Energy Efficient Relaying Games in Cooperative Wireless Transmission Systems

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Abstract-In wireless networks, intermediate nodes are often used as relays to reduce the transmission energy required to deliver a message to an intended destination. The selfishness of autonomous nodes, however, raises concerns about their willingness to expend energy to relay information for others. This paper considers the effect of selfishness on energy efficiency using a non-cooperative game theoretic approach. A two-source relaying game is formulated for both non-fading and fading scenarios. We show that cooperative transmission with optimum energy allocation is a Nash Equilibrium in non-fading channels when the sources are sufficiently patient. In fading channels, cooperative transmission with optimum energy allocation is also a Nash Equilibrium when a ceiling is applied to the relay energy of each source. Simulation results show that sources acting in their own self-interest can achieve an energy efficiency close to that of centrally optimized energy allocation in many cases.

I. INTRODUCTION

In energy limited wireless networks, sources often rely on intermediate users to serve as relays in order to improve performance or reduce energy consumption [1], [2]. The popularity of self-organizing ad hoc networks and the emergence of programmable wireless devices raise doubts on the assumption that each node will always be willing to offer help when needed. Some work in this area has considered a game theoretic framework to analyze the trade off between an individual source's interest and system performance [3]. Especially, the problem of whether cooperation can exist without incentive mechanisms is considered in [4]-[9]. Most of the work in this area has focused on the network layer. It is claimed in [4] that cooperation can be stimulated provided that no node has to forward more traffic than it generates. The nodes are classified into different energy classes and an energy efficient Nash Equilibrium strategy is proposed in [5]. Based on game theory and graph theory, the conditions when cooperation solely based on the nodes' self-interest can exist are proposed in [6]. A joint analysis of cooperation stimulation and security is given in [7] and a set of reputation-based cheat-proof and attach-resistant cooperation stimulation strategies are derived.

A few work investigates the effect of nodes' selfishness in the physical layer. It is shown in [8] that a mutually cooperative Nash Equilibrium can always be obtained when convex utility functions are used in Rayleigh fading channels for decode-and-forward protocol. Lai and Gamal [9] prove that full cooperation is possible by using a vanishingly small fraction of altruistic nodes .

In this paper, we investigate the problem of whether cooperation can exist without incentive mechanisms or altruistic

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nodes of the two-source amplify-and-forward protocol in both non-fading and fading scenarios. In both cases the sources are required to satisfy a instantaneous SNR constraint and are assumed to be rational and self-interested. The utility of each source is based on its own energy consumption. We model both scenarios as an infinitely repeated two-source relaying game. In the non-fading scenario, our results show that cooperation with optimum energy allocation can exist given sources are sufficiently patient. In the fading scenario, we propose a conditional trigger cooperative strategy and show that this strategy is a Nash Equilibrium of the infinitely repeated game. An important feature of the conditional trigger strategy is that the sources cooperate using optimum resource allocation but with a ceiling placed on the optimized relay energy. If either source is asked to transmit with relay energy greater than their ceiling in a stage game, both sources use direct transmission in that stage game. We show that this ceiling goes to infinity as the sources become more patient. Our results show that sources using the conditional trigger strategy can often achieve an overall system energy efficiency close to that of a centrally-optimized system, especially when the sources are patient.

II. System Model

We consider the 4 node cooperative transmission system illustrated in Figure 1. Source node 1 and source node 2 take turns to transmit to destination node 1 and 2 respectively. Each source node can potentially help the other by relaying the transmission to the intended destination. The quantities G_{ir} , G_{is} and H denote the amplitudes of the squared channel gains, normalized with respect to channel noise.



Fig. 1. System model.

We assume that the two sources cooperate through the halfduplex "amplify and forward" (AF) protocol shown in Figure 2. In this protocol, the *transmission session* is divided into two *transmission intervals* for source 1 and source 2 respectively. Each transmission interval is further divided into two *time slots* of equal duration. In S_1 's transmission interval, S_1 will first send out a relay request to S_2 . At this point, one of two things can happen:

- 1) S_2 responds to the relay request within some waiting time t, indicating that it will cooperate. In this case, S_1 transmits using the optimum cooperative source energy \mathcal{E}_{1s}^* in time slot 1. At the same time S_2 listens. In time slot 2, S_2 forwards to D_1 the signal overheard in the first time slot with the optimized relay energy \mathcal{E}_{2r}^* .
- 2) S_2 does not respond to the relay request within the waiting time t. In this case, S_1 will assume that S_2 is not willing to cooperate. S_1 transmits with direct transmission energy \mathcal{E}_1 .

The same sequence of events applies in S_2 's transmission interval.



Fig. 2. Two-source amplify and forward protocol.

To facilitate analysis in the following sections, we make the following assumptions:

- A1: Both sources must satisfy the SNR constraint ρ in each transmission session.
- A2: The squared channel amplitudes in each transmission interval are known at both sources and destinations. This implies that the sources can dynamically allocate transmit energies according to the instantaneous channel state and that the destinations can combine the two observations using maximal ratio combining to maximize the SNR.
- A3: Fading channels are assumed to be quasi-static in the sense that the channel realization remains constant over the duration of a transmission interval but is random for different transmission sessions.

