of the hard-decision PIC iteration.

Lemma 1. *Given $\boldsymbol{d} \in \{-1,+1\}^K$, $\boldsymbol{d}$ is a fixed point of (2) iff $\boldsymbol{d}$ is a local maximum (neighborhood size of Hamming distance one) of the likelihood function $2\boldsymbol{b}^\top\boldsymbol{A}\boldsymbol{y} - \boldsymbol{b}^\top\boldsymbol{A}\boldsymbol{R}\boldsymbol{A}\boldsymbol{b}$.*

Lemma 2. *Suppose (2) enters a period-2 limit cycle such that $\boldsymbol{d}(m) = \boldsymbol{d}(m-2) \neq \boldsymbol{d}(m-1)$ for all $m \geq M$. The Hamming distance between $\boldsymbol{d}(m)$ and $\boldsymbol{d}(m-1)$ is at least 2 for all $m \geq M$.*

The proofs of these lemmas are straightforward and are omitted for space.

Despite the conceptual simplicity of the hard-decision PIC iteration in (2), little is actually known about the regions of attraction of the iteration's fixed points and period-2 limit cycles. For example, initializing the PIC iteration at a point Hamming distance one from a fixed point of the iteration does not guarantee convergence to that fixed point. Example 1 explicitly demonstrates this by showing a case where the optimum fixed point is only Hamming distance one from both limit cycle points.

A graphical analysis showing regions in $\boldsymbol{y}$-space leading to fixed point and period-2 limit cycle attractors was presented for the two-user case under the initializations $\boldsymbol{d}(0) = \mathrm{sgn}(\boldsymbol{y})$ and $\boldsymbol{d}(0) = \mathrm{sgn}(\boldsymbol{R}^{-1}\boldsymbol{y})$ in [9]. This analysis makes it possible to predict the behavior of the hard-decision multistage PIC detector before the application of any iterations and, if necessary, *proactively* modify the initialization and/or the "interconnection matrix" $(\boldsymbol{R}-\boldsymbol{I})\boldsymbol{A}$ to achieve desirable convergence behavior. Such an analysis appears to be difficult for the general $K \geq 2$ case, however. This realization, combined with the fact that each hard-decision PIC iteration has low computational complexity, suggests that *reactive* techniques for improving the convergence behavior of the hard-decision multistage PIC detector may be more appropriate. Moreover, since period-2 limit cycles are (a) the primary cause of poor performance (see Section IV) and (b) simple to identify at the output of the hard-decision PIC iteration, reactive methods for limit cycle mitigation should provide significant performance gains with minimal additional complexity. With this motivation, the following section develops a reactive limit cycle mitigation algorithm that corrects for period-2 limit cycle behavior without affecting other modes of convergence.

III. Mitigation of Limit Cycles

In this section, we develop a reactive technique for limit cycle mitigation that, unlike the stabilization techniques proposed in [23], does not modify the original hard-decision PIC iteration or its initialization. The limit cycle mitigation algorithm described below retains the desirable properties of the hard-decision PIC iteration while correcting only for undesirable limit cycle behavior.

The essential idea behind the limit cycle mitigation algorithm is that it is a simple task for a receiver, in the process of performing the hard-decision PIC iteration, to identify the onset of a period-2 limit cycle or convergence to a fixed point. On the other hand, it will often be difficult for a receiver to determine if convergence was to the optimum fixed point or a non-optimum fixed point without significant computational expense. These facts, combined with the evidence in Section IV suggesting that limit cycle behavior tends to occur with much higher probability than convergence to non-optimum fixed points, lead to the following limit cycle mitigation algorithm.

Limit Cycle Mitigation Algorithm

1. In each bit interval, hard-decision multistage PIC detection is applied until the iteration converges to a fixed point or a period-2 limit cycle begins.
2. If the multistage PIC detector converges to a fixed point, this fixed point is used as the output (bit estimates) of the detector. No special action is taken.
3. If the multistage PIC detector enters a period-2 limit cycle, the following limit cycle correction steps are taken.
   (a) Of the K tentative decisions at the output of the PIC iteration, let $2 \le \kappa \le K$ denote the Hamming distance between the states in the limit cycle.
   (b) Partition the tentative decision vector $\boldsymbol{d}$ into a limit cycle part and a fixed part. Since the indexing of the users is arbitrary, we can reorder the users for conceptual clarity such that the limit cycle part is the first κ elements of $\boldsymbol{d}$ and the fixed part is the last $K-\kappa$ elements of $\boldsymbol{d}$. Denote $\boldsymbol{d} = [\boldsymbol{d}_{LC}^\top \ \boldsymbol{d}_{FIXED}^\top]^\top$ and set $\boldsymbol{d}_{FIXED}$ equal to the fixed part of the tentative decision vector.
   (c) Conditioning on the fixed bits $\boldsymbol{d}_{FIXED}$, perform a conditional joint maximum likelihood (CJML) optimization on the limit cycle bits $\boldsymbol{d}_{LC}$, i.e.,

$$\hat{\boldsymbol{d}}_{LC-CJML} = \arg \max_{\boldsymbol{d}_{LC} \in \{-1,+1\}^\kappa} 2\boldsymbol{d}^\top \boldsymbol{A}\boldsymbol{y} - \boldsymbol{d}^\top \boldsymbol{A}\boldsymbol{R}\boldsymbol{A}\boldsymbol{d}. \quad (3)$$

   (d) Form the output (bit estimates) of the detector as

$$\hat{\boldsymbol{b}} = \begin{bmatrix} \hat{\boldsymbol{d}}_{LC-CJML} \\ \boldsymbol{d}_{FIXED} \end{bmatrix}.$$

We reiterate that the algorithm posed above does not attempt to rectify convergence to non-optimum fixed points but only corrects for limit cycle behavior due to their relative frequency and the ease at which they can be identified. Intuitively, the limit cycle mitigation algorithm assumes that the fixed bits in a period-2 limit cycle have a high probability of being correct and that one or more of the bits toggling in the limit cycle have a low probability of being correct. The algorithm exploits this fact to perform a *conditional* joint maximum likelihood optimization only on the toggling bits, typically at a much lower computational cost than a full, unconditional maximum likelihood search.

We also note that the limit cycle mitigation algorithm described above is similar in spirit to the group detector proposed in [25]. In this context, the group size corresponding to the original hard-decision multistage PIC detector is one. If a limit cycle occurs, then the users are partitioned into two groups (the users with toggling bits and the users with fixed bits) of size κ and $K-\kappa$ and a final "group detection" iteration is performed on the κ users with toggling bits. Unlike [25], however, the members, and consequently the sizes, of the groups in the limit cycle mitigation algorithm proposed above are dynamically determined by the properties of the limit cycle.

The following section presents numerical examples that demonstrate the effectiveness of the limit cycle mitigation algorithm in several operating scenarios.

IV. Numerical Results

In this section, we present new simulation results on the convergence behavior of the original hard-decision multistage PIC detector and on the gain in performance obtained through the limit cycle mitigation algorithm developed in Section III. All of the simulations in this section assume that the users are received with equal power (i.e., $\boldsymbol{A} = a\boldsymbol{I}$), that the spreading codes are random, independent, and binary valued with spreading gain $N = 16$, and that the hard-decision PIC iteration is initialized such that $\boldsymbol{d}(0) = \mathrm{sgn}(\boldsymbol{y})$. The output of the hard-decision PIC detector is compared after each stage to the output of the prior stage to see if the iteration has converged to a fixed point. If not, the output is compared to the output of the stage twice prior to determine if the iteration has entered a period-2 limit cycle. If either of these results occurs, the iteration is terminated and the bit estimates are set equal to the output of the final stage.

A Convergence Behavior of Hard-Decision PIC

Figure 1 plots the relative probability of each mode of convergence of the hard-decision multistage PIC detector. These results show that (2) often converges to the optimum fixed point, especially for smaller values of K, and that the probability of convergence to a non-optimum fixed point tends to be relatively small the cases shown. These results also demonstrate that period-2 limit cycles tend to occur with much greater probability than convergence to non-optimum fixed points over the entire range of cases considered.

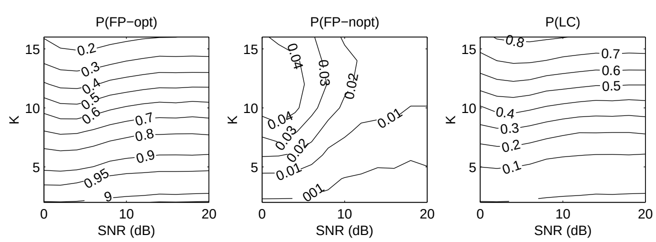

Fig. 1: Probability of the modes of convergence for hard-decision multistage PIC detection. Notation: "FP-opt": optimum fixed point convergence, "FP-nopt": non-optimum fixed point convergence, "LC": period-2 limit cycle.

Denoting M as the number of PIC iterations required to achieve fixed point convergence or enter a period-2 limit cycle, we note that M is a discrete valued random variable. At least one PIC iteration is required to detect a fixed point and at least two PIC iterations are required to detect a period-2 limit cycle. Figure 2 plots the probability mass function (pmf) of M and the probability that the PIC iteration converges to a fixed point as a function of M. These results suggest that the number of required PIC iterations tends to increase with K and corroborate the results in Figure 1 by showing that the probability of overall fixed point convergence tends to decrease significantly as K increases. The results shown here are also used in the complexity analysis of Section V.

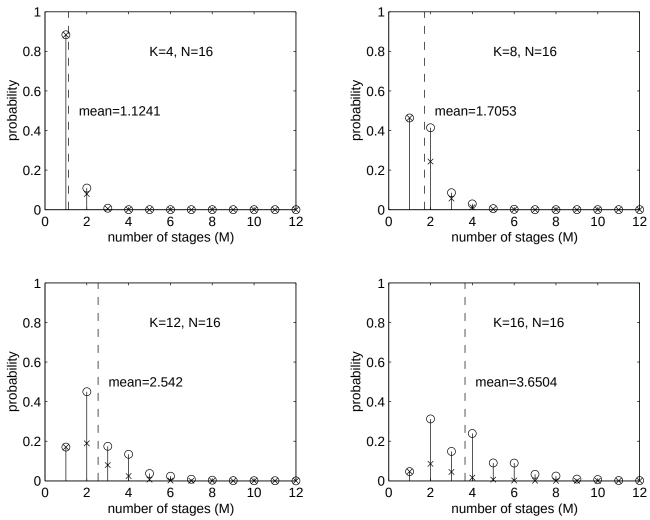

Fig. 2: Probability mass function of M denoted by '$\circ$'. Overall (optimum and non-optimum) fixed point convergence probability as a function of M denoted by '$\times$'.

Figure 3 plots the pmf of κ, the Hamming distance between the last two states at the output of the hard-decision multistage PIC detector upon termination. The event $\kappa = 0$ is equivalent to the event that (2) converges to a fixed point (optimum or non-optimum). These results are consistent with the results of Figures 1–2 and show that the overall fixed point convergence probability decreases significantly as K increases. Moreover, the average number of toggling bits in a period-2 limit cycle also tends to increase with K, especially as K approaches N. These results also demonstrate Lemma 2 by showing that no period-2 limit cycles of the hard-decision PIC iteration have Hamming distance one.

