On Global Channel State Estimation and Dissemination in Ring Networks

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***Abstract*—This paper studies global channel state information (CSI) in time-slotted wireless ring networks with time-varying reciprocal channels. Lower bounds on maximum and average staleness of global CSI are derived, and efficient protocols that achieve the bounds are developed. Two extreme scenarios are considered with either (i) one node transmitting at a time or (ii) the maximum number of nodes transmitting at a time without collisions. In addition, the amount of CSI disseminated per packet is varied between two extremes. Simulation results confirm the analysis and quantify staleness in terms of the network parameters.**

***Index Terms*—Wireless networks, time-varying channels, global channel state information, channel estimation, data dissemination.**

I. Introduction

Channel state information at the transmitter (CSIT), can be used to improve the performance of wireless networks by efficiently using available resources, i.e., power and bandwidth [1]–[3]. But, in scenarios such as cooperative and distributed communications, *global* channel state information (CSI), i.e., knowledge of all channels in the network at all nodes provides higher gains in terms of the network's performance, compared to the case of just CSIT [4]–[11]. Another scenario where global CSI can be used is wireless sensor networks to determine minimum energy routes [12], [13]. However, in all of these scenarios, available global CSI is often assumed without an evaluation of its feasibility in practical settings.

Recent works [14], [15] have studied the problem of providing global CSI in fully-connected networks with time-varying reciprocal channels. A new *staleness* metric for the usefulness of the estimates of all channels throughout the network was proposed and bounds on *maximum* and *average* staleness of the channel estimates were evaluated. Also, optimal deterministic protocols that achieve these lower bounds were developed. While [14] provides bounds on the best possible staleness that can be achieved by any protocol with a deterministic data and CSI dissemination approach, the performance of opportunistic protocols is evaluated in [15].

Due of path loss and fading effects in wireless networks, nodes may only be able to communicate with other nodes within a certain range in practice, and not all nodes may be connected to each other. Ring networks are an example of a scenario where each node can only communicate with its neighbors [16], [17]. Another important factor in design of wireless networks is energy efficiency. This becomes even more critical in sensor networks with limited battery life. In these cases, where energy efficient communication is important, minimum energy routing requires each node to know the CSI of all nodes [12], [13]. As an example, consider a four node ring network, where node 1 wants to send a packet to node 3. There is no direct link from node 1 to node 3, so nodes 2 and 4 can act as relays. If node 1 has CSIT, it knows its channel with nodes 2 and 4, and it might try to send the packet through the best link as determined by the CSIT. But node 1 does not know the channel from nodes 2 and 4 to node 3. Now if the chosen node, i.e., either node 2 or 4, to relay the packet, has a poor channel with node 3, the total energy cost of delivering the packet might be high. So, if node 1 obtains an estimate of all channels, it can send the packet through the best overall link.

In this paper, the problem of global CSI dissemination in ring networks with time-varying reciprocal channels is considered. Closed-form expressions for the staleness bounds are derived and deterministic protocols are developed to achieve these lower bounds. Our TDD model is based on a time-slotted assumption where nodes typically transmit one at a time. However, our assumption of a ring network implies that not all nodes are connected, raising the possibility of having multiple nodes transmitting simultaneously without interference. Thus, we consider both single transmitter operation [14], [15], and we also consider multiple simultaneous transmissions which leads to considerable improvements in staleness. Simulation results are provided to verify the efficiency of the protocols.

II. System Model

Consider a ring network with N single-antenna nodes communicating over time-varying reciprocal channels. The complex channel gain between two adjacent nodes i and j at time n is denoted by $h_{i,j}[n]$ and assuming reciprocity, we have $h_{i,j}[n] = h_{j,i}[n]$. The ring network's topology is described by a cycle graph, i.e., a connected, 2-regular graph with $N \geq 3$ vertices and $L_C = \{(1,2),(2,3),\dots,(N-1,N),(N,1)\}$ represents the set of all channels in the network, i.e., the edges in the graph. Figure 1 represents a general structure of a cycle

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graph C_N where K nodes transmit simultaneously during each time slot and M channel state estimates are *disseminated* by each transmitting node. Each node in the network maintains its own local table of estimates of the N channels.

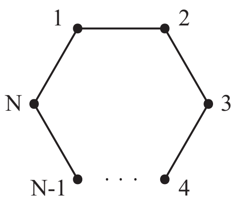

Fig. 1: A graph representation of general ring networks.

