Characterizing the Effect of Channel Estimation Error on Distributed Reception with Hard Decision Exchanges

Sabah Razavi and D. Richard Brown III
Department of Electrical and Computer Engineering
Worcester Polytechnic Institute
100 Institute Rd, Worcester, MA 01609
Email: {srazavi,drb@wpi.edu}

***Abstract*—This paper considers a distributed wireless reception system with M-ary phase shift keyed (M-PSK) modulated signals in which two or more receivers exchange quantized information about their observations to approximate a receive beamformer and improve the signal to noise ratio with respect to single-user reception. To reduce the throughput requirements of distributed reception, previous work on this problem considered the exchange of hard decisions among the nodes in the receive cluster with a simple form of linear combining called "pseudo-beamforming". The previous work also assumed that the nodes in the receive cluster had perfect channel estimates. This paper generalizes the previous work by analyzing the performance of pseudo-beamforming with imperfect channel estimates. While channel estimation error degrades the performance of both ideal receive beamforming (no quantization) and pseudo-beamforming, the asymptotic analysis in this paper reveals the somewhat surprising result that the SNR ratio between ideal receive beamforming and pseudo-beamforming does not depend on the amount of channel estimation error. The SNR ratio with channel estimation error is identical to the previously derived SNR ratio without channel estimation error. Numerical examples with a finite number of receivers are also presented to confirm the analysis.**

***Index Terms*—Distributed reception, receiver cooperation, beamforming, channel estimation error, asymptotic analysis.**

I. Introduction

This paper considers a distributed reception system as illustrated in Fig. 1. A distant transmitter emits digitally modulated signals which are received by the N nodes in the receive cluster. Quantized versions of the received signals at each node in the receive cluster are then exchanged among the nodes through a wireless local area network and are subsequently processed and combined at one or more nodes in the receive cluster to increase the diversity and signal to noise ratio [1]–[3]. While distributed reception has been used in a variety of contexts, e.g., aperture synthesis for radio astronomy and sensor fusion, the focus of this paper is on the use of distributed reception for improving the reception of digitally modulated signals.

A difficulty in practically implementing distributed reception systems, even with a modest number of receive nodes, is that the throughput requirements of the local area network (LAN) used by the receive cluster to exchange quantized versions of the received signal can be prohibitive. For example, suppose that the distant transmitter employs 8-PSK modulation with an information rate of 1 Mbit/s and a rate $r = 2/3$ code. The symbols are received at each receiver at a rate of 500 Ksymbols/s. For "ideal" receive beamforming, assuming $N = 10$ receivers and 16-bit quantization of the in-phase and quadrature observations, the LAN would need to support a throughput of at least $500 \cdot 10^3 \times 16 \times 2 \times 10 = 160$ Mb/s, not including overhead. Motivated by this difficulty, [1]–[3] considered the exchange of hard decisions among the nodes in the receive cluster. Under the same parameters as the previous example, the required LAN throughput for exchanging hard decisions is only $500 \cdot 10^3 \times 3 \times 10 = 15$ Mb/s.

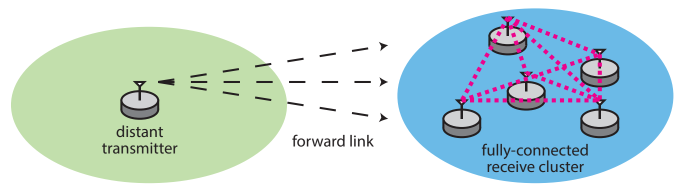

Fig. 1. Distributed reception scenario.

It has been shown that this dramatic reduction in LAN/backhaul throughput requirements can be achieved without a significant loss of performance. For the low per-node SNR regime of interest with large receive clusters, asymptotic analysis of a suboptimal combining technique termed "pseudo-beamforming" showed that distributed reception with hard decision exchanges performs within 1-2 dB of ideal receive beamforming, depending on the digital modulation format [3].

A critical assumption in all of the past work is that the receive cluster has perfect channel estimates to facilitate combining of the hard decisions. This assumption is clearly optimistic, especially in the low per-node SNR regime of interest with large receive clusters. It is of interest to understand the effect of channel estimation error on distributed reception. The effect of channel estimation error on distributed reception systems with hard decision exchanges and M-ary phase shift

This work was supported by National Science Foundation awards CCF-1302104 and CCF-1319458.

keyed modulation is particularly interesting since the channel estimation errors have two effects: (i) channel *phase* errors cause increased likelihood of hard decision errors and (ii) channel *magnitude* errors cause combining errors. This is the problem considered in this paper.

The effect of channel estimation error on digital communication systems is a classic problem [4]–[6], and continues to be considered in a variety of contexts. For example, [7] investigates the effect of imperfect channel estimation on the bit-error-rate (BER) performance of amplify-and-forward (AF) relay-assisted cooperative transmission. Related to the M-PSK focus of this paper, several prior papers have investigated the effect of imperfect carrier phase recovery on the performance of M-PSK [8]–[16]. Since our focus is on distributed reception and on combining demodulated and quantized M-PSK signals, our analysis accounts for the effects of *phase and magnitude* error in the channel estimates at the receivers.

The main contribution of this paper is an asymptotic analysis of the effect of imperfect channel estimation in a distributed reception system with M-ary PSK modulation. Our analysis provides closed-form SNR expressions for both ideal receive beamforming and a suboptimal hard-decision combining technique termed "pseudo-beamforming". Our analysis shows the expected effect that distributed reception performance is degraded by channel estimation error, but also reveals the somewhat surprising result that the SNR ratio between ideal receive beamforming and pseudo-beamforming does not depend on the amount of channel estimation error. Specifically, for QPSK, the SNR ratio is $\frac{2}{\pi}$, corresponding to a loss of approximately 2 dB. For M-PSK with $M \to \infty$, the SNR ratio is $\frac{\pi}{4}$, corresponding to a loss of approximately 1 dB. Since these SNR ratios are independent of the amount of channel estimation error (and are identical to the SNR ratios with no channel estimation error), our analysis reveals that channel estimation error causes the identical amounts of performance degradation in ideal beamforming and pseudo-beamforming systems despite the fact that the channel estimation errors manifests themselves quite differently in both systems.

The rest of the paper is organized as follows. Section II describes the system model for the proposed scenario. In section III we derive the channel estimation at the receiver. Section IV describes our asymptotic SNR analysis for each technique with and without channel estimation error. In section V the numerical results from simulation is presented and section VI is about the conclusions. The proofs for the lemma 1 and corollary 1 are given in Appendix A and B.