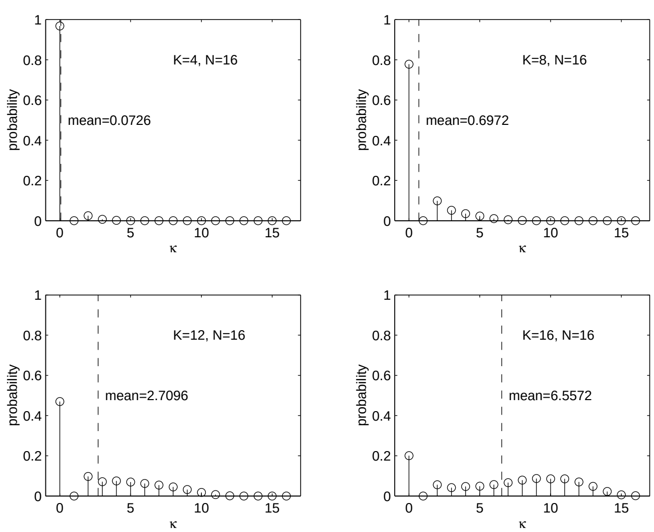

Fig. 3: Probability mass function of κ.

B Performance of PIC with Limit Cycle Mitigation

Figure 4 shows the probability that the limit cycle mitigation algorithm developed in Section III is able to convert a period-2 limit cycle to the optimum fixed point. Although these results suggest that the limit cycle mitigation algorithm converts a large majority of the period-2 limit cycles to the optimum fixed point, these results also show that optimum fixed point conversion is not guaranteed, in general. An explanation for this is that successful limit cycle conversion requires the fixed bits in the conditional joint maximum likelihood optimization of (3) to be optimum. This requirement is satisfied in most cases but clearly not with probability 1.

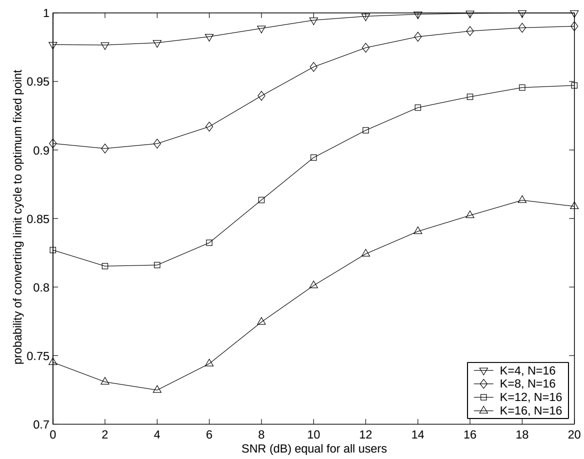

Fig. 4: Probability that a period-2 limit cycle is converted to optimum (joint maximum likelihood) decisions via the limit cycle mitigation algorithm.

Figure 5 demonstrates the gain in bit error rate performance obtained through limit cycle mitigation with respect to the original hard-decision multistage PIC detector, the matched filter detector, and the optimum (joint maximum likelihood) detector. These results show that the limit cycle mitigation algorithm may indeed significantly improve the bit error rate performance of the original hard-decision multistage PIC detector, especially when the number of users is small and the SNR is high.

V. Complexity Analysis

In this section, we evaluate and compare the computational complexity of the hard-decision multistage PIC detector with limit cycle mitigation to the original multistage PIC detector and the optimum (joint maximum likelihood) multiuser detector. The results in this section provide insight into the tradeoff between the improved performance demonstrated in the prior section and the increased implementation complexity of the limit cycle mitigation algorithm.

For the purposes of complexity comparison, we assume that the following values are precomputed and available to the multiuser detectors without any computational cost:

$$\begin{aligned} \boldsymbol{G} &= (\boldsymbol{R} - \boldsymbol{I})\boldsymbol{A}, \\ \boldsymbol{H} &= \boldsymbol{A}\boldsymbol{R}\boldsymbol{A}, \text{ and} \\ \boldsymbol{z} &= 2\boldsymbol{A}\boldsymbol{y}. \end{aligned}$$

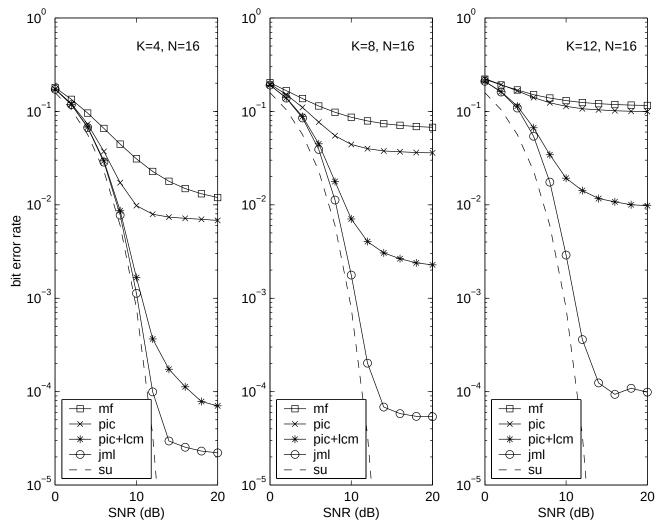

Fig. 5: Bit error rate performance with equipower users and random spreading codes. Notation: "mf": conventional matched filter, "pic": original hard-decision multistage PIC, "pic+lcm": original hard-decision multistage PIC with limit cycle mitigation, "jml": joint maximum likelihood (optimum) detection, "su": single user bound.

Also to facilitate the comparison, we define a *complexity unit* as one binary multiplication[1] and one signed addition.

A Multistage PIC Detection

The computational complexity of each stage of interference cancellation for the original multistage PIC detector can be determined by inspection of (2). Computation of $\boldsymbol{G}\boldsymbol{d}(m)$ requires K^2 binary multiplications and $K(K-1)$ signed additions. Subtracting $\boldsymbol{G}\boldsymbol{d}(m)$ from $\boldsymbol{y}$ also requires K signed additions, resulting in a total of K^2 signed additions. We assume that the $\operatorname{sgn}(\cdot)$ operation in (2) and the binary comparisons required to determine if the iteration has converged require no additional computational complexity.

Since the number of PIC iterations required before termination is random (see Figure 2), the complexity of the hard-decision multistage PIC detector is also random. The prior analysis implies that the average complexity of the hard-decision multistage PIC detector is equal to $\mathrm{E}[M]K^2$ complexity units.

B Joint Maximum Likelihood Detection

Under the previously established notation, the optimum joint maximum likelihood detector can be written as a combinatorial optimization of the likelihood function

$$\hat{\boldsymbol{b}} = \arg \max_{\boldsymbol{d}\in\{-1,+1\}^K} 2\boldsymbol{d}^\top \boldsymbol{z} - \boldsymbol{d}^\top \boldsymbol{H}\boldsymbol{d}. \qquad (4)$$

Following a similar method of analysis as before with the additional assumption that each real valued comparison involved in finding the maximum of (4) is computationally equivalent to one signed addition, we find that the joint maximum likelihood detector can be implemented with $2^K(K^2+K)$ binary multiplications and $2^K(K^2+K)-1$ signed additions. For purposes of comparison, we can apply a slight approximation to these results to conclude that the (deterministic) complexity of the joint maximum likelihood detector is $2^K(K^2+K)$ complexity units.

[1] By "binary multiplication", we mean the multiplication of a real valued number by ± 1.

C Multistage PIC Detection with Limit Cycle Mitigation

Since the limit cycle mitigation algorithm is executed only when a limit cycle terminates the PIC iteration, the prior results imply that the average computational complexity of the multistage PIC detector with limit cycle mitigation may be expressed as $\mathrm{E}\left[MK^2 + 2^\kappa(\kappa^2+\kappa)\right]$ complexity units. Clearly, when $\kappa = 0$, no limit cycle occurred and no additional complexity results with respect to the original multistage PIC detector. On the other hand, when a period-2 limit cycle does occur, there are $2 \leq \kappa \leq K$ toggling bits and the conditional joint maximum likelihood optimization for these κ bits requires the expense of $2^\kappa(\kappa^2+\kappa)$ additional complexity units with respect to the original multistage PIC detector.

D Numerical Results

Figure 6 combines the experimental distribution results obtained in Figures 2 and 3 with the analytical complexity results developed in this section to compare the overall computational complexity of the multistage PIC detector with limit cycle mitigation to the original multistage PIC and optimum multiuser detectors. The simulations used to obtain these results assume that the users are received with equal power (i.e., $\boldsymbol{A} = a\boldsymbol{I}$) at 10dB SNR and that the spreading codes are random, independent, and binary valued with spreading gain $N = 16$. These results show that, when K is small, the computational complexity of multistage PIC detection with limit cycle mitigation is very similar to that of the original multistage PIC detector. Intuitively, this is a reasonable result since period-2 limit cycles tend to occur infrequently in this case and, when they do occur, they tend to have have small Hamming distance. On the other hand, the complexity of multistage PIC detection with limit cycle mitigation tends to track the complexity of the optimum detector for larger values of K. This is a result of the increased frequency of period-2 limit cycles and the increased Hamming distance per limit cycle when K is large. Figure 6 also shows that, although the computational complexity of the multistage PIC detector with limit cycle mitigation tends to be roughly two orders of magnitude less than that of the optimum detector at large values of K in the cases considered, the incremental complexity per user of both detectors is similar (on a logarithmic scale) for large K. This implies that a multistage PIC detector using the limit cycle mitigation technique developed in this paper will tend to exhibit average computational complexity significantly less than the optimum multiuser detector but still exponential in K for large values of K.

VI. Conclusions

This paper presents a new technique for improving the performance of multistage PIC detection. Our results suggest that limit cycles are the largest cause of poor performance in the hard-decision multistage PIC detector, hence we propose a heuristic algorithm to reactively correct for limit cycle behavior. The proposed limit cycle mitigation algorithm retains the desirable properties of the original hard-decision multistage PIC iteration while detecting and correcting only for period-2 limit cycles. Simulation results suggest that the limit cycle

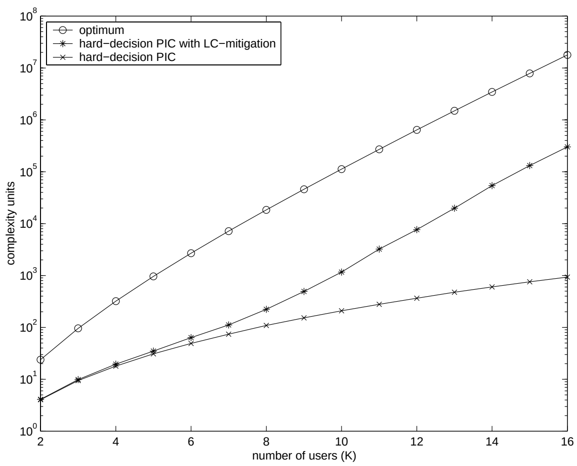

Fig. 6: Implementation complexity comparison between optimum (joint maximum likelihood), multistage PIC with limit cycle mitigation, and original multistage PIC detection.

mitigation algorithm is often successful at converting limit cycles to optimum fixed point convergence. Consequently, limit cycle mitigation tends to significantly improve the bit error rate performance of the hard-decision multistage PIC detector in a variety of operating scenarios with the greatest improvements observed when the number of users is small with respect to the spreading gain and when the SNR is high.

The computational complexity of the limit cycle mitigation algorithm developed in this paper, although similar to the original hard-decision PIC detector when K is small, becomes exponential in K as K becomes large. A potential topic for future research would be the development of lower complexity limit cycle mitigation algorithms that offer significant performance gains with complexity polynomial in K.

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