Simultaneous Distributed Beamforming and Nullforming with Adaptive Positioning

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***Abstract*—This paper considers a wireless communication system with simultaneous distributed transmit beamforming and nullforming. In addition to the usual method of controlling the phases of the transmistters to provide directivity, one or more nodes in the distributed array also adapt their position(s) to improve the quality of the beam without loss of null quality. This paper presents an adaptive algorithm with simple feedback to facilitate node positioning such that optimal beamforming is achieved to one desired receiver while simultaneously steering nulls to one or more protected receivers. The efficacy of the adaptive algorithm is demonstrated via simulation.**

***Index Terms*—Cooperative communication, beamforming, nullforming, zero-forcing, mobility.**

I. Introduction

Beamforming has traditionally been studied in the context of multiple input multiple output (MIMO) communications, in which central antenna arrays are utilized to mitigate the multipath fading effects of wireless channels and improve certain characteristics of the network such as quality of service and capacity [1]–[5]. However, due to the cost and circuitry complexity, multiple-antenna structures may not be practical in energy limited networks with small nodes. A solution to this problem is deploying virtual antenna arrays by a network of single-antenna nodes [6]. This considerably enhances energy and delay efficiency in wireless sensor networks [7], [8].

Subsequently, a lot of attention has been drawn by distributed beamforming, where a number of single-antenna nodes are utilized to form a virtual antenna array with the goal of cooperatively sending a common message to a destination [9]–[14]. A simple distributed beamforming scheme based on one-bit feedback from the receiver is proposed in [10]. Each transmitter independently makes a small random adjustment to its phase, while the receiver provides a single bit of feedback, indicating if the signal to noise ratio has improved or worsened after the current iteration. It was shown that this random phase adjustment procedure leads to asymptotic phase coherence with probability one and the random phase adjustments can be chosen such that the convergence time is linear in the number of collaborating nodes. In [11], a network of N nodes with the goal of forming a strong beam towards a receiver, with only phase adjustments, is considered based on feedback from the receiver, while [12] utilizes only the transmitters' mobility to form beams towards the receiver. In [13] and [14], in addition to multiple transmitters and a single beam receiver, a number of protected receivers towards which nulls are simultaneously steered, are considered. The feedback from all receivers are used to compute the optimal zero-forcing beamforming vector at the transmitters' side in order to minimize the received power at the protected receivers while maximizing the received power at the beam receiver.

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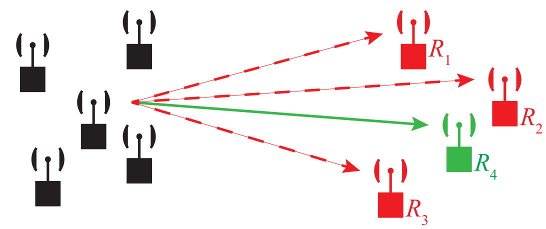

Fig. 1: A typical node topology, where $N = 5$ transmitters, $M = 3$ protected receivers and the beam receiver are represented by black, red and green nodes, respectively. The green and red arrow lines represent the beam and the nulls.

To the best of our knowledge, the problem of considering both phase adjustments and utilizing mobility of the nodes to simultaneously perform beamforming and nullforming has not been studied previously. In this paper, adaptive algorithms are proposed where, in addition to the phase adjustments at the transmitters to maintain perfect nulls at a number of protected receivers, a number of mobile nodes alter their positions to improve the received power at a single beam receiver.

The rest of the paper is organized as follows. Section II represents the system model. The proposed distributed algorithms are discussed in section III. Section IV provides some simulation results and section V concludes the paper.

II. System Model

We consider a system of N transmitters, T_i, $1 \leq i \leq N$ and $M+1$ receivers R_j, $1 \leq j \leq M+1$, $M < N$, where R_j's, $1 \leq j \leq M$ are static protected receivers, and R_{M+1} is a beam receiver. Figure 1 represents a typical setting for the nodes' positions where, all nodes are placed randomly. All nodes are equipped with a half-duplex single antenna. In practice, the local oscillators of the transmitters have inherent frequency offsets and behave stochastically, causing phase offset variations in each "effective" channel from the transmitters to a receiver even when the propagation channels are time invariant [8]. Here, we ideally assume all frequency offset values are equal to zero. All transmitters send a common message $m(t)$ with carrier frequency f_c. All channels are modeled as narrowband and linear. The effective channel from transmit node i to receiver j is modeled as [14]

$$h_{i,j} = g_{i,j} e^{\iota \tau_{i,j}}.$$

Here, ι, λ, $d_{i,j}$, $g_{i,j}$, and $\tau_{i,j} = e^{\frac{\iota 2\pi d_{i,j}}{\lambda}}$ represent the imaginary unit, wavelength, distance, channel gain, and propagation delay between T_i and R_j, respectively. The overall channel matrix from the transmitters to the protected receivers is defined as

$$\mathbf{H}_{N\times M} = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,M} \\ h_{2,1} & h_{2,2} & \dots & h_{2,M} \\ \vdots & \vdots & \dots & \vdots \\ h_{N,1} & h_{N,2} & \dots & h_{N,M} \end{bmatrix}.$$

Note that column j, $1 \le j \le M$ of $\mathbf{H}$ represents the channel coefficients from the transmitters to receiver j. The overall channel vector from the transmitters to the beam receiver is represented by $\mathbf{h}_{N\times 1}$.

III. Distributed Adaptive Algorithm

In this section we present distributed adaptive algorithms for two cases: (i) only the beam receiver is mobile and (ii) only some of the transmitters are mobile. For both cases, the phases of the transmitters and the position of the mobile node(s) are updated based on feedback received from the beam receiver in order to continuously improve the beam receiver's power while at the same time steering perfect nulls towards the protected receivers. For the positioning part, the mobile node(s) moves to a new randomly generated position with the goal of improving the received power at the single beam receiver. For the phase adjustments part, the transmitters apply zero-forcing to insure the formation of perfect nulls. Rather than just simply moving the mobile node(s) randomly, a form of memory is included based on the *heavy-ball* method [15]. The heavy-ball method was originally proposed by Polyak [16], and the idea is to consider the result of the previous iteration during the current iteration to increase the convergence speed. This means if direction of the movement of the mobile node(s) during the previous iteration increased the received power at the beam receiver, the mobile node(s) keeps moving in that direction during the current iteration. In the following, the adaptive algorithms based on mobility of the nodes are represented.

The following represents the distributed adaptive beamforming and nullforming (DABN) algorithm with mobile beam receiver.

DABN with mobile beam receiver

step I: Initialize phases of the transmitters and position of the beam receiver.

step II: Generate a random displacement vector $\Delta = |\Delta| e^{\iota \angle \Delta}$, where step size $|\Delta|$ is fixed and $\angle \Delta$ is uniformly distributed between 0 and 2π. Move the beam receiver to the new position. Considering the heavy-ball method, if $\angle \Delta$ during the previous iteration improved the received power at the beam receiver, the beam receiver moves in the same direction as the previous iteration without generating a new Δ.

step III: All $M+1$ receivers provide feedback so the transmitters know all channels $\mathbf{H}$ and $\mathbf{h}$ exactly.

step IV: Form perfect nulls towards all the protected receivers. To form perfect nulls at the protected receivers, the zero-forcing method from [14] is adopted to compute the phases of the transmitters.

step V: Check the received power $P_b(k)$ at the beam receiver.

* If $P_b(k) \ge P_b(k-1)$, the beam receiver remains at the new position.
* If $P_b(k) < P_b(k-1)$, the beam receiver goes back to its previous position.

step VI: Repeat from step II for a maximum number of iterations.

Note that k represents the current iteration's index.

The following represents the distributed adaptive beamforming and nullforming algorithm with N' mobile transmitters, $1 \le N' \le N$.

DABN with mobile transmitters

step I: Initialize positions and phases of the mobile transmitters.

step II: Generate a random displacement vector $\Delta_i = |\Delta_i| e^{\iota \angle \Delta_i}$ for every mobile transmitter T_i, $i \in \{1, 2, \dots, N'\}$, where step size $|\Delta_i|$ is fixed and $\angle \Delta_i$ is uniformly distributed between 0 and 2π. Move all the mobile transmitters to the new positions. Considering the heavy-ball method, if $\angle \Delta$ during the previous iteration improved the received power at the beam receiver, all the transmitters move in the same directions as the previous iteration without generating a new Δ.

step III: All $M+1$ receivers provide feedback so the transmitters know all channels $\mathbf{H}$ and $\mathbf{h}$ exactly.

step IV: Form perfect nulls towards all the protected receivers.

step V: Check the received one-bit feedback $F(k)$ transmitted by the beam receiver based on its received power $P_b(k)$, where

* $F(k) = 1$ if $P_b(k) \ge P_b(k-1)$, all the mobile transmitters remain at the new positions.
* $F(k) = 0$ if $P_b(k) < P_b(k-1)$, all the mobile transmitters go back to their previous positions.

step VI: Repeat from step II for a maximum number of iterations.

The zero-forcing vector $\mathbf{w}$ at the transmitters' side is computed as the following [14].

$$\mathbf{w}_{N\times 1} = \left[\mathbf{I} - \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\right]\mathbf{h}$$

where $\{\cdot\}^H$ represents the hermitian operation.

IV. Numerical Results

In this section some simulation results are presented to demonstrate efficiency of the proposed algorithms. Unless stated otherwise, in the following results the carrier frequency is set to $f_c = 1$ GHz and the step size is $\lambda/200$, where $\lambda = 0.3$ is the wavelength and all points on the (x, y) plane are in meters. The algorithm for each case is run for 1000 iterations and the results are averaged over 500 independent Monte-Carlo simulations. All channel amplitudes are ideally assumed to be equal to $|h_{i,j}| = |g_{i,j}| = 1$, which means path loss is equal to 0 dB. Note that path loss just scales the received power values without changing positions of the local minimum and maximum points. Therefore, since the main concern is finding the optimum positions, not the real received power values, it is acceptable to set path loss equal to 0 dB.

Figure 2 represents a contour plot of the received power at the beam receiver as a function of the points on the (x, y) plane for $N = 4$ static transmitters and $M = 3$ static protected receivers. The transmitters are placed at $(-30, 0)$, $(0, 30)$, $(30, 0)$, $(0, -30)$ and the protected receivers are placed at $(-30, -10)$, $(-30, 0)$ and $(-30, 10)$. Here only the beam receiver is mobile and its trail for four different initial positions (marked by the black asterisks) is depicted. In all four cases, the beam receiver approaches a point at which its received power is locally maximized, i.e. the received power is almost equal to coherent power, which is equal to $10 \log(N^2) = 12.0412$ dB.

Figure 3 represents the received power at the beam receiver for the case of mobile beam receiver with the initial positions in Figure 2 versus the number of iterations. The rest of the parameters are the same as Figure 2. It can be seen that the received power at the beam receiver is constantly increasing and based on the initial position, it converges to a local maximum close to the coherent power with different speeds.

Figure 4 represents a contour plot of the received power at the beam receiver as a function of the points on the (x, y) plane for $N = 4$ transmitters and $M = 3$ static protected receivers. The transmitters and the protected receivers are placed at the same points as Figure 2 and the static beam receiver is placed at $(0, 0.3)$. Here only the transmitter placed at $(0, -30)$ is mobile and its trail for four different initial positions (marked by the black asterisks) is depicted. In all the four cases, the mobile transmitter approaches a point at which the beam receiver's power is locally maximized.

Figure 5 represents the received power at the beam receiver for the case of mobile transmitter with the initial positions in Figure 4 versus the number of iterations. The rest of the parameters are the same as Figure 4. It can be seen that the received power at the beam receiver is constantly increasing and based on the initial position, it converges to a local maximum close to the coherent power with different speeds.

Figure 6 represents the beam receiver's power for $N \in \{5, 10\}$ mobile transmitters, $M = 1$ static null receiver and 1 static beam receiver. The transmitters are initially placed on the x axis from $-N\lambda$ to $N\lambda$ with equal space. The null and the beam receivers are placed at $(-30, -10)$ and $(0, 0.3)$, respectively. Note that here all transmitters are mobile. It can be seen that for both cases decreasing the step size from $\lambda/200$ to $\lambda/10$ increases the convergence speed considerably.

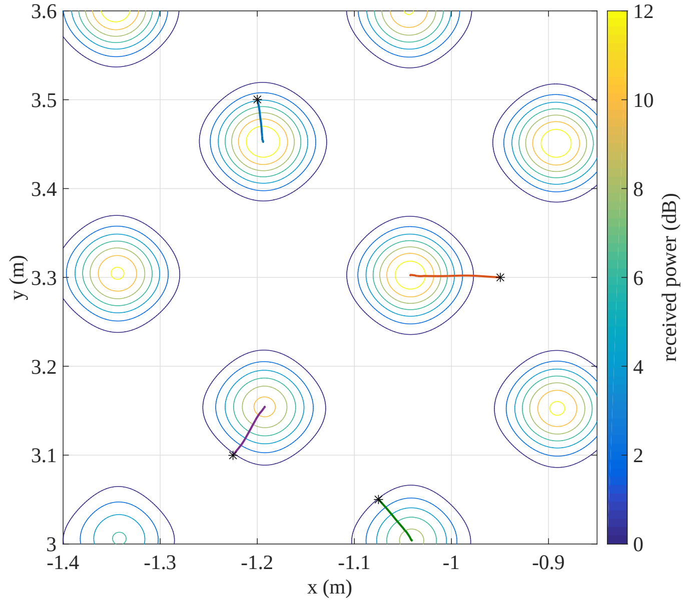

Fig. 2: Trail of the mobile beam receiver for four different initial positions.

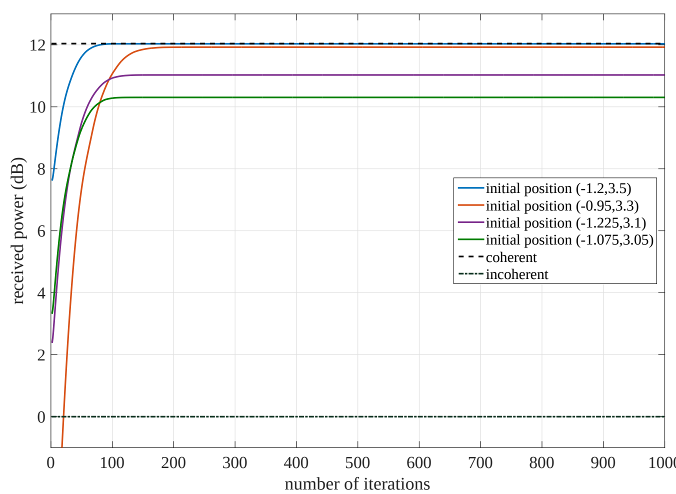

Fig. 3: Received power for a system with $N = 4$ static transmitters, $M = 3$ static protected receivers and 1 mobile beam receiver.

V. Conclusion

This paper presented distributed adaptive algorithms for simultaneous beamforming and nullforming by a number of transmitters towards a single beam receiver and a number of protected receivers. The transmitters update their phases to maintain perfect nulls at the protected receivers at all times, while the mobile nodes change their positions to provide an improved beam to the beam receiver. The numerical results

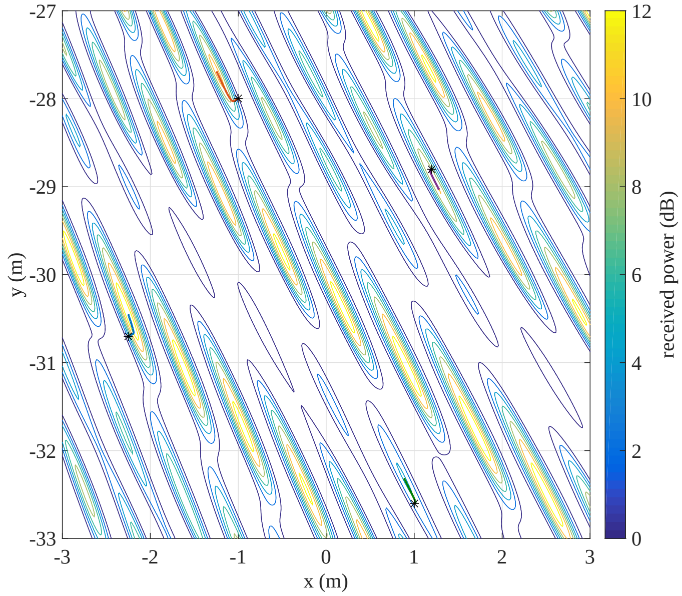

Fig. 4: Trail of the mobile transmitter for four different initial positions.

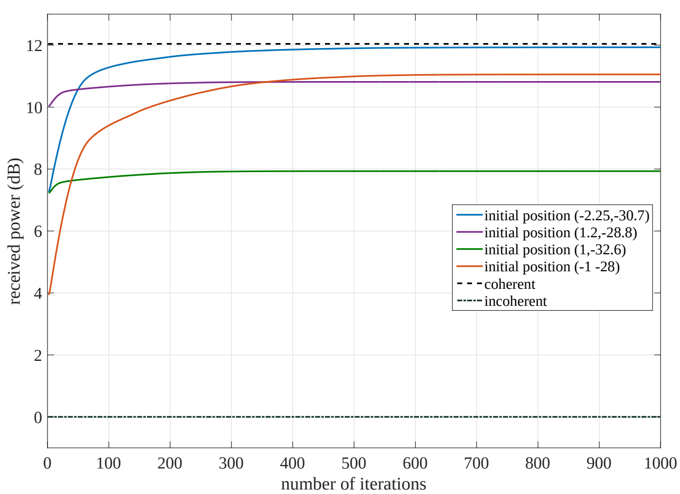

Fig. 5: Received power for a system with $N = 4$ transmitters, one of which is mobile, $M = 3$ static protected receivers and 1 static beam receiver.

show that the proposed algorithms provide the beam receiver with a locally maximized power, while maintaining perfect nulls at the protected receivers.

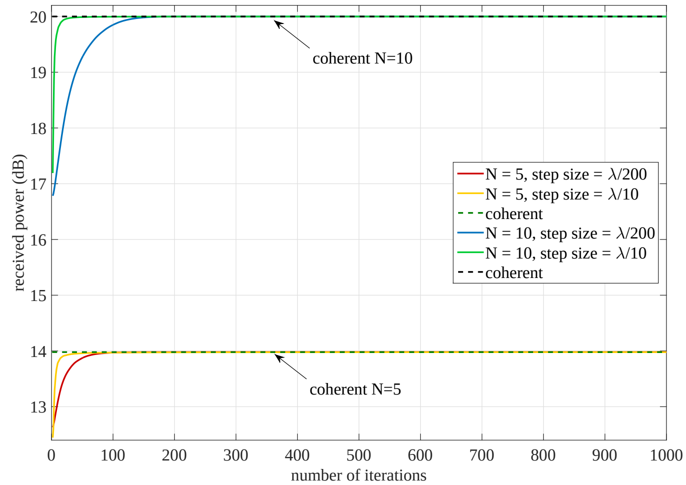

Fig. 6: Received power for a system with $N \in \{5, 10\}$ mobile transmitters, $M = 1$ static protected receivers and 1 static beam receiver for two different step sizes $\{\lambda/200, \lambda/10\}$.

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