On Linear Parallel Interference Cancellation

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Abstract — **The performance of the linear parallel interference cancellation (LPIC) receiver in a synchronous multiuser CDMA system with binary signaling is studied. We show that there exist conditions under which the LPIC receiver underperforms other receivers and characterize its asymptotic behavior.**

I. Introduction and Motivation

The linear parallel interference cancellation (LPIC) receiver has been been studied in the literature recently due to its low computational complexity and good performance under certain operating conditions. In this paper, we compare the performance of the LPIC receiver to the hard parallel interference cancellation (HPIC) and conventional matched filter (MF) receivers.

We assume the standard discrete synchronous CDMA system model [1] with K users using binary (± 1) spreading sequences of length N and binary signaling over an additive white Gaussian noise (AWGN) channel with variance σ^2. Let $\boldsymbol{R}$ be the $K \times K$ normalized spreading sequence cross-correlation matrix and $\boldsymbol{A}$ be the $K \times K$ diagonal matrix of positive real amplitudes. If the spreading sequences are chosen randomly, we resort to large system techniques [2, 3] to get analytical results. By "large system", we mean that $K \to \infty$ and $N \to \infty$ but $K/N \to \beta$, for some constant β.

In parallel interference cancellation (PIC), the desired user's decision statistic is formed by subtracting an estimate of the multiple access interference (MAI) from the original observation of the desired user. PIC lends itself to a multistage implementation in which M stages can be used to generate the final decision statistics. The HPIC receiver generates hard bit decisions at each stage to be used in subsequent stages, while the LPIC receiver passes on soft information.

The goal of this paper is to develop a better understanding of the behavior and performance of the LPIC receiver. Some authors have previously noted the performance limitations of the LPIC and others have suggested improvements. We do not propose to fix the LPIC receiver but rather to understand it better so that we can bound the operating regions where the LPIC receiver exhibits good or bad performance. In that spirit, we present a collection of related analytical results which expose the behavior of the LPIC receiver. We refer to [4] for detailed proofs.

II. Results and Conclusions

Our main results are as follows:

1. Let $\mathrm{MSE}^{(\ell)}_{\mathsf{LPIC}}$ and $\mathrm{MSE}^{(\ell)}_{\mathsf{HPIC}}$ be the mean squared error of the ℓ^{th} user's MAI estimate for the two stage LPIC and two stage HPIC receivers respectively. Let $\mathrm{AMSE}^{(\ell)}_{\mathsf{HPIC}}$ be the approximate MSE derived by using a Gaussian approximation for the MAI. Then for any $\boldsymbol{R}$, σ, $\boldsymbol{A}$, K, and ℓ, we show that $\mathrm{MSE}^{(\ell)}_{\mathsf{LPIC}} > \mathrm{AMSE}^{(\ell)}_{\mathsf{HPIC}}$.

2. Let $P^{(k)}_{\mathsf{LPIC}}(M)$ and $P^{(k)}_{\mathsf{MF}}$ be the error probabilities for the M-stage LPIC and the MF respectively for the k^{th} user. Then for any k, M, $\boldsymbol{R} \neq \boldsymbol{I}$, $\sigma > 0$, and interfering user amplitudes $a^{(\ell)}\ \forall \ell \neq k$, there exists an amplitude threshold $a^* < \infty$ such that $P^{(k)}_{\mathsf{LPIC}}(M) > P^{(k)}_{\mathsf{MF}}$ for $a^{(k)} > a^*$.

3. For any user k in a system with $K > 2$ users, odd M, equal amplitude users such that $\boldsymbol{A} = a\boldsymbol{I}$ and $\frac{a}{\sigma} > 0$, there exists $\boldsymbol{R}$ such that $P^{(k)}_{\mathsf{LPIC}}(M) > 0.5$. We say that the k^{th} user *suffers*.

4. Consider the behavior of the LPIC for large M. Let $\rho(\boldsymbol{R})$ be the spectral radius of $\boldsymbol{R}$, i.e., the maximum magnitude of all eigenvalues of $\boldsymbol{R}$. It is well known that if $\rho(\boldsymbol{R}) < 2$, the LPIC converges to the decorrelating detector. Our result is as follows. If $\rho(\boldsymbol{R}) > 2$, there exists M^* and *at least one* k such that $P^{(k)}_{\mathsf{LPIC}}(M) > 0.5$ for all odd integer values of $M \geq M^*$.

5. An extra constraint on $\boldsymbol{R}$ allows us to show that all users can suffer. Suppose $\rho(\boldsymbol{R}) > 2$ and $\boldsymbol{R}$ has an eigenvector, associated with an eigenvalue greater than two, with all nonzero entries. Then there exists M^* such that *for all* k, $P^{(k)}_{\mathsf{LPIC}}(M) > 0.5$ for all odd integer values of $M \geq M^*$.

6. For randomly chosen spreading sequences and large systems, we show that for any σ, $\boldsymbol{A}$, and ℓ, $E[\mathrm{MSE}^{(\ell)}_{\mathsf{LPIC}}] > E[\mathrm{MSE}^{(\ell)}_{\mathsf{HPIC}}]$. Note that, unlike Result 1, we need not rely upon the Gaussian approximation for the MAI.

7. For randomly chosen spreading sequences and large systems, we show that if $\beta = K/N > (\sqrt{2}-1)^2 \approx 0.17$, then $\rho(\boldsymbol{R}) > 2$ almost surely. Result 4 then indicates that at least one user will suffer in each bit interval for large odd M. More precisely, we note that the misperforming user may be different for each realization of $\boldsymbol{R}$, and so no one user need suffer on average. Numerical experiments suggest that, on average, all users may indeed suffer as $M \to \infty$, but a proof of this conjecture is left as an open problem.

The results in this paper, which indicate that the LPIC has the potential to misperform, are intended to fill in some of the gaps in our understanding of PIC receivers. We hope they can serve as cautionary guidelines concerning the application of LPIC receivers to CDMA communication systems.

References

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