II. System Model

We assume a block transmission scenario with blocks of length n as in [3] and let N denote the number of receive nodes in the cluster. The complex forward link channel to receive node i in block m is denoted as $h_i[m]$ for $i = 1, \dots, N$ and the vector channel for block m is denoted as $\boldsymbol{h}[m] = [h_1[m], \dots, h_N[m]]^\top$. Over each block, the forward link channels are assumed to be constant but may change block to block.

For clarity of exposition and to explore the effects of channel phase and magnitude errors on distributed reception, we assume M-PSK modulation in the forward link. The ℓ^{th} symbol in block m is denoted as $X[m,\ell]$ for $\ell = 1, \dots, n$ and is assumed to be drawn equiprobably from the PSK alphabet $\mathcal{X} = \{x_1, \cdots, x_M\} = \left\{a, ae^{j2\pi/M}, ae^{j4\pi/M}, \cdots, ae^{j(M-1)2\pi/M}\right\}$. The energy per transmitted signal is denoted as $\mathcal{E}_s = |X[m,\ell]|^2 = a^2$. Given an additive white Gaussian noise channel (AWGN) with power spectral density $N_0/2$ in the real and imaginary dimensions, the complex baseband signal received at the i^{th} receive node for the ℓ^{th} symbol of block m can be written as

$$U_i[m,\ell] = h_i[m]X[m,\ell] + W_i[m,\ell] \tag{1}$$

for $i = 1, \dots, N$ and $\ell = 1, \dots, n$ where $W_i[m,\ell] \sim \mathcal{CN}(0, N_0)$ is spatially and temporally independent and identically distributed (i.i.d.) proper complex Gaussian baseband noise. We assume the noise variance is identical at each receive node. The quantity $\rho_i[m] = \frac{|h_i[m]|^2 \mathcal{E}_s}{N_0}$ corresponds to the signal-to-noise ratio (SNR) at receive node i for symbols received in block m.

To facilitate distributed reception, it is assumed that the receive cluster has an established LAN backhaul, either adhoc or through infrastructure such as an access point, and that LAN transmissions are reliable. The LAN is also assumed to support broadcast transmission in which any single node can send a message to all other nodes simultaneously. To prevent any interruption in transmission over forward link, it is assumed that LAN and forward link operating frequencies differ from each other which enables the receive cluster to send and receive over the LAN, and also receive signals from transmitter at the same time. The LAN is also assumed to support a sufficient throughput for the exchange of hard decisions among all nodes in the receive cluster.

III. Channel Estimation

Unlike the prior work in [1]–[3], we do not assume $h_i[m]$ is known perfectly here. To facilitate estimation of $h_i[m]$ at receiver i, we assume some of the symbols in each transmitted block are known. Suppose $X[m,1], \dots, X[m,P]$ are known, where $P \leq n$. Then node i can estimate $h_i[m]$ by computing a least squares solution to

$$\begin{bmatrix} U_i[m,1] \\ \vdots \\ U_i[m,P] \end{bmatrix} = \begin{bmatrix} X[m,1] \\ \vdots \\ X[m,P] \end{bmatrix} h_i[m] \tag{2}$$

$$\boldsymbol{U}_i[m] = \boldsymbol{X}[m] h_i[m] \tag{3}$$

such that

$$\hat{h}_i[m] = \frac{\boldsymbol{X}^H[m]\boldsymbol{U}_i[m]}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} \tag{4}$$

Substituting $\boldsymbol{U}_i[m] = h_i[m]\boldsymbol{X}[m,\ell] + \boldsymbol{W}_i[m]$, we can write

$$\begin{aligned}\hat{h}_i[m] &= \frac{h_i[m]\boldsymbol{X}^H[m]\boldsymbol{X}[m] + \boldsymbol{X}^H[m]\boldsymbol{W}_i[m]}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} \\ &= h_i[m] + \frac{\boldsymbol{X}^H[m]\boldsymbol{W}_i[m]}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} \\ &= h_i[m] + \tilde{h}_i[m] \end{aligned} \tag{5}$$

where $\tilde{h}_i[m] \sim \mathcal{CN}(0, 2\rho)$ is a proper complex Gaussian random variable with variance ρ in the real and imaginary dimensions. Since the training sequence $\boldsymbol{X}[m]$ is known, we can determine 2ρ by computing

$$\begin{aligned}\mathrm{var}(\tilde{h}_i[m]) &= \mathrm{E}\left\{ \left| \frac{\boldsymbol{X}^H[m]\boldsymbol{W}_i[m,\ell]}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} \right|^2 \mid \boldsymbol{X}[m] \right\} \\ &= \frac{\boldsymbol{X}^H[m]}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} (\boldsymbol{I}N_0) \frac{\boldsymbol{X}[m]}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} \\ &= \frac{N_0}{\boldsymbol{X}^H[m]\boldsymbol{X}[m]} \\ &= \frac{N_0}{P\mathcal{E}_s} \end{aligned} \tag{6}$$

where the last result follows from our M-PSK assumption and the fact that the length of $\boldsymbol{X}[m]$ is P.

IV. Asymptotic SNR Analysis

In this section, we consider the case where $N \to \infty$ and the per-node SNR goes to zero at a rate of $\frac{1}{N}$ so that the SNR of an ideal receive beamformer combiner is finite and bounded away from zero. We can suppress the block/symbol indices and consider the scalar observation at receive node i as

$$U_i = h_i X + W_i \tag{7}$$

where X is drawn from an M-PSK constellation with $|X|^2 = \mathcal{E}_s$. For our asymptotic analysis, we will assume signal energy $\mathcal{E}_s = \mathcal{E}_s^{(1)}/N$, i.e., the transmit power scales as $1/N$, where $\mathcal{E}_s^{(1)}$ is the per-symbol transmit energy with one receiver. We also assume $P = NP^{(1)}$, i.e., the training signal length scales with N, where $P^{(1)}$ is the training signal length with one receiver. Under this assumption, note that $P\mathcal{E}_s$ is a constant. Since N_0 is also fixed, the variance of the channel estimation errors is constant.

The following subsections analyze the performance of ideal distributed receive beamforming and a suboptimal combining technique called "pseudo-beamforming" with and without channel estimation error.

A. Ideal Receive Beamforming: Perfect Channel Estimation

The output of ideal receive beamformer at node i is realized by using unquantized observations U_j and is defined as

$$Y_{bf} \equiv Y_i = \sum_{j\in\mathcal{P}} \sqrt{\rho_i} U_j = \alpha \sum_{j\in\mathcal{P}} |h_j| U_j \tag{8}$$

where $\rho_i = \frac{|h_i|^2\mathcal{E}_s}{N_0}$ and $\alpha = \sqrt{\frac{\mathcal{E}_s}{N_0}}$.