III. OPTIMUM ENERGY ALLOCATION FOR AF COOPERATION

Using the results in [10], we know that, for $i, j \in \{1, 2\}$ with $j \neq i$, the SNR of an AF cooperative communications system can be expressed as

$$\mathsf{SNR}_i = G_{is}\mathcal{E}_{is} + \frac{H\mathcal{E}_{is}G_{jr}\mathcal{E}_{jr}}{1 + H\mathcal{E}_{is} + G_{jr}\mathcal{E}_{jr}} \tag{1}$$

where \mathcal{E}_{is} is the source transmission energy and \mathcal{E}_{jr} is the relay transmission energy. Under a fixed SNR constraint ρ , this result can be used to compute the energy reduction at S_i obtained through optimum cooperative energy allocation, which is denoted as α_i .

$$\alpha_{i} = \underbrace{\frac{\rho}{G_{is}}}_{\mathcal{E}_{i}} - \underbrace{\frac{\rho}{H + G_{is}} - \frac{(\rho H)^{1/2} (G_{is} + (1 + \rho)H)^{1/2}}{(H + G_{is}) \left(H(G_{jr} - G_{is}) + G_{is}G_{jr}\right)^{1/2}}_{\mathcal{E}^{*}_{is}}$$

for sources $i, j \in \{1, 2\}$ with $j \neq i$. \mathcal{E}_i denotes the direct transmission energy required to satisfy the SNR threshold. \mathcal{E}_{is}^* denotes the optimum source energy to satisfy the SNR threshold when S_j agrees to help. The quantity $\beta_i = \mathcal{E}_{ir}^*$ is the optimized relaying energy used by a source to cooperate (the cost of cooperation). β_j can be computed from (1) and \mathcal{E}_{is}^* .

IV. TWO-SOURCE RELAYING GAME

We can consider the system in Figure 1 in the context of a two-source relaying game. Source 1 and source 2 formulate the player set $S = \{S_1, S_2\}$. They each have the strategy space $\Theta_i = \{R, N\}$, where "R" denotes "relay" at the optimum resource allocation relay energy and "N" denotes "do not relay". We present the payoff function using the payoff matrix shown in Table I. The quantity $\alpha_i = \mathcal{E}_i - \mathcal{E}_{is}^*$ denotes the energy reduction obtained by S_i with respect to the direct transmission energy required to satisfy the SNR constraint when the other source cooperates with S_i . Clearly $\alpha_i \geq 0$ and $\beta_i \geq 0$. Finally, the quantity $\alpha_i - \beta_i$ is the net energy gain of S_i under optimum energy allocation.

	$\theta_2 = N$	$\theta_2 = \mathbf{R}$
$\theta_1 = N$	0, 0	$\alpha_1, -\beta_2$
$\theta_1 = \mathbf{R}$	$-\beta_1, \alpha_2$	$\alpha_1 - \beta_1, \ \alpha_2 - \beta_2$

TABLE I Two-source relaying game payoff matrix

In the *n*-source game, the set of strategies $(\theta_1^*, ..., \theta_n^*)$ is a Nash Equilibrium (NE) if, for each source i, θ_i^* is source i's best response to θ_{-i}^* , where θ_{-i} denotes the strategies of all the sources except source i. It can be presented as [11]

$$\pi_i(\theta_i^*, \theta_{-i}^*) \ge \pi_i(\theta_i, \theta_{-i}^*)$$

where π_i is the payoff function (utility) of S_i .

The concept of Nash Equilibrium is intuitively explained as follows: if all of the sources are following NE strategies, no source can increase their payoff by deviating from the NE strategy. Our goal is to find Nash Equilibria of the two-source relaying game in the case of non-fading and fading channels and to identify the conditions under which an equilibrium based on cooperation exists.

A. Non-fading Channels

Denote the static two-source relaying game as the stage game of the repeated game. In our system model, a stage game corresponds to a transmission session. In the non-fading channels, when the two-source relaying game is repeated, the payoff matix in Table I will remain the same in each stage game. It is easy to check that for the one-shot or finite repeated two-source relaying game, the only NE is $\theta^* = (N, N)$ in each stage game. The situation changes, however, when the game is repeated infinitely. The goal of the sources now is to maximize the payoff that they accumulate over time, that is, they are willing to secure a high-payoff in the next stage by cooperating. Denote the payoff of S_i of t^{th} stage game as π_{it} . Assume that for each time t, the outcomes of the t-1 preceding plays of the stage game are known before the present stage begins. Then we are ready to define the accumulated payoff $\overline{\Pi}_i$ of S_i as

$$\overline{\Pi}_i = \sum_{t=1}^{\infty} \delta^{t-1} \pi_{it}, \qquad (2)$$

where $0 \leq \delta \leq 1$ and t is the time index of the game. The discount factor δ represents the degree to which the payoff of each transmission session is discounted relative to the previous transmission session. We assume both sources have the same δ . In our case, δ can be looked as a measure of the mobility, which is the probability of still having the other source in the neighborhood after the current transmission session. It is

reasonable to assume that when the sources begin the game, they do not know when the game will end. Thus we can model the finite repeated game with an unpredictable end as an infinite repeated game with discounted future payoffs.

Proposition 1: There exists a Nash Equilibrium based on cooperation with optimum energy allocation for the infinitely repeated two-source relaying game in non-fading channels if and only if

$$\forall i \in \{1, 2\} \quad \max_{i \in \{1, 2\}} \left(\frac{\beta_i}{\alpha_i}\right) \le \delta \le 1.$$
(3)

The proof is quite similar with those in [11](pp. 88-92). Here we omit the proof due to space limitations.

Proposition 1 has implied that $\alpha_i \geq \beta_i$ for both sources. It denotes the *mutual beneficial* channel conditions that both sources can save energy by cooperation with optimum energy allocation. A source that can not have individual benefit by cooperation will always play N regardless of the interest of others. Thus cooperation can not be stimulated unless the mