

ON THE AVERAGE STALENESS OF GLOBAL CHANNEL STATE INFORMATION IN WIRELESS NETWORKS WITH RANDOM TRANSMIT NODE SELECTION

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ABSTRACT

This paper studies the average staleness of global channel state information (CSI) in fully-connected time-slotted wireless networks with time-varying reciprocal channels where, in each time slot, one node is equiprobably selected to transmit and disseminate CSI. The proposed protocols are compatible with the random nature of transmissions and they embed CSI in existing network traffic. A staleness framework is developed for quantifying the usefulness of global CSI and closed-form expressions are derived for the average staleness of two protocols with different amounts of overhead: (i) dissemination of a single channel estimate in each packet, and (ii) dissemination of all estimated CSI in each packet. Analysis shows the average staleness scales as $\mathcal{O}(N^2)$ in both cases but “all CSI dissemination” provides better average staleness except when the amount of data in each packet is small. Simulation results confirm the analysis and quantify average staleness in terms of the network parameters.

Index Terms— Wireless networks, time-varying channels, global channel state information (CSI), channel estimation, data dissemination.

1. INTRODUCTION

This paper considers the problem of tracking global channel state information in a fully-connected wireless network with reciprocal channels. It is well-known that nodes in wireless networks can exploit channel state information (CSI) to improve one or more performance characteristics of the network, e.g., data rate, interference, and/or energy efficiency. Availability of the channel state information at the transmitter (CSIT) can improve performance through techniques such as Tomlinson-Harashima precoding [1], waterfilling [2, 3], and/or adaptive transmission over fading channels [4]. CSIT is also beneficial in MIMO channels and can enable coherent MIMO transmission techniques like beamforming. CSIT can also provide multiplexing gains [5, 6] and facilitate interference mitigation, e.g., zero-forcing beamforming [7], nullforming [8], and interference alignment [9]. The effect of stale or outdated CSIT has been considered in [10–15].

While the benefits of CSIT are well-understood, there are also scenarios in which nodes in a wireless network can benefit from a more comprehensive view of the channel states in the network beyond just CSIT. These scenarios include cooperative relaying [16–20], distributed communication [21–27], and cross-layer design for multihop networks [28–30]. Typically, these scenarios require the nodes to have estimates of channels to which they are not directly

connected, e.g., the source may benefit from knowledge of the channel between the relays and the destination in a cooperative-relaying scenario. In general, a more comprehensive view of the channel states in a network allows the nodes in the network to dynamically adapt their roles, form efficient cooperative structures, and effectively use the available network resources.

This paper considers the problem of maintaining *global* CSI knowledge at each node in an N -node fully-connected wireless network with time-varying reciprocal channels. The notion of “global” here means that nodes maintain estimates of all $L = \frac{N(N-1)}{2}$ channels in the network including channels to which they are not directly connected. To inform nodes about these indirect channels, we consider protocols in which nodes disseminate channel state information by embedding one or more CSI estimates into each transmission. Since the network is fully connected, all nodes receive this disseminated channel state information and update their CSI tables accordingly. Nodes also estimate the channels to which they are directly connected through standard channel estimation techniques.

To analyze and compare the performance of channel state dissemination protocols, we first present a framework for quantifying the average staleness of the CSI across all nodes in the network. We then analyze the average CSI staleness for protocols with random transmit node selection where, in each timeslot, one random node in the network transmits a packet and, as part of this transmission, disseminates one or more CSI estimates. These protocols correspond to a scenario in which the CSI dissemination process is effectively embedded in existing network traffic. We then derive closed-form expressions for the average staleness in two regimes with different amounts of overhead: (i) dissemination of a single channel estimate in each packet, and (ii) dissemination of all estimated CSI in each packet. Our analysis shows that the average staleness scales as $\mathcal{O}(N^2)$ in both cases but that “all” CSI dissemination provides better average staleness except when the amount of data in each packet is very small. The numerical results confirm the analysis and quantify the average staleness in terms of the network parameters.

2. SYSTEM MODEL

We consider a fully-connected wireless network with N single-antenna nodes communicating over time-varying reciprocal channels. The complex channel gain between nodes i and j at time n is denoted by $h_{i,j}[n]$ and the reciprocity assumption implies $h_{i,j}[n] = h_{j,i}[n]$. Each node in the network maintains its own local table of L global CSI estimates.

Fig. 1 represents the general structure of a packet assumed to be exchanged among the nodes in the network. All packets are assumed to be received reliably. Each fixed-length packet contains overhead, data, and M channel state estimates that are *disseminated* by the

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transmitting node to all other nodes in the network. Since node k cannot estimate a channel to which it is not directly connected, i.e., the channel between nodes i and j for $i \neq j \neq k$, it uses the disseminated CSI information embedded in the transmitted packets by either node i or j to obtain an estimate of the (i, j) channel. Assuming a length of D words for the data plus overhead, each packet has a length of $P = M + D$ words. Although Fig. 1 shows a particular packet structure, the position of the overhead, data, and disseminated CSI within any packet does not affect our analysis. We assume the



Fig. 1. Example fixed-length packet showing overhead, data, and CSI dissemination. The CSI dissemination consists of $M \in \{1, \dots, N - 1\}$ channel estimates and each channel estimate has a length of one word. The data and overhead consists of D words. The total packet length is $P = M + D$ words.

network is fully-connected, which means when node i transmits a packet at any time n , all other nodes $j \neq i$ are able to receive the packet¹. Each node $j \neq i$ that receives the transmitted packet by node i does two things:

1. It directly estimates the channel $h_{i,j}[n]$, which can be obtained via a known training sequence in the packet, e.g., a known preamble embedded in the overhead, and/or through blind channel estimation techniques.
2. It extracts the disseminated CSI and uses it to update any “staler” CSI in its local table.

Each node keeps its own table of estimates of the state of all L channels in the network. The k^{th} node’s estimate of the (i, j) channel, obtained during the packet transmitted at time n is denoted by $\hat{h}_{i,j}^{(k)}[n]$. Note that in any node’s table, $N - 1$ of the CSI estimates are *directly* obtained via channel estimation in step 1 above (for $i = k$ or $j = k$), and the remaining $L - N + 1$ channel state estimates are *indirectly* obtained via disseminated CSI in step 2 above (for $i, j \neq k$). Thus, in total there are $N(N - 1) = 2L$ directly estimated parameters, and $N(L - N + 1) = L(N - 2)$ indirectly estimated parameters in the network.

The following definitions formalize the staleness metrics considered for the remainder of this paper.

Definition 1 (Staleness). *The staleness $s_{i,j}^{(k)}[n]$ of the CSI estimate $\hat{h}_{i,j}^{(k)}[n']$ at time $n \geq n'$ is $(n - n')P$ words.*

Definition 2 (Average staleness). *The average staleness S_{avg} of a protocol is defined as*

$$S_{avg} = \frac{1}{LN} \mathbb{E} \left[\sum_{i,j,k} s_{i,j}^{(k)}[n] \right]$$

where the expectation is over $n \geq \bar{n}$ for \bar{n} sufficiently large such that all nodes have complete CSI tables.

Finally, as part of our system model, we assume a random node is selected equiprobably out of the set $\{1, 2, \dots, N\}$ to transmit in each timeslot $n = 0, 1, \dots$

¹The packet from node i may have data directed to a particular node but we assume all nodes can receive the packet due to the broadcast nature of the wireless network. We also assume the disseminated CSI is not encrypted.

3. CSI DISSEMINATION PROTOCOLS

In this section, the average staleness of CSI dissemination with equiprobable random node selection is analyzed for two protocols: (i) dissemination of the single freshest channel estimate in each packet, and (ii) dissemination of all directly estimated CSI in each packet. These cases correspond to $M = 1$ and $M = N - 1$, respectively.

3.1. Nodes Disseminate Freshest Single CSI ($M = 1$)

For $n \geq 1$, the transmitting node i_n disseminates its single freshest CSI estimate. By “freshest”, we mean the channel state estimate with the least staleness. This freshest CSI estimate at time n corresponds to the (i_{n-1}, i_n) channel directly estimated by node i_n at time $n - 1$. This CSI estimate has a staleness of P words (one packet).

Theorem 1. *The average staleness of freshest CSI dissemination with equiprobable transmit node selection is equal to*

$$S_{avg} = \frac{N(N - 1)}{2} (D + 1). \quad (1)$$

Proof. Since the network is assumed to be fully connected and nodes transmit equiprobably, the staleness statistics are identical at each node in the network. Hence, we focus specifically on the staleness of channel estimates from the perspective of node i . Consider the staleness of the direct channel estimate (i, j) at node i for $j \neq i$. The staleness of this channel estimate follows

$$s_{i,j}^{(i)}[n] = \begin{cases} 0 & \text{w.p. } \frac{1}{N} \\ s_{i,j}^{(i)}[n - 1] + P & \text{w.p. } \frac{N-1}{N} \end{cases}$$

where the first case corresponds to node j transmitting at time n and the second case corresponds to any node except node j transmitting at time n . Observe that the staleness of the (i, j) channel estimate from the perspective of node i is a Markov chain with an infinite number of states. Let $q \in \{0, 1, 2, \dots\}$ be the state index corresponding to the staleness $s_{i,j}^{(i)}[n] = qP$ of the (i, j) channel estimate at node i . Denoting $\pi_q = \text{Prob}(s_{i,j}^{(i)}[n] = qP)$, the probability of state 0 can be computed as $\pi_0 = \sum_{q=0}^{\infty} \frac{1}{N} \pi_q = \frac{1}{N}$, since $\sum_{q=0}^{\infty} \pi_q = 1$ by definition of the state probabilities. The remaining state probabilities π_q for $q \in \{1, 2, 3, \dots\}$ can be straightforwardly computed from the fact that $\pi_q = \frac{N-1}{N} \pi_{q-1}$. Hence, the steady-state distribution of the staleness states of the direct channel estimate (i, j) at node i is

$$\pi_q = \frac{(N - 1)^q}{N^{q+1}}$$

for $q \in \{0, 1, 2, \dots\}$. The average staleness of the direct channel estimate (i, j) at node i follows as

$$S_{avg,direct} = \sum_{q=0}^{\infty} q \pi_q P = (N - 1)P. \quad (2)$$

Consider now the staleness of the indirectly estimated (j, k) channel at node i for $j \neq k \neq i$. Observe that the staleness of this channel estimate can only be reduced at node i if node j or node k disseminates the CSI corresponding to the (j, k) channel. This event can only occur in the freshest CSI dissemination protocol when either node j transmits immediately after node k or vice-versa. If this event occurs, the staleness of the (j, k) channel estimate at node i becomes one, otherwise the staleness of the (j, k) channel estimate

at node i increments. Fig. 2 shows a Markov chain representation of the state transitions of the staleness of the (j, k) channel estimate at node i . For notational convenience, for $\ell \in \{1, \dots, N\}$, denote

$$\pi_{q,\ell} = \text{Prob} \left(s_{j,k}^{(i)}[n] = qP, i_n = \ell \right) \quad (3)$$

$$\pi_{q,\star} = \text{Prob} \left(s_{j,k}^{(i)}[n] = qP, i_n \neq j, i_n \neq k \right). \quad (4)$$

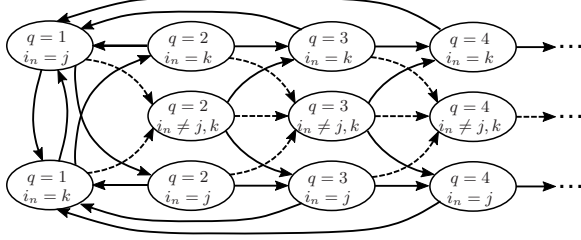


Fig. 2. A Markov chain representation of the staleness of the (j, k) channel estimate from the perspective of node i , where solid lines represent transition probability of $\frac{1}{N}$ and dashed lines represent transition probability of $\frac{N-2}{N}$. Also, q represents the state index corresponding to the staleness $s_{j,k}^{(i)}[n] = qP$ of the (j, k) channel estimate at node i and i_n represents the index of the transmitting node at time n .

Further, the steady state probability distribution of the staleness states is defined as

$$\pi_q = \sum_{\ell=1}^N \pi_{q,\ell} = 2\pi_{q,j} + \pi_{q,\star} \quad (5)$$

where the second equality results from the fact that $\pi_{q,j} = \pi_{q,k}$ for all $q \in \{1, 2, \dots\}$, which can be seen from the symmetry of the states in Fig. 2. From Fig. 2, for $q = 1$, we can write

$$\pi_{1,j} = \frac{1}{N} \sum_{q=1}^{\infty} \pi_{q,j}, \quad \pi_{1,\star} = 0 \quad (6)$$

and hence $\pi_1 = 2\pi_{1,j}$. For $q \in \{2, 3, 4, \dots\}$, we can write

$$\pi_{q,j} = \frac{1}{N} (\pi_{q-1,j} + \pi_{q-1,\star}) \quad (7)$$

$$\pi_{q,\star} = \frac{N-2}{N} (2\pi_{q-1,j} + \pi_{q-1,\star}). \quad (8)$$

Combining the linear equations (3)-(8), the final steady state probabilities can then be computed as $\pi_0 = 0$, $\pi_1 = 2\pi_{1,j} = 2/N^2$, $\pi_2 = 2\pi_{2,j} + \pi_{2,\star} = 2(N-1)/N^3$ and $\pi_q = 2\pi_{q,j} + \pi_{q,\star} = \mathbf{A}\mathbf{B}^{q-2}\mathbf{C} \quad \forall q \geq 3$, where

$$\mathbf{A} = \begin{bmatrix} \frac{1}{N^3} & \frac{2(N-2)}{N^3} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{1}{N} & \frac{2(N-2)}{N} \\ \frac{1}{N} & \frac{N-2}{N} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

The average staleness of the indirect estimate of the (j, k) channel at node i follows as

$$S_{avg,indirect} = \sum_{q=0}^{\infty} q\pi_q P = \frac{(N-1)(N+2)P}{2}. \quad (9)$$

Since the staleness statistics are identical for all nodes in the network, we can use (2) and (9) to compute the average staleness as

$$\begin{aligned} S_{avg} &= \frac{2LS_{avg,direct} + L(N-2)S_{avg,indirect}}{LN} \\ &= \frac{N(N-1)}{2}P \end{aligned}$$

where the result in (1) follows from the fact that $P = D + 1$ with single-CSI dissemination. \square

3.2. Nodes Disseminate All Directly Estimated CSI ($M = N - 1$)

For $n \geq 1$, the transmitting node i_n disseminates *all* of its directly estimated CSI ($M = N - 1$ words of CSI dissemination). Note that node i_n does not disseminate indirectly estimated CSI since the staleness of indirectly estimated CSI at node i_n is the same or worse than the staleness of these channel estimates at all other nodes in the fully-connected network.

Theorem 2. *The average staleness of all directly estimated CSI dissemination with equiprobable transmit node selection is equal to*

$$S_{avg} = \frac{3N-4}{2}(D+N-1). \quad (10)$$

Proof. Similar to the case with single freshest CSI dissemination, we consider the average staleness of the directly estimated and indirectly estimated CSI separately. The staleness of the direct estimate (i, j) at node i for $j \neq i$ is the same as the freshest CSI dissemination case since the staleness of these channels do not depend on the disseminated CSI. Hence, the average staleness of the (i, j) channel, considered from the perspective of node i , is identical to (2).

Consider now the staleness of the indirect estimates (j, k) at node i for $j \neq k \neq i$. To facilitate analysis, define the vector state $[s_{j,k}^{(i)}[n], m[n]]^T$ where $m[n]$ denotes the number of packets since either node j or node k last transmitted at time n . Under our equiprobable transmit node assumption, the vector state follows

$$[s_{j,k}^{(i)}[n], m[n]] = \begin{cases} \begin{bmatrix} s_{j,k}^{(i)}[n-1] + P \\ m[n-1] + 1 \end{bmatrix} & \text{w.p. } \frac{N-2}{N} \\ \begin{bmatrix} s_{j,k}^{(i)}[n-1] + P \\ 0 \end{bmatrix} & \text{w.p. } \frac{1}{N} \\ \begin{bmatrix} (m[n-1] + 1)P \\ 0 \end{bmatrix} & \text{w.p. } \frac{1}{N} \end{cases}$$

where the first case corresponds to neither node j nor node k transmitting at time n . The second case corresponds to node j (resp. node k) transmitting, but node j (resp. node k) was the most recent node to transmit among node j and node k . This case does not immediately reduce the staleness at node i because the disseminating node is not disseminating anything new about the (j, k) channels. The third case corresponds to node j (resp. node k) transmitting, and node k (resp. node j) was the most recent node to transmit among node j and node k . When this event occurs, the staleness of channel estimate (j, k) at node i becomes $s_{j,k}^{(i)}[n] = (m[n-1] + 1)P$ since node j or node k disseminates all of its directly estimated CSI at time n and the (j, k) channel estimate has staleness $(m[n-1] + 1)P$ at the disseminating node at time n . Fig. 3 shows a Markov chain representation of the staleness of the (j, k) channel estimate from the perspective of node i . For notational convenience, define

$$\pi_{q,m} = \text{Prob} \left(m[n] = m, s_{j,k}^{(i)}[n] = qP \right).$$

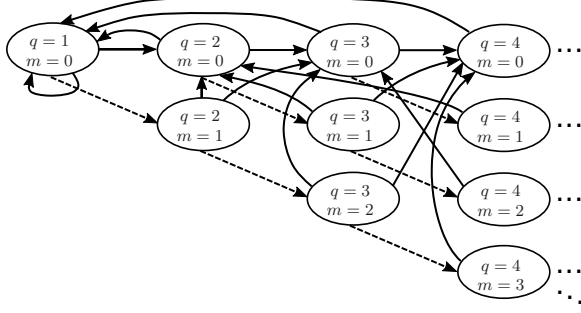


Fig. 3. A Markov chain representation of the staleness of the indirect estimate (j, k) from the perspective of node i , where solid lines represent transition probability of $\frac{1}{N}$ and dashed lines represent transition probability of $\frac{N-2}{N}$. The quantity q represents the state index corresponding to the staleness $s_{j,k}^{(i)}[n] = qP$ of the (j, k) channel estimate at node i and the quantity m represents the number of packets since either node j or node k last transmitted.