For the ideal receive beamformer, we have the vector observation

$$\boldsymbol{U} = \boldsymbol{h}X + \boldsymbol{W}. \tag{9}$$

Assuming no channel estimation error, the ideal receive beamformer output is given as

$$Y_{bf} = \boldsymbol{h}^H\boldsymbol{U} = \boldsymbol{h}^H\boldsymbol{h}X + \boldsymbol{h}^H\boldsymbol{W}. \tag{10}$$

The SNR of ideal receive beamforming (conditioned on the channel realizations) can be computed as

$$\begin{aligned}\mathsf{SNR}_{bf} &= \frac{\left(\mathrm{E}\left\{\boldsymbol{h}^H\boldsymbol{h}X + \boldsymbol{h}^H\boldsymbol{W} \mid X\right\}\right)^2}{\mathrm{var}\left\{\boldsymbol{h}^H\boldsymbol{h}X + \boldsymbol{h}^H\boldsymbol{W} \mid X\right\}} \\ &= \frac{\|\boldsymbol{h}\|^4\mathcal{E}_s}{\boldsymbol{h}^H\mathrm{E}\{\boldsymbol{W}\boldsymbol{W}^H\}\boldsymbol{h}} \\ &= \frac{\|\boldsymbol{h}\|^2\mathcal{E}_s}{N_0}. \end{aligned} \tag{11}$$

If we further assume an i.i.d. Rayleigh fading channel such that $h_i \sim \mathcal{CN}(0, 2\lambda)$, then asymptotically we have $\lim_{N\to\infty} \frac{\|\boldsymbol{h}\|^2}{N} = 2\lambda$. The asymptotic SNR is then

$$\mathsf{SNR}_{bf} \to \frac{2N\lambda\mathcal{E}_s}{N_0} = \frac{2\lambda\mathcal{E}_s^{(1)}}{N_0}. \tag{12}$$

B. Ideal Receive Beamformer: Noisy Channel Estimation

Now we consider ideal receive beamforming with channel estimates of the form

$$\hat{\boldsymbol{h}} = \boldsymbol{h} + \tilde{\boldsymbol{h}} \tag{13}$$

where $\tilde{\boldsymbol{h}} \sim \mathcal{CN}(0, 2\rho\boldsymbol{I})$. The ideal receive beamformer output with channel estimation error is given as

$$\begin{aligned} Y_{bfe} &= \hat{\boldsymbol{h}}^H\boldsymbol{U} = \hat{\boldsymbol{h}}^H(\boldsymbol{h}X + \boldsymbol{W}) \\ &= \left(\boldsymbol{h} + \tilde{\boldsymbol{h}}\right)^H(\boldsymbol{h}X + \boldsymbol{W}) \\ &= \boldsymbol{h}^H(\boldsymbol{h}X + \boldsymbol{W}) + \tilde{\boldsymbol{h}}^H(\boldsymbol{h}X + \boldsymbol{W}) \\ &= Y_{bf} + \tilde{Y}_{bf}. \end{aligned} \tag{14}$$

Then, the SNR of ideal receive beamforming with channel estimation error (conditioned on the channel realizations) can be computed as

$$\mathsf{SNR}_{bfe} = \frac{\left(\mathrm{E}\left\{Y_{bf} + \tilde{\boldsymbol{h}}^H(\boldsymbol{h}X + \boldsymbol{W}) \mid X\right\}\right)^2}{\mathrm{var}\left\{Y_{bf} + \tilde{\boldsymbol{h}}^H(\boldsymbol{h}X + \boldsymbol{W}) \mid X\right\}}. \tag{15}$$

Note that $\tilde{\boldsymbol{h}}$ is independent of $\boldsymbol{h}$ and X. Since the channel estimates were generated from different observations than the ones used in the SNR calculations, $\tilde{\boldsymbol{h}}$ is also independent of $\boldsymbol{W}$. Hence,

$$\begin{aligned}\mathrm{E}\left\{Y_{bf} + \tilde{\boldsymbol{h}}^H(\boldsymbol{h}X + \boldsymbol{W}) \mid X\right\} &= \mathrm{E}\left\{Y_{bf} \mid X\right\} \\ &= \|\boldsymbol{h}\|^2\sqrt{\mathcal{E}_s} \end{aligned} \tag{16}$$

and the numerator of this expression is unchanged from the case with no channel estimation error. As for the denominator, since $Y_{\rm bf}$ and $\tilde{Y}_{\rm bf}$ are independent, we have

$$\begin{aligned}\text{var}\left\{Y_{bf}+\tilde{Y}_{bf}\,|\,X\right\} &= \text{var}\left\{Y_{bf}\,|\,X\right\} \\ &+ \text{var}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\,|\,X\right\} \\ &= \|\boldsymbol{h}\|^2 N_0 + \text{var}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\,|\,X\right\} \end{aligned} \tag{17}$$

We can compute the second term as

$$\begin{aligned}\text{var}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\,|\,X\right\} &= \text{E}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\right. \\ &\times \left.(\boldsymbol{h}X+\boldsymbol{W})^H\tilde{\boldsymbol{h}}\,|\,X\right\} - \left|\text{E}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\,|\,X\right\}\right|^2 \\ &= \text{E}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\times(\boldsymbol{h}X+\boldsymbol{W})^H\tilde{\boldsymbol{h}}\,|\,X\right\} \end{aligned} \tag{18}$$

where the second equality follows from the fact that $\tilde{\boldsymbol{h}}$ is zero mean and independent of the other terms in the expectation. We can further compute

$$\begin{aligned}\text{var}\left\{\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\,|\,X\right\} &= \mathcal{E}_s\text{E}\left\{\tilde{\boldsymbol{h}}^H\boldsymbol{h}\boldsymbol{h}^H\tilde{\boldsymbol{h}}\,|\,X\right\} \\ &+ \text{E}\left\{\tilde{\boldsymbol{h}}^H\boldsymbol{W}\boldsymbol{W}^H\tilde{\boldsymbol{h}}\,|\,X\right\} \\ &= \mathcal{E}_s\boldsymbol{h}^H\text{E}\left\{\tilde{\boldsymbol{h}}\tilde{\boldsymbol{h}}^H\,|\,X\right\}\boldsymbol{h} + \text{E}\left\{\tilde{\boldsymbol{h}}^H\boldsymbol{W}\boldsymbol{W}^H\tilde{\boldsymbol{h}}\,|\,X\right\} \\ &= \mathcal{E}_s\|\boldsymbol{h}\|^2 2\rho + \text{E}\left\{\tilde{\boldsymbol{h}}^H\boldsymbol{W}\boldsymbol{W}^H\tilde{\boldsymbol{h}}\,|\,X\right\} \\ &= \frac{\|\boldsymbol{h}\|^2 N_0}{P} + \text{E}\left\{\tilde{\boldsymbol{h}}^H\boldsymbol{W}\boldsymbol{W}^H\tilde{\boldsymbol{h}}\,|\,X\right\} \end{aligned} \tag{19}$$

The final expectation can be solved with iterated expectations since $\tilde{\boldsymbol{h}}$ and $\boldsymbol{W}$ are independent. We can write

$$\begin{aligned}\text{E}\left\{\tilde{\boldsymbol{h}}^H\boldsymbol{W}\boldsymbol{W}^H\tilde{\boldsymbol{h}}\,|\,X\right\} &= \text{E}\left\{\tilde{\boldsymbol{h}}^H\text{E}\left\{\boldsymbol{W}\boldsymbol{W}^H\,|\,X,\tilde{\boldsymbol{h}}\right\}\tilde{\boldsymbol{h}}\,|\,X\right\} \\ &= \text{E}\left\{\tilde{\boldsymbol{h}}^H(N_0\boldsymbol{I})\tilde{\boldsymbol{h}}\,|\,X\right\} \\ &= N_0\text{E}\left\{\tilde{\boldsymbol{h}}^H\tilde{\boldsymbol{h}}\,|\,X\right\} = N_0 N 2\rho \\ &= \frac{N_0^2 N}{P\mathcal{E}_s}. \end{aligned} \tag{20}$$

Putting it all together, we have

$$\begin{aligned}&\text{var}\left\{Y_{bf}+\tilde{\boldsymbol{h}}^H(\boldsymbol{h}X+\boldsymbol{W})\,|\,X\right\} = \\ &\|\boldsymbol{h}\|^2 N_0 + \frac{\|\boldsymbol{h}\|^2 N_0}{P} + \frac{N_0^2 N}{P\mathcal{E}_s}. \end{aligned} \tag{21}$$

and hence

$$\mathsf{SNR}_{bfe} = \frac{\|\boldsymbol{h}\|^2\mathcal{E}_s}{N_0 + \frac{N_0}{P} + \frac{N_0^2 N}{\|\boldsymbol{h}\|^2 P\mathcal{E}_s}}. \tag{22}$$

Asymptotically, since P grows proportionally with N and $P\mathcal{E}_s$ is fixed, the middle term in the denominator vanishes. So for large N with vanishing per-node SNR we can write

$$\mathsf{SNR}_{bfe} \to \frac{\|\boldsymbol{h}\|^2\mathcal{E}_s}{N_0 + \frac{N_0^2 N}{\|\boldsymbol{h}\|^2 P\mathcal{E}_s}}. \tag{23}$$

Moreover, since $\lim_{N\to\infty}\frac{\|\boldsymbol{h}\|^2}{N} = 2\lambda$, $\mathcal{E}_s = \frac{\mathcal{E}_s^{(1)}}{N}$, and $P = NP^{(1)}$, it can be easily obtained that

$$\mathsf{SNR}_{bfe} \to \frac{2\lambda\mathcal{E}_s^{(1)}}{N_0\left(1+\frac{N_0}{2\lambda P^{(1)}\mathcal{E}_s^{(1)}}\right)}. \tag{24}$$

The results in (11) and (23) allow us to compute the penalty of channel estimation error in an ideal receive beamformer as $N\to\infty$ as

$$\mathcal{P}_{bf} = \frac{\mathsf{SNR}_{bf}}{\mathsf{SNR}_{bfe}} \to 1 + \frac{N_0}{2\lambda P^{(1)}\mathcal{E}_s^{(1)}}. \tag{25}$$

C. Pseudo-beamforming: Perfect Channel Estimation

Pseudo-beamforming is a simple but sub-optimal combining technique where (8) is performed on the *hard decisions* from each node. Specifically, the pseudo-beamformer combiner output is

$$Y_{pbf} \equiv Y_i = \sum_{j\in\mathcal{P}}\sqrt{\rho_i}V_j = \alpha\sum_{j\in\mathcal{P}}|h_j|V_j \tag{26}$$

where $V_j \in \mathcal{X}$ for all j and are conditionally independent given the transmitted symbol.

The asymptotic SNR of pseudo-beamforming for various modulation formats was analyzed in [3]. The main results are summarized here. First, the conditional mean of M-PSK hard decisions can be calculated as

$$\text{E}[V_j|X=x_l] = \left(\frac{M\rho_j\sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi}}\right)x_l. \tag{27}$$

Second, the conditional variance of with M-PSK hard decisions in the low per-node SNR regime can be calculated as

$$\text{var}[V_j|X=x_l] \approx a^2. \tag{28}$$

These results allow us to compute the conditional mean and variance of the pseudo-beamformer output with M-PSK hard decisions. The conditional mean can be computed as

$$\text{E}[Y_{pbf}|X=x_l] = \alpha\frac{aM\sin\left(\frac{\pi}{M}\right)}{2\sqrt{N_0\pi}}||\boldsymbol{h}||^2 x_l. \tag{29}$$

Similarly, the conditional variance of the pseudo-beamformer output with M-PSK hard decisions in the low per-node SNR regime can be computed as

$$\text{var}[Y_{pbf}|X=x_l] = \alpha^2 a^2||\boldsymbol{h}||^2 \tag{30}$$

where we used the facts that $\rho_j = \frac{|h_j|a}{\sqrt{N_0}}$ and $\sum_j|h_j|^2 = ||\boldsymbol{h}||^2$. These results then imply

$$\begin{aligned}\mathsf{SNR}_{pbf} &= \frac{\text{E}[Y_{pbf}|X=x_l]^2}{\text{var}[Y_{pbf}|X=x_l]} \\ &= \frac{M^2\sin^2\left(\frac{\pi}{M}\right)||\boldsymbol{h}||^2\mathcal{E}_s}{4N_0\pi} \end{aligned} \tag{31}$$

$$= \frac{M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi}\mathsf{SNR}_{bf}. \tag{32}$$

With QPSK, we have $M=4$ and $\frac{M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi}=\frac{2}{\pi}$. This then implies

$$\mathsf{SNR}_{pbf}^{\text{QPSK}} \approx \frac{2}{\pi}\mathsf{SNR}_{bf}. \tag{33}$$

For large M, we can use small angle approximation which means we can say $\sin\left(\frac{\pi}{M}\right)=\frac{\pi}{M}$ and it results in $\frac{M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi}\to\frac{\pi}{4}$. Hence

$$\lim_{M\to\infty}\mathsf{SNR}_{pbf}^{M-\text{PSK}} \approx \frac{\pi}{4}\mathsf{SNR}_{bf}. \tag{34}$$

D. Pseudo-beamforming: Noisy Channel Estimation

The effect of channel estimation error on pseudo-beamforming has two effects: (i) channel *phase* errors cause increased likelihood of hard decision errors and (ii) channel *magnitude* errors cause combining errors. To model the effect of channel estimation error on the decision variable at an individual receiver, we first define the perfect and noisy channel estimate, respectively, as

$$h_j = |h_j|e^{j\theta} \tag{35}$$

$$\hat{h}_j = |\hat{h}_j|e^{j\hat{\theta}}. \tag{36}$$

Lemma 1 provides expressions for the conditional mean and variance of hard decisions at an individual receiver with low per-node SNR in presence of channel estimation error.

Lemma 1. *For a forward link with $M-PSK$ modulation with $M\geq 4$ and even, at low per-node SNR we have*

$$\mathrm{E}[V_j|X=x_l] = \left(\frac{M\rho_j|h|\sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi}\mathrm{E}[|\hat{h}|]}\right)x_l \tag{37}$$

and the variance is

$$\mathrm{var}[V_j|X=x_l] \approx a^2. \tag{38}$$

A proof for this lemma is given in Appendix A. As it can be seen from the proof, the calculation of mean and variance of hard decisions do not depend on any specific phase error distribution. In a low per-node SNR regime and for a large N, since ρ_j becomes very small, it is expected that the mean goes to zero. Also, the variance given in (38) serves as an upper bound for the variance of hard decisions as N gets large.

The next step is to find the mean and variance of the pseudo-beamformer output in order to be able to calculate its SNR performance. The pseudo-beamformer output with imperfect channel estimates is given as

$$Y_{pbfe} = \alpha\sum_{j\in N}|\hat{h}_j|V_j. \tag{39}$$

Corollary 1 uses the results obtained from lemma 1 to provide expressions for the conditional mean and variance of the pseudo-beamformer output.

Corollary 1. *Given the channel information h_j and input symbol $X=x_l$, the mean and variance of pseudo-beamformer output can be computed as*

$$\mathrm{E}[Y_{pbfe}|X=x_l] = \alpha\frac{aM\sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi N_0}}||\boldsymbol{h}||^2x_l \tag{40}$$

and the variance is

$$\mathrm{var}[Y_{pbfe}|X=x_l] = \alpha^2a^2\left(||\boldsymbol{h}||^2+\frac{N_0N}{P\mathcal{E}_s}\right) \tag{41}$$

A proof for this corollary is provided in Appendix B. With the results of Corollary 1, we now can compute the SNR of pseudo-beamforming with channel estimation error as

$$\begin{aligned}\mathsf{SNR}_{pbfe} &= \frac{|\mathrm{E}[Y_{pbfe}|X=x_l]|^2}{\mathrm{var}[Y_{pbfe}|X=x_l]}\\ &= \frac{\alpha^2\frac{a^2M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi N_0}||\boldsymbol{h}||^4x_l^2}{\alpha^2a^2\left(||\boldsymbol{h}||^2+\frac{N_0N}{P\mathcal{E}_s}\right)}\\ &= \frac{M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi}\frac{||\boldsymbol{h}||^2\mathcal{E}_s}{N_0+\frac{N_0^2N}{||\boldsymbol{h}||^2P\mathcal{E}_s}} \quad (42)\\ &\to \frac{M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi}\frac{2\lambda\mathcal{E}_s^{(1)}}{N_0\left(1+\frac{N_0}{2\lambda P^{(1)}\mathcal{E}_s^{(1)}}\right)} \quad (43)\end{aligned}$$

where the final result assumes $N\to\infty$ with correspondingly vanishing per-node SNR. In light of (24), we can write

$$\mathsf{SNR}_{pbfe} = \frac{M^2\sin^2\left(\frac{\pi}{M}\right)}{4\pi}\mathsf{SNR}_{bfe}. \tag{44}$$

Hence, the SNR gaps between pseudo-beamforming with channel estimation error and ideal receive beamforming with channel estimation error are identical to the cases without channel estimation error.

Using (31) and (43), the SNR penalty of channel estimation error with pseudo-beamforming can be expressed as

$$\mathcal{P}_{pbf} = \frac{\mathsf{SNR}_{pbf}}{\mathsf{SNR}_{pbfe}} = 1+\frac{N_0}{2\lambda P^{(1)}\mathcal{E}_s^{(1)}} \tag{45}$$

which is identical to (25). This shows the somewhat surprising result that the SNR ratio between ideal receive beamforming and pseudo-beamforming does not depend on the amount of channel estimation error. In other words, the SNR ratio with channel estimation error is identical to the SNR ratio without channel estimation error, as derived in [3].

V. Numerical Results

In this section the results from the simulation are presented. In this simulation a QPSK modulation is chosen for the forward link between single transmitter and the receive cluster. The number of receive nodes inside the cluster are $N=[10\ 20\ 40\ 80\ 160\ 320\ 640\ 1280\ 2560\ 5120\ 7680]$. The number of iterations for each channel/noise realization is chosen to be 1000 and the per-symbol transmit energy with one receiver $\mathcal{E}_s^{(1)}=10$ and training signal length with one receiver $P^{(1)}=1$. The magnitude of each symbol is $a=\sqrt{2}$ and the number of payload symbols per block is $Q=100$. The total noise power $N_0=5$ and channel variance in real and imaginary dimension is $\lambda=2$.

Fig. 2 shows the comparison of the SNRs between ideal receive beamforming and pseudo-beamforming each with and without channel estimation error.

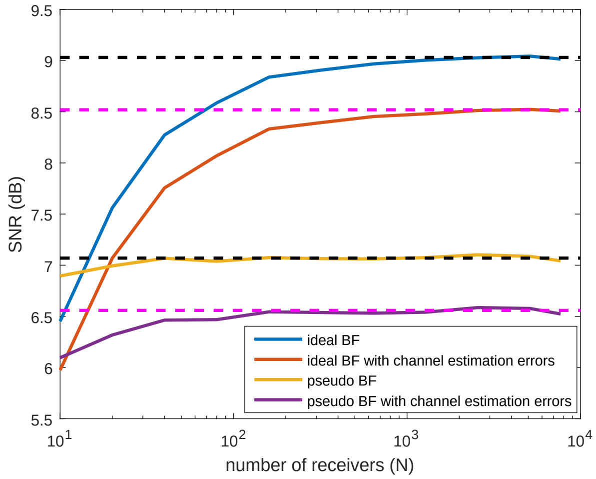

Fig. 2. Comparison of the SNRs between ideal receive beamforming and pseudo-beamforming with and without channel estimation error. The dotted lines are the calculated SNRs for large N in each scenario.

The results from Fig. 2 confirms our proofs that the ratio of the SNRs between ideal receive beamforming and pseudo-beamforming in both case of perfect and noisy channel estimation are equal to $\frac{2}{\pi}$ and the SNR in each case converges to the calculated limit for large N.

Fig. 3 shows the comparison of the penalties between the ideal and pseudo beamformer. It can be seen that, the penalty term in both cases converges to the same number since the SNR ratios in each case, as shown in equation (25) for an ideal beamformer and (45) for a pseudo beamformer are the same.

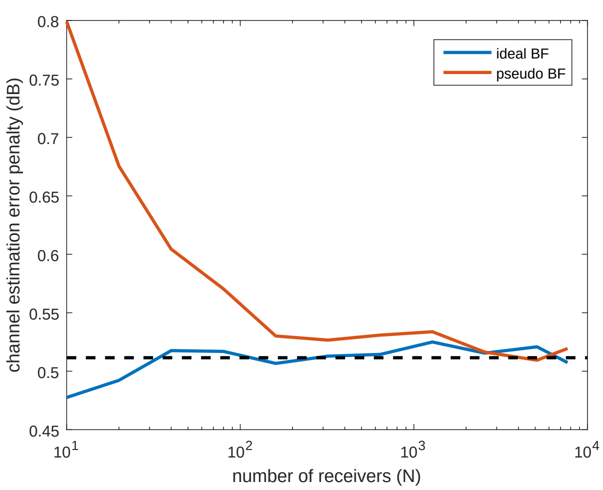

Fig. 3. Comparison of the penalties between the ideal and pseudo beamformer.

Fig. 4 shows the mean and variance of the hard decisions when there is channel estimation error. The results from the figure show that, the calculated mean of the hard decisions closely follows the numerical results. Also, the variance of the hard decisions approaches to the upper bound obtained from the theoretical results.

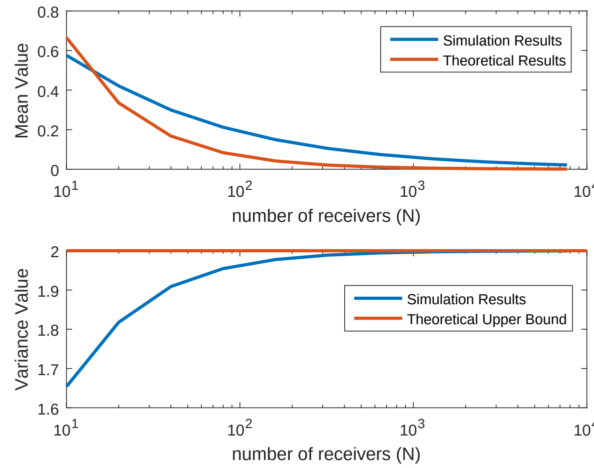

Fig. 4. Mean and variance of the hard decisions when there is channel estimation error.

VI. Conclusion

In this paper we used theoretical calculations, asymptotic analysis and numerical results from simulation, to obtain and characterize the effect of imperfect channel estimation in a distributed reception system with M-PSK modulation. As mentioned in the paper, channel estimation error had two effects, channel phase error and channel magnitude error, which our analysis had accounted for both of these effects in the channel estimation process at the receiver. In our analysis, phase error did not have a specific distribution and our results are valid for any phase error distribution. Using theoretical computations, we derived closed-form expressions for the SNR of both ideal receive beamforming and pseudo-beamforming. As it was expected, the results of our analysis shows that channel estimation error degrades the performance of distributed reception with both ideal and pseudo-bemforming techniques by almost $0.5\ dB$. The interesting outcome of our analysis was that, the SNR ratio between ideal receive beamforming and pseudo-beamforming does not depend on the amount of channel estimation error and are identical to the SNR ratios with no channel estimation error. So, our analysis shows, channel estimation error causes the same amounts of performance degradation in ideal beamforming and pseudo-beamforming systems despite the fact that the channel estimation errors manifests themselves quite differently in both systems. Also, simulation results confirmed our calculations for the mean and variance of hard decisions with channel estimation error and also, our results for the penalty term in both ideal and pseudo-beamforming systems.

Appendix A
Proof of Lemma 1

Proof. To be able to find the mean and variance of hard decisions, the distribution of decision variable phase at the receiver should be calculated. The decision variable with no estimation error would be

$$\overline{U}_j = e^{-j\theta} h_j X + e^{-j\theta} W_j \tag{46}$$

and with estimation error would be

$$\overline{U}_{j_e} = e^{-j\hat{\theta}} h_j X + e^{-j\hat{\theta}} W_j \tag{47}$$

if we define $\theta_e = \theta - \hat{\theta}$ and replace the h_j with its polar format defined in (35) we get

$$\overline{U}_{j_e} = (|h_j| X + e^{-j\theta} W_j) e^{j\theta_e} = \overline{U} j \times e^{j\theta_e} \tag{48}$$

If we replace the decision variables with their polar formats we get

$$\theta_{\overline{U}_{j_e}} = \theta_{\overline{U}_j} + \theta_e \tag{49}$$

Since we already have the distribution of $\theta_{\overline{U}_j}$ from (11) in [17], we can derive the distribution of $\theta_{\overline{U}_{j_e}}$ by convolving the distributions of $\theta_{\overline{U}_j}$ and θ_e. So,

$$\begin{aligned} & f(\theta_{\overline{U}_{j_e}} | X = x_1) = f(\theta_{\overline{U}_j} | X = x_1) * f(\theta_e) \\ & = \int_{-\infty}^{\infty} \left(\frac{1}{2\pi} e^{-\rho_j^2} + \frac{\rho_j}{\sqrt{\pi}} \cos(\theta_{\overline{U}_{j_e}} - \theta_e) e^{-\rho_j^2 \sin^2(\theta_{\overline{U}_{j_e}} - \theta_e)} \right. \\ & \quad \left. \left(1 - Q(\sqrt{2\rho_j^2 \cos^2(\theta_{\overline{U}_{j_e}} - \theta_e)})\right) \right) \times f(\theta_e) d\theta_e \end{aligned} \tag{50}$$

where $f(\theta_e)$ is the distribution of θ_e and could have any distribution.

Using $\theta_{\overline{U}_{j_e}}$ distribution, transition probability or the probability of deciding $V_j = x_m$ given $X = x_1$, can be expressed as

$$p_{m,1} = \int_{\frac{(2m-3)\pi}{M}}^{\frac{(2m-1)\pi}{M}} f(\theta_{\overline{U}_{j_e}} | X = x_1) d\theta_{\overline{U}_{j_e}} \tag{51}$$

In a low per-node SNR regime, we can calculate a first-order Taylor series expansion of $p_{m,1}$ at $\rho_j = 0$ by computing

$$\begin{aligned} p_{m,1}\big|_{\rho_j=0} &= \int_{\frac{(2m-3)\pi}{M}}^{\frac{(2m-1)\pi}{M}} f(\theta_{\overline{U}_{j_e}} | X = x_1)\Big|_{\rho_j=0} d\theta_{\overline{U}_{j_e}} \\ &= \int_{\frac{(2m-3)\pi}{M}}^{\frac{(2m-1)\pi}{M}} \frac{1}{2\pi} \left[\int_{-\infty}^{\infty} f(\theta_e) d\theta_e \right] d\theta_{\overline{U}_{j_e}} \\ &= \frac{1}{M} \end{aligned} \tag{52}$$

The expression in the brackets is equal to 1 since it is the integral of a distribution. For the second term we have

$$\begin{aligned} & \frac{\partial}{\partial \rho_j} p_{m,1}\big|_{\rho_j=0} = \int_{\frac{(2m-3)\pi}{M}}^{\frac{(2m-1)\pi}{M}} \frac{\partial}{\partial \rho_j} f(\theta_{\overline{U}_{j_e}} | X = x_1)\Big|_{\rho_j=0} d\theta_{\overline{U}_{j_e}} \\ & = \int_{\frac{(2m-3)\pi}{M}}^{\frac{(2m-1)\pi}{M}} \int_{-\infty}^{\infty} \frac{\cos(\theta_{\overline{U}_{j_e}} - \theta_e)}{2\sqrt{\pi}} f(\theta_e) d\theta_e d\theta_{\overline{U}_{j_e}} \\ & = \int_{\frac{(2m-3)\pi}{M}}^{\frac{(2m-1)\pi}{M}} \left[\frac{1}{2\sqrt{\pi}} \left(\cos(\theta_{\overline{U}_{j_e}}) \mathrm{E}[\cos(\theta_e)] \right. \right. \\ & \left. \left. + \sin(\theta_{\overline{U}_{j_e}}) \mathrm{E}[\sin(\theta_e)] \right) \right] d\theta_{\overline{U}_{j_e}} \end{aligned} \tag{53}$$

Channel estimation $\hat{h}$ in polar format can be written as $|\hat{h}| e^{j\hat{\theta}} = |h| e^{j\theta} + |\tilde{h}| e^{j\tilde{\theta}}$. Also, h is given and $\tilde{h} \sim \mathcal{CN}(0, 2\rho)$. Then, from expectation of real and imaginary part of $\hat{h}$, respectively, we have

$$\mathrm{E}[\cos(\hat{\theta})] = \frac{|h| \cos(\theta)}{\mathrm{E}[|\hat{h}|]} \tag{54}$$

$$\mathrm{E}[\sin(\hat{\theta})] = \frac{|h| \sin(\theta)}{\mathrm{E}[|\hat{h}|]} \tag{55}$$

Using $\theta_e = \theta - \hat{\theta}$ and getting the expectation of $\cos(\theta_e)$, we would have

$$\mathrm{E}[\cos(\theta_e)] = \frac{|h|}{\mathrm{E}[|\hat{h}|]} \tag{56}$$

$$\mathrm{E}[\sin(\theta_e)] = 0 \tag{57}$$

Now if we substitute these result into equation (53) we would have

$$\begin{aligned} \frac{\partial}{\partial \rho_j} p_{m,1}\big|_{\rho_j=0} &= \int_{(2m-3)\pi/M}^{(2m-1)\pi/M} \left[\frac{1}{2\sqrt{\pi}} \cos(\theta_{\overline{U}_{j_e}}) \frac{|h|}{\mathrm{E}[|\hat{h}|]} \right] d\theta_{\overline{U}_{j_e}} \\ &= \frac{|h| \sin\left(\frac{\pi}{M}\right)}{\sqrt{\pi} \mathrm{E}[|\hat{h}|]} \left[\cos\left(\frac{2\pi(m-1)}{M} \right) \right] \end{aligned} \tag{58}$$

So, in a low pre-node SNR regime with ρ_j small, we have

$$p_{m,1} \approx \frac{1}{M} + \frac{|h| \sin\left(\frac{\pi}{M}\right)}{\sqrt{\pi} \mathrm{E}[|\hat{h}|]} \left[\cos\left(\frac{2\pi(m-1)}{M} \right) \right] \rho_j \tag{59}$$

Under the assumption that $M \geq 4$ is even, we can compute the conditional expectation as follow

$$\begin{aligned} \mathrm{E}[V_j | X = x_1] &= \sum_{m=1}^{M} x_m p_{m,1} \\ &\approx \sum_{m=1}^{M} a e^{j2\pi(m-1)/M} \left\{ \frac{1}{M} + \frac{|h| \sin\left(\frac{\pi}{M}\right)}{\sqrt{\pi} \mathrm{E}[|\hat{h}|]} \right. \\ & \quad \left. \times \left[\cos\left(\frac{2\pi(m-1)}{M} \right) \right] \rho_j \right\} \\ &= \frac{2a\rho_j |h| \sin\left(\frac{\pi}{M}\right)}{\sqrt{\pi} \mathrm{E}[|\hat{h}|]} \\ & \quad \times \left[\sum_{m=1}^{M/2} \cos^2\left(\frac{2\pi(m-1)}{M} \right) \right] \\ &= \left(\frac{M\rho_j |h| \sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi} \mathrm{E}[|\hat{h}|]} \right) x_1 \end{aligned} \tag{60}$$

The conditional variance can be computed similarly as

$$\begin{aligned} \mathrm{var}[V_j | X = x_1] &= \mathrm{E}[|V_j|^2 | X = x_1] - |\mathrm{E}[V_j | X = x_1]|^2 \\ &\approx a^2 - \left(\frac{M\rho_j |h| \sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi} \mathrm{E}[|\hat{h}|]} \right)^2 a^2 \end{aligned} \tag{61}$$