Figure 2 represents the general structure of a packet assumed to be exchanged among the nodes in the network. All packets are assumed to be received reliably. Each fixed-length packet contains overhead, data, and M channel state estimates. Since node k cannot estimate a channel to which it is not directly connected, i.e., the channel between nodes i and j for $i \neq j \neq k$, it uses the disseminated CSI information embedded in the transmitted packets by other nodes that form a path from either nodes i or j to node k, to obtain an estimate of the (i,j) channel. Assuming a length of D words for the data plus overhead, each packet has a length of $P = D + M$ words. Although Fig. 2 shows a particular packet structure, the position of the overhead, data, and disseminated CSI within any packet does not affect our analysis.

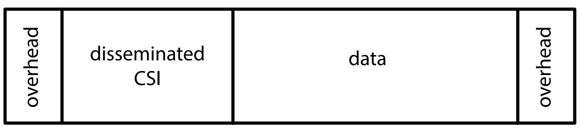

Fig. 2: Example fixed-length packet showing overhead, data, and CSI dissemination. The CSI dissemination consists of M channel estimates and each channel estimate has a length of one word. The data and overhead consists of D words. The total packet length is $P = D + M$ words.

We assume each node requires an estimate of all N complex channel gains in the network. Each node that receives the transmitted packet by node i does two things:

1) It directly estimates the channel $h_{i,j}[n]$, which can be obtained via a known training sequence in the packet, e.g., a known preamble embedded in the overhead, and/or through blind channel estimation techniques.
2) It extracts the disseminated CSI and uses it to update any "staler" CSI in its local table.

We denote the k^{th} node's estimate of the (i,j) channel during the packet transmitted at time n as $\hat{h}_{i,j}^{(k)}[n]$. Note that 2 of a node's estimates are *directly* obtained via channel estimation in step 1 above (for $i = k$ or $j = k$). The remaining $N-2$ estimates are *indirectly* obtained via disseminated CSI in step 2 above (for $i, j \neq k$). Thus, the network contains a total of $2N$ directly estimated parameters, and $N(N-2)$ indirectly estimated parameters.

The following definitions are considered for the metrics that are used to establish the results in section III.

Definition 1 (Staleness)**.** *The staleness $s_{i,j}^{(k)}[n]$ of the CSI estimate $\hat{h}_{i,j}^{(k)}[n']$ at time $n \geq n'$ is $(n-n')P$ words.*

Definition 2 (Maximum staleness)**.** *The maximum staleness S_{max} of a deterministic protocol is defined as*

$$S_{max} = \max_{i,j,k,n \geq \bar{n}} s_{i,j}^{(k)}[n]$$

for $\bar{n}$ sufficiently large such that all nodes have complete CSI tables.

Definition 3 (Average staleness)**.** *The average staleness S_{avg} of a protocol is defined as*

$$S_{avg} = \frac{1}{LN} \mathrm{E}\left[\sum_{i,j,k} s_{i,j}^{(k)}[n]\right]$$

where the expectation is over $n \geq \bar{n}$ for $\bar{n}$ sufficiently large such that all nodes have complete CSI tables.

We define a *protocol* as a sequence of transmitting nodes and the channel indexes they disseminate. In this paper we only focus on deterministic protocols.

Definition 4 (Efficient protocol)**.** *An efficient deterministic protocol is a valid protocol that simultaneously achieves maximum staleness $S^*_{max} + g_{max}P$, while also achieving an average staleness of at most $S^*_{avg} + g_{avg}P$ for constants g_{max}, g_{avg} and all N.*

The modulus operator

$$\sigma_k^m(i) = i + mk \pmod N$$

is used to simplify the notation. Also, if $\sigma_k^m(i) = 0$, set $\sigma_k^m(i) = N$, and argument i can be a vector, in which case σ operates element-wise.

III. CSI DISSEMINATION PROTOCOLS

In this section we provide lower bounds on the maximum and average staleness of any deterministic protocol and develop efficient protocols for dissemination of global CSI. The staleness performance of the ring network is evaluated for two extreme cases that (i) $K = 1$ and (ii) $K = K_{max} = \lfloor \frac{N}{3} \rfloor$ nodes transmit during each time slot. Also, for each case dissemination of the single freshest channel estimate in each packet, $M = 1$, and dissemination of $M = N-1$ CSI estimates in each packet are considered.

Table I represents lower bounds on the maximum and average staleness, S^*_{max} and S^*_{avg}, for different choices of K and M. Proofs are omitted due to the lack of space, however, in the following we mention some of the lemmas that are used to obtain the lower bounds.

Lemma 1 (Minimum staleness of the (i,j) channel ($(i,j) \in L_C$))**.** *At any time, at most one node in the network can have an estimate of the (i,j) channel with staleness zero and this estimate must be observed directly.*

TABLE I: Lower bounds on the maximum and average staleness.

K	M	P	S^*_{max}	S^*_{avg}	Protocol
1	1	$D+1$	$(\frac{2N^2-3N-4}{2})P$	$(\frac{2N^2-7N+8}{4})P$	1-a,b,c,d
1	$N-1$	$D+N-1$	$(2N-4)P$	$(\frac{2N^2-5N+4}{2N})P$	2
K_{max}	1	$D+1$	$(\frac{7N-16}{2})P$	$(\frac{7N^2-28N+36}{4N})P$	3-a,b,c
K_{max}	$N-1$	$D+N-1$	$(\frac{2N}{3})P$	$(\frac{2N^2-N+6}{6N})P$	4

During any time slot, if node i (j) transmits, node j (i) makes a direct estimate of the (i,j) channel with staleness zero, so there exists only one node that has a fresh estimate of the (i,j) channel. Otherwise, if neither node i nor node j transmits during a time slot, the (i,j) channel estimates throughout the network have staleness of at least one packet.

Lemma 2 (Minimum number of transmissions to disseminate a channel to all nodes)**.** *The number of disseminations to provide an estimate of the (i,j) channel to all nodes is lower bounded by $N-2$.*

Note that each channel, say the (i,j) channel should be disseminated by at least $N-4$ nodes that indirectly estimate it and to disseminate a fresh estimate of the (i,j) channel, at least one transmission is required by each of nodes i and j, which gives a minimum of $N-2$ disseminations.

Lemma 3 (Maximum number of simultaneous transmissions without collisions)**.** *The maximum number of nodes that can simultaneously transmit without collisions is $K_{max} = \lfloor \frac{N}{3} \rfloor$.*

To have no collisions, any receiving node in the network must be adjacent to only one or zero transmitting nodes. A receiving node also may not transmit while receiving. Hence, if node $j,\ 1 \leq j \leq N$ is transmitting, nodes $\sigma_0^0(\{j-2, j-1, j+1, j+2\})$ are not permitted to transmit.

The basic steps for deriving the given bounds involve separately considering the directly and indirectly estimated parameters throughout the network, deriving the staleness statistics for each group of parameters, and subsequently obtaining the maximum and average staleness bounds over all parameters. For some choices of parameters (i.e., amount of CSI disseminated per packet, and number of simultaneously transmitting nodes), the underlying combinatorics are such that development of bounds and efficient protocols achieving the bounds requires splitting the number of nodes N into different cases. For example, in the first theorem below where $K = M = 1$, four separate cases of N are required: N odd and $N \neq 6k-3$, N odd and $N = 6k-3$, N even and $N \neq 4k$, N even and $N = 4k$, where we define $k \in \mathbb{Z}^+$.

Theorem 1 (Achievability of the lower bound on the maximum and average staleness of C_N for $K=1$ and $M=1$)**.** *The following protocols achieve within P time slots of S^*_{max}. In each protocol, $H = \{H_0, H_1, \ldots, H_{2N-1}\}$ denotes the node transmission order for $N(N-2)$ time slots and must be repeated with period $N(N-2)$ to maintain its achievable lower bound on the maximum and average staleness.*

Protocol 1-a (N odd and $N \neq 6k-3$, $K=1$, $M=1$)
Define $H_m = \sigma^m_{\frac{N-1}{2}}(\{1, 2, \ldots, \frac{N-1}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^m_{\frac{N-1}{2}}((N,1))$ channel for $m = 0, 1, \ldots, N-1$. Define $H_{N+m} = \sigma^{-m}_{\frac{N-3}{2}}(\{N-1, N-2, \ldots, \frac{N+3}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^{-m}_{\frac{N-3}{2}}((N-1,N))$ channel for $m = 0, 1, \ldots, N-1$.

Protocol 1-a achieves maximum staleness of $S_{max} = S^*_{max} + P/2$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Protocol 1-b ($N = 6k-3$, $K=1$, $M=1$)
Define $H_m = \sigma^m_{\frac{N-3}{2}}(\{1, 2, \ldots, \frac{N-3}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^m_{\frac{N-3}{2}}((N,1))$ channel for $m = 0, 1, \ldots, \ell-1$, which $\ell = \frac{N}{3}$. Define $H_{\ell+m} = \sigma^m_{\frac{N-3}{2}}(\{N, 1, \ldots, \frac{N-5}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^m_{\frac{N-3}{2}}((N-1,N))$ channel for $m = 0, 1, \ldots, \ell-1$. Define $H_{2\ell+m} = \sigma^{-m}_{\frac{N-1}{2}}(\{N-2, N-3, \ldots, \frac{N-1}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^{-m}_{\frac{N-1}{2}}((N-2, N-1))$ channel for $m = 0, 1, \ldots, N-3$. Define $H_{2\ell+N-2+m} = \sigma^m_{\frac{N-3}{2}}(\{N-1, N, \ldots, \frac{N-7}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^m_{\frac{N-3}{2}}((N-2, N-1))$ channel for $m = 0, 1, \ldots, \ell-1$. Define $H_{3\ell+N-2+m} = \sigma^{-m}_{\frac{N-1}{2}}(\{N-3, N-4, \ldots, \frac{N-3}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^{-m}_{\frac{N-1}{2}}((N-3, N-2))$ channel for $m = 0, 1$.

Protocol 1-b achieves maximum staleness of $S_{max} = S^*_{max} + P/2$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Protocol 1-c (N even and $N \neq 4k$, $K=1$, $M=1$)
Define $H_n = \sigma^m_{N-2}(\{1, 2, \ldots, \frac{N}{2}\})$, which forms the node transmission order for dissemination of the $\sigma^m_{N-2}((N,1))$ channel for $n = 2m = 0, 2, \ldots, N-2$. Define $H_\ell = \sigma^m_{N-2}(\{\frac{N+2}{2}, \frac{N+4}{2}, \ldots, N-2\})$, which forms the node transmission order for dissemination of the $\sigma^m_{N-2}((\frac{N}{2}, \frac{N+2}{2}))$ channel for $\ell = 2m+1 = 1, 3, \ldots, N-1$, and $m = 0, 1, \ldots, \frac{N-2}{2}$. Define $H_{N+n} =$

$\sigma_{N-2}^{-m}(\{N-1, N-2, \ldots, \frac{N}{2}\})$, which forms the node transmission order for dissemination of the $\sigma_{N-2}^{-m}((N-1, N))$ channel for $n = 2m = 0, 2, \ldots, N-2$. Define $H_{N+\ell} = \sigma_{N-2}^{-m}(\{\frac{N-2}{2}, \frac{N-4}{2}, \ldots, 2\})$, which forms the node transmission order for dissemination of the $\sigma_{N-2}^{-m}((\frac{N-2}{2}, \frac{N}{2}))$ channel for $\ell = 2m+1 = 1, 3, \ldots, N-1$, and $m = 0, 1, \ldots, \frac{N-2}{2}$.

Protocol 1-c achieves maximum staleness of $S_{max} = S^*_{max} + P$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Protocol 1-d ($N = 4k$, $K = 1$, $M = 1$)
Define $H_m = \sigma_{\frac{N-2}{2}}^{m}(\{1, 2, \ldots, \frac{N-2}{2}\})$, which forms the node transmission order for dissemination of the $\sigma_{\frac{N-2}{2}}^{m}((N, 1))$ channel for $m = 0, 1, \ldots, N-1$. Define $H_{N+m} = \sigma_{\frac{N-2}{2}}^{-m}(\{N-1, N-2, \ldots, \frac{N+2}{2}\})$, which forms the node transmission order for dissemination of the $\sigma_{\frac{N-2}{2}}^{-m}((N-1, N))$ channel for $m = 0, 1, \ldots, N-1$.

Protocol 1-d achieves maximum staleness of $S_{max} = S^*_{max}$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Figure 3 shows the achieved staleness for this efficient protocol at each time instant when $N = 4$ and $D = 0$. We see that over one period of the protocol, i.e., $8 \leq n \leq 15$, the instantaneous maximum staleness is equal to 8, thus achieving S^*_{max}. Also, we note that $S_{avg} = 3.125 = S^*_{avg} + 1/8$. Thus, Fig. 3 shows that the protocol is efficient.

Theorem 2 (Achievability of the lower bound on the maximum and average staleness of C_N for $K = 1$ and $M = N-1$)**.**

Protocol 2 ($N \geq 3$, $K = 1$, $M = N-1$)
The node transmission order for the first block of length N is $H = \{1, 2, 3, \ldots, N\}$, where each node i_n disseminates all of its table except the estimate of the channel between nodes i_{n+1} and i_{n+2}. Note that H denotes the node transmission order for N time slots and must be repeated with period N to maintain its achievable lower bound on the maximum and average staleness.

Protocol 2 achieves maximum staleness of $S_{max} = S^*_{max}$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Theorem 3 (Achievability of the lower bound on the maximum and average staleness of C_N for $K = K_{max}$ and $M = 1$)**.** *The following protocols achieve S^*_{max}. Note that $H = \{H_0, H_1, \ldots, H_5\}$ denotes the node transmission order for $3(N-2)$ time slots and must be repeated with period $3(N-2)$ to maintain its achievable lower bound on the maximum and average staleness.*

Protocol 3-a ($N = 6k$, $K = K_{max}$, $M = 1$)
Let $h^{\ell}_{1+m} = \sigma^1_{m+\ell\frac{(N-2)}{2}}(\{1, 4, \ldots, N-2\})$ be the simultaneous transmitters' indices for dissemination of the $\sigma^1_{\ell\frac{(N-2)}{2}}(\{(N,1), (3,4), \ldots, (N-3, N-2)\})$ channels, respectively, during time $1 + m + \ell\frac{(N-2)}{2}$ for $m = \{0, 1, \ldots, \frac{N-4}{2}\}$. Define $H_\ell = \{h^\ell_1, h^\ell_2, \ldots, h^\ell_{\frac{N-2}{2}}\}$ for $\ell = \{0, 1, 2\}$. Let $h^{\ell}_{1+\frac{3(N-2)}{2}+m} = \sigma^1_{-m-\ell\frac{(N-2)}{2}}(\{2, 5, \ldots, N-1\})$ be the simultaneous transmitters' indices for dissemination of the $\sigma^1_{-\ell\frac{(N-2)}{2}}(\{(2,3), (5,6), \ldots, (N-1, N)\})$ channels, respectively, during time $1 + m + (\ell+3)\frac{(N-2)}{2}$ for $m = \{0, 1, \ldots, \frac{N-4}{2}\}$. Define $H_{\ell+3} = \{h^\ell_{1+\frac{3(N-2)}{2}}, h^\ell_{2+\frac{3(N-2)}{2}}, \ldots, h^\ell_{2(N-2)}\}$ for $\ell = \{0, 1, 2\}$.

Protocol 3-a achieves maximum staleness of $S_{max} = S^*_{max}$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Protocol 3-b ($N = 6k-3$, $K = K_{max}$, $M = 1$)
Let $h^{2\ell}_{1+m} = \sigma^1_{m+\ell}(\{1, 4, \ldots, N-2\})$ be the simultaneous transmitters' indices for dissemination of the $\sigma^1_{m+\ell}(\{(N,1), (3,4), \ldots, (N-3, N-2)\})$ channels, respectively, during time $1 + m + \ell(N-2)$ for $m = \{0, 1, \ldots, \frac{N-3}{2}\}$. Define $H_{2\ell} = \{h^{2\ell}_1, h^{2\ell}_2, \ldots, h^{2\ell}_{\frac{N-1}{2}}\}$ for $\ell = \{0, 1, 2\}$. Let $h^{2\ell+1}_{1+m} = \sigma^1_{-m+\ell}(\{3, 6, \ldots, N\})$ be the simultaneous transmitters' indices for dissemination of the $\sigma^1_{-m+\ell}(\{(3,4), (6,7), \ldots, (N,1)\})$ channels, respectively, during time $\frac{N-1}{2} + 1 + m + \ell(N-2)$ for $m = \{0, 1, \ldots, \frac{N-5}{2}\}$. Define $H_{2\ell+1} = \{h^{2\ell+1}_1, h^{2\ell+1}_2, \ldots, h^{2\ell+1}_{\frac{N-3}{2}}\}$ for $\ell = \{0, 1, 2\}$.

Protocol 3-b achieves maximum staleness of $S_{max} = S^*_{max} + P/2$ and average staleness of $S_{avg} \leq S^*_{avg} + P/2$.

Protocol 3-c ($N \neq 3k$, $N \geq 7$, $K = K_{max}$, $M = 1$)
Let $h^{2\ell}_{1+m} = \sigma^1_{m+\ell}(\{1, 4, \ldots, 1 + 3(K_{max}-1)\})$ be the simultaneous transmitters' indices for dissemination of the $\sigma^1_{m+\ell}(\{(N,1), (3,4), \ldots, (N + 3(K_{max}-1), 1 + 3(K_{max}-1))\})$ channels, respectively, during time $1 + m + \ell(N-2)$ for $m = \{0, 1, \ldots, n_1 - 1\}$. Define $H_{2\ell} = \{h^{2\ell}_1, h^{2\ell}_2, \ldots, h^{2\ell}_{\frac{N-1}{2}}\}$ for $\ell = \{0, 1, \ldots, \frac{N}{\gcd(N, N-2)} - 1\}$. Let $h^{2\ell+1}_{1+m} = \sigma^1_{-m+\ell}(\{n_1 - 1, n_1 + 2, \ldots, n_1 - 1 + 3(K_{max}-1)\})$ be the simultaneous transmitters' indices for dissemination of the $\sigma^1_{-m+\ell}(\{(n_1-1, n_1), (n_1+2, n_1+3), \ldots, (n_1 - 1 + 3(K_{max}-1), n_1 + 3(K_{max}-1))\})$ channels, respectively, during time $n_1 + 1 + m + \ell(N-2)$ for $m = \{0, 1, \ldots, n_2 - 1\}$. Define $H_{2\ell+1} = \{h^{2\ell+1}_1, h^{2\ell+1}_2, \ldots, h^{2\ell+1}_{\frac{N-3}{2}}\}$ for $\ell = \{0, 1, \ldots, \frac{N}{\gcd(N, N-2)} - 1\}$. Here, $n_1 = \frac{N-1}{2}$ and $n_2 = \frac{N-3}{2}$, when N is odd, and $n_1 = \frac{N}{2}$ and $n_2 = \frac{N-4}{2}$, when N is even.

For Protocol 3-c $H = \{H_0, H_1, \ldots, H_{\frac{2N}{\gcd(N,N-2)}-1}\}$ denotes the node transmission order for $[\frac{N}{\gcd(N,N-2)}](N-2)$ time slots and must be repeated with period $[\frac{N}{\gcd(N,N-2)}](N-2)$ to maintain its achievable lower bound on the maximum and average staleness. When $N \neq 3k$, at least 4 time slots are

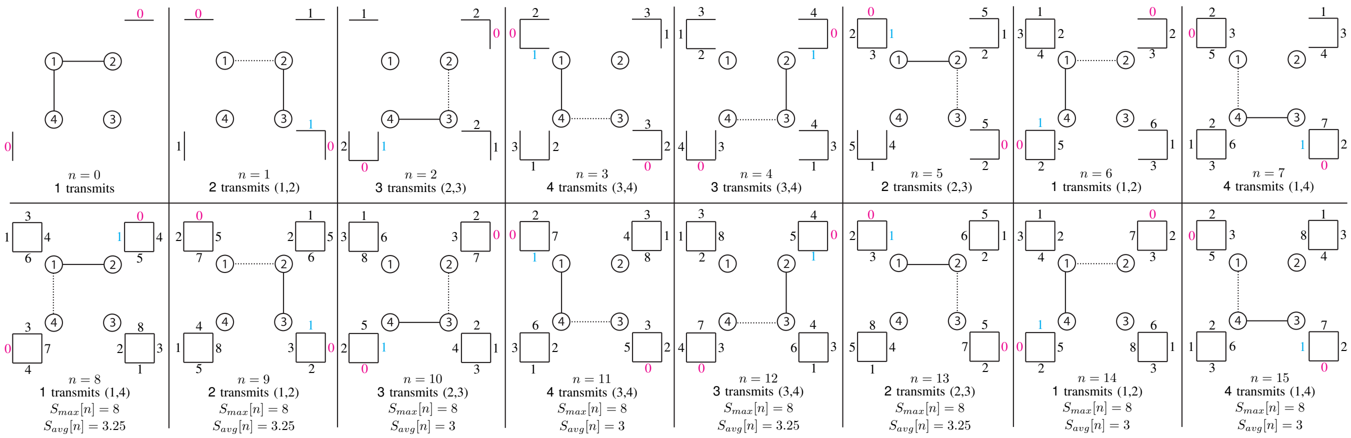

Fig. 3: Efficient CSI dissemination protocol for $N = 4$, $K = 1$ and $M = 1$ case. Numbers on edges indicate staleness of CSI estimates locally at each node; red numbers indicate CSI estimates have been refreshed through direct estimation, blue numbers indicate CSI refreshed through dissemination, and black numbers indicate no update to CSI since the last packet.

required so that all nodes can transmit at least once with no collisions, while for $N = 3k$ only 3 time slots is enough. Thus, any protocol with $N \neq 3k$ has to include some "leftover" dissemination rounds by a single node, which causes the staleness to increase with N. Also, considering fairness among the nodes, for $N \neq 3k$ at least $\frac{N(N-2)}{\gcd(N,N-2)}$ time slots are required to have all nodes transmit an equal number of times. Based on this, we conjecture that Protocol 3-c achieves the lowest maximum and average staleness compared to any other protocol. However, Protocol 3-c does not achieve within a constant gap of S^*_{max} or S^*_{avg}.

Theorem 4 (Achievability of the lower bound on the maximum staleness of C_N for $K = K_{max}$ and $M = N - 1$)**.**

Protocol 4 ($N \geq 3$, $K = K_{max}$, $M = N - 1$)
Let $H_0 = \{1, 4, \ldots, 1 + 3(K_{max} - 1)\}$ of length K_{max} denote the first group of nodes that simultaneously transmit without collisions and each node i_n disseminates its table except the estimate of the channel between nodes i_{n+1} and i_{n+2}. Define $H_\ell = \sigma_1^\ell(H_0)$, which forms the group of simultaneous transmitters for $\ell = 0, 1, 2, \ldots, N - 1$. Note that $H = \{H_0, H_1, H_2, \ldots, H_{N-1}\}$ denotes the node transmission order for N time slots and must be repeated with period N to maintain its achievable lower bound on the maximum and average staleness.

Protocol 4 achieves maximum staleness of $S_{max} \leq S^*_{max} + 8P/3$ and average staleness of $S_{avg} \leq S^*_{avg} + 6P/5$.

Table II shows the node transmission order and their disseminated CSI, i.e. Protocol 3-a for $N = 6$.

TABLE II: Protocol 3-a for $N = 6,\ K = 2,\ M = 1$.

time slot	Tx_1	disseminated CSI	Tx_2	disseminated CSI
$n = 0$	1	$(1, 6)$	4	$(3, 4)$
$n = 1$	2	$(1, 6)$	5	$(3, 4)$
$n = 2$	3	$(2, 3)$	6	$(5, 6)$
$n = 3$	4	$(2, 3)$	1	$(5, 6)$
$n = 4$	5	$(4, 5)$	2	$(1, 2)$
$n = 5$	6	$(4, 5)$	3	$(1, 2)$
$n = 6$	5	$(5, 6)$	2	$(2, 3)$
$n = 7$	4	$(5, 6)$	1	$(2, 3)$
$n = 8$	3	$(3, 4)$	6	$(1, 6)$
$n = 9$	2	$(3, 4)$	5	$(1, 6)$
$n = 10$	1	$(1, 2)$	4	$(4, 5)$
$n = 11$	6	$(1, 2)$	3	$(4, 5)$

IV. Numerical Results

This section provides numerical examples to verify the analysis in the previous section and to quantify the maximum and average staleness as a function of the network parameters N and D. Figure 4 plots the maximum and average staleness of the protocols for C_N, versus the number of nodes N for $D \in \{0, 10\}$. The $D = 0$ case can be considered a protocol with no data or overhead where each packet is dedicated solely to CSI dissemination. These results show that for large N, when $D = 0$, the $K = K_{max}$ and $M = 1$ case provides the minimum maximum and average staleness, and when $D = 10$, the $K = K_{max}$ and $M = N - 1$ case provides the minimum maximum and average staleness.

Figure 5 plots the maximum and average staleness of the protocols for C_N, versus the packet data and overhead D for $N \in \{6, 25\}$. The results show for $D \geq 6$, the $K = K_{max}$ and $M = N - 1$ case provides the minimum maximum and average staleness.

Next, an example applying the derived staleness bounds to a practical wireless setting is considered. For global CSI to

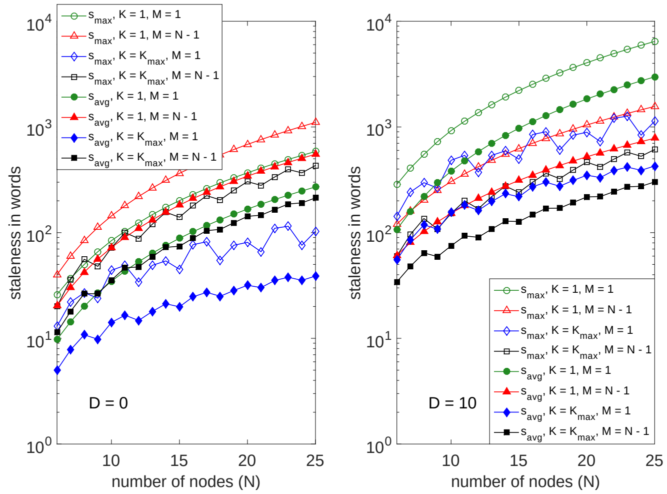

Fig. 4: Achievable maximum and average staleness versus N.

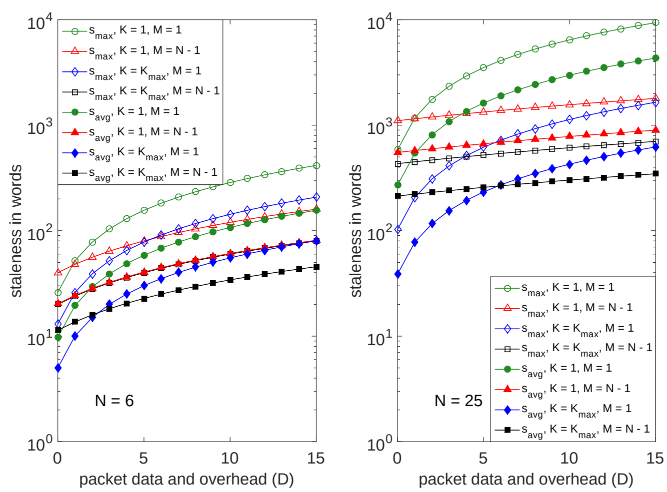

Fig. 5: Achievable maximum and average staleness versus D.

be useful at all nodes, the maximum staleness lower bound in seconds must be less than the coherence time of the channel. For a carrier frequency of f_c and an average relative speed of nodes equal to v, the resulting Doppler spread is $f_D = \frac{vf_c}{c}$ where c is the speed of light, and the coherence time is $T_c = \frac{0.423}{f_D} = \frac{0.423c}{vf_c}$ [18]. Consider mobile transmission of voice using LTE [19], for example, with a data rate of $R_b = 25$ Mbps, 1200 bits of data plus overhead per packet, 32 bits per word, a carrier frequency of $f_c = 1900$ MHz, relative speed of $v = 100$ m/s, $K = K_{max}$ simultaneous transmitters without collisions and $M = N - 1$ CSI estimates per packet, the bounds tell us that if the node cluster size exceeds $N > 15$, global CSI dissemination is infeasible.

V. Conclusion

This paper analyzed lower bounds on the staleness of deterministic protocols for global CSI estimation and dissemination in wireless ring networks with packet-based transmission and time-varying reciprocal channels. Efficient protocols that achieve these bounds within a small constant gap were developed. The analysis showed that for both cases of disseminating a single channel estimate and $N-1$ CSI estimates, the maximum and average staleness bounds scale as $\mathcal{O}(N^2)$, except when the maximum number of nodes transmit without collisions and a single channel estimate gets disseminated per packet, in which case the maximum and average staleness bounds scale as $\mathcal{O}(N)$. Also, for small and large amounts of data plus overhead compared to the number of nodes, a single channel estimate and $N-1$ CSI estimates should be disseminated per packet, respectively, to minimize the maximum and average staleness.

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