With knowledge of the transition probabilities, the steady state probability distribution of the staleness states are calculated as

$$\pi_q = \sum_{m=0}^{q-1} \pi_{q,m} = \frac{2}{N} \left\{ \left(\frac{N-1}{N} \right)^q - \left(\frac{N-2}{N} \right)^q \right\}$$

for all $q \in \{1, 2, 3, \dots\}$ with $\pi_0 = 0$. The average staleness of the (j, k) channel considered from the view of node i is obtained as

$$S_{avg,indirect} = \sum_{q=0}^{\infty} q\pi_q P = \frac{(3N-2)P}{2}. \quad (11)$$

Finally, since the staleness statistics are identical for all nodes in the network, using (2) and (11), the average staleness is computed as

$$\begin{aligned} S_{avg} &= \frac{2LS_{avg,direct} + L(N-2)S_{avg,indirect}}{LN} \\ &= \frac{(3N-4)}{2}P. \end{aligned}$$

where the result in (10) follows from the fact that $P = D + N - 1$ when nodes disseminate all directly estimated CSI. \square

4. NUMERICAL RESULTS

This section provides numerical examples to verify the analysis in the previous section and to quantify the average staleness as a function of the network parameters D and N . Figure 4 plots the average staleness of single/all CSI dissemination versus the number of nodes N for $D \in \{0, 10\}$. The $D = 0$ case can be considered a protocol with no data or overhead where each packet is dedicated solely to CSI dissemination. These results show that single CSI dissemination ($M = 1$) provides better staleness when $D = 0$ but all directly estimated CSI dissemination ($M = N - 1$) provides better staleness when $D = 10$. Figure 5 plots the average staleness versus the packet data and overhead D for $N \in \{5, 25\}$. These results show that single CSI dissemination ($M = 1$) tends to be more efficient only for very small values of D , especially in the $N = 25$ case. Intuitively, when the amount of data and overhead in each packet is large, it is more efficient to disseminate all directly estimated CSI

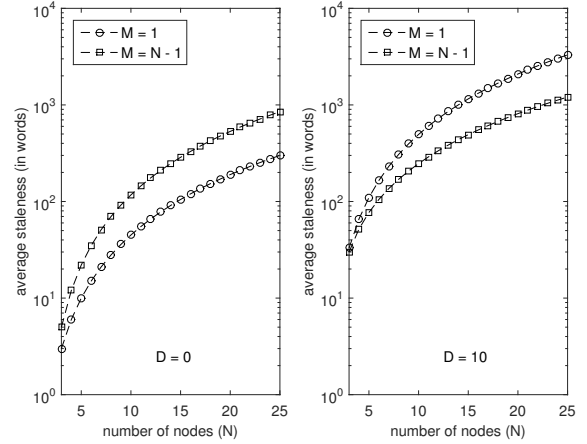


Fig. 4. Average staleness versus number of nodes N .

($M = N - 1$) since the additional incurred staleness is relatively small. In fact, in the $N = 25$ case, we see that “all” CSI dissemination provides better average staleness than single CSI dissemination for $D \geq 3$.

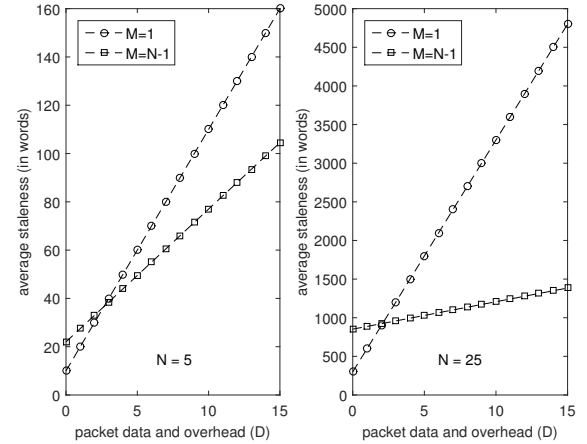


Fig. 5. Average staleness versus packet data and overhead D .

5. CONCLUSION

This paper analyzed the average staleness of global CSI for fully-connected wireless networks with packet-based transmission with random node selection and reciprocal channels. The analysis showed that the average staleness scales as $\mathcal{O}(N^2)$ but that “all” CSI dissemination provides better average staleness than “single” channel estimate dissemination except when the amount of data in each packet is very small. A potentially useful extension of this work would be to generalize the analysis to non-fully-connected networks.

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