Since ρ_j is small under low per-node SNR assumption, we can discard the term with ρ_j^2, so we get

$$\mathrm{var}[V_j | X = x_1] \approx a^2 \tag{62}$$

□

Appendix B
Proof of Corollary 1

Proof. By having the mean and variance of hard decision we can now calculate the mean and variance of pseudo-beamformer output. The pseudo-beamformer uses the estimated channel magnitudes to compute the combiner output

$$Y_{pbfe} = \alpha \sum_{j \in N} |\hat{h}_j| V_j \tag{63}$$

Therefore the mean of the pseudo-beamformer output is

$$\begin{aligned} \mathrm{E}[Y_{pbfe}|X = x_l] &= \alpha \sum_{j \in N} \mathrm{E}[|\hat{h}_j||X = x_l] \mathrm{E}[V_j|X = x_l] \\ &= \alpha \frac{M \sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi}} \sum_{j \in N} \left(\rho_j |h_j|\right) x_l \end{aligned} \tag{64}$$

by replacing $\rho_j := \frac{|h_j| a}{\sqrt{N_0}}$ and setting $\sum_{j \in N} |h_j|^2 = ||\boldsymbol{h}||^2$ we would have

$$\mathrm{E}[Y_{pbfe}|X = x_l] = \alpha \frac{a M \sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi N_0}} ||\boldsymbol{h}||^2 x_l \tag{65}$$

Also, the variance of the pseudo-beamformer output can calculate as follow

$$\begin{aligned} \mathrm{var}[Y_{pbfe}|X = x_l] &= \alpha^2 \sum_{j \in N} \mathrm{var}[|\hat{h}_j| V_j|X = x_l] \\ &= \alpha^2 \sum_{j \in N} \left(\mathrm{E}[|\hat{h}_j|^2] \mathrm{E}[V_j^2|X = x_l] - \mathrm{E}[|\hat{h}_j|]^2 \mathrm{E}[V_j|X = x_l]^2 \right) \end{aligned} \tag{66}$$

the second term can be set equal to zero since in a low per-node SNR regime $E[V_j|X = x_l]^2 \approx 0$. Then we would have

$$\mathrm{var}[Y_{pbfe}|X = x_l] = \alpha^2 a^2 \sum_{j \in N} \mathrm{E}[|\hat{h}_j|^2] \tag{67}$$

To obtain $\mathrm{E}[|\hat{h}_j|^2]$ we have to use the fact that

$$|\hat{h}_j|^2 = \left(|h_j| \cos(\theta_j) + |\tilde{h}_j| \cos(\tilde{\theta}_j)\right)^2 \tag{68}$$

$$+ \left(|h_j| sin(\theta_j) + |\tilde{h}_j| \sin(\tilde{\theta}_j)\right)^2 \tag{69}$$

After simplifying the above equation and getting the expectation of both side, we have

$$\mathrm{E}[|\hat{h}_j|^2] = |h_j|^2 + \mathrm{E}[|\tilde{h}_j|^2] = |h_j|^2 + \frac{N_0}{P \mathcal{E}_s} \tag{70}$$

by replacing it back in the equation (67) the variance of pseudo-beamformer output is obtained.

$$\mathrm{var}[Y_{pbfe}|X = x_l] = \alpha^2 a^2 \left(||\boldsymbol{h}||^2 + \frac{N_0 N}{P \mathcal{E}_s} \right) \tag{71}$$

□

References

[1] D. R. Brown, M. Ni, U. Madhow, and P. Bidigare, "Distributed reception with coarsely-quantized observation exchanges," in *Information Sciences and Systems (CISS), 2013 47th Annual Conference on*, March 2013, pp. 1–6.

[2] R. Wang, D. R. Brown, M. Ni, U. Madhow, and P. Bidigare, "Outage probability analysis of distributed reception with hard decision exchanges," in *Signals, Systems and Computers, 2013 Asilomar Conference on*, Nov 2013, pp. 597–601.

[3] D. Brown, U. Madhow, M. Ni, M. Rebholz, and P. Bidigare, "Distributed reception with hard decision exchanges," *Wireless Communications, IEEE Transactions on*, vol. 13, no. 6, pp. 3406–3418, June 2014.

[4] M. Stojanovic, J. G. Proakis, and J. A. Catipovic, "Analysis of the impact of channel estimation errors on the performance of a decision-feedback equalizer in fading multipath channels," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 877–886, Feb 1995.

[5] X. Tang, M. S. Alouini, and A. J. Goldsmith, "Effect of channel estimation error on m-qam ber performance in rayleigh fading," *IEEE Transactions on Communications*, vol. 47, no. 12, pp. 1856–1864, Dec 1999.

[6] M. Medard, "The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel," *IEEE Transactions on Information Theory*, vol. 46, no. 3, pp. 933–946, May 2000.

[7] S. Han, S. Ahn, E. Oh, and D. Hong, "Effect of channel-estimation error on BER performance in cooperative transmission," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 4, pp. 2083–2088, May 2009.

[8] A. Chandra, D. Biswas, and C. Bose, "BER performance of coherent PSK in rayleigh fading channel with imperfect phase estimation," in *Recent Trends in Information, Telecommunication and Computing (ITC), 2010 International Conference on*, March 2010, pp. 130–134.

[9] V. K. Prabhu, "PSK performance with imperfect carrier phase recovery," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-12, no. 2, pp. 275–286, March 1976.

[10] W. C. Lindsey, "Phase-shift-keyed signal detection with noisy reference signals," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-2, no. 4, pp. 393–401, July 1966.

[11] P. Y. Kam, S. K. Teo, Y. K. Some, and T. T. Tjhung, "Approximate results for the bit error probability of binary phase shift keying with noisy phase reference," *IEEE Transactions on Communications*, vol. 41, no. 7, pp. 1020–1022, Jul 1993.

[12] M. K. Simon and M. S. Alouini, "Simplified noisy reference loss evaluation for digital communication in the presence of slow fading and carrier phase error," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 2, pp. 480–486, Mar 2001.

[13] I. A. Falujah and V. K. Prabhu, "Performance analysis of PSK systems in the presence of slow fading, imperfect carrier phase recovery, and AWGN," in *Electrical and Computer Engineering, 2005. Canadian Conference on*, May 2005, pp. 1859–1862.

[14] K. Bucket and M. Moeneclaey, "Effect of random carrier phase and timing errors on the detection of narrowband M-PSK and bandlimited DS/SS M-PSK signals," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 1260–1263, Feb 1995.

[15] C. M. Lo and W. H. Lam, "Error probability of binary phase shift keying in nakagami-m fading channel with phase noise," *Electronics Letters*, vol. 36, no. 21, pp. 1773–1774, Oct 2000.

[16] F. Fan and L.-M. Li, "Effect of noisy phase reference on coherent detection of band-limited offset-QPSK signals," *IEEE Transactions on Communications*, vol. 38, no. 2, pp. 156–159, Feb 1990.

[17] M. Gursoy, "On the low-SNR capacity of phase-shift keying with hard-decision detection," in *Information Theory, 2007. ISIT 2007. IEEE International Symposium on*, June 2007, pp. 166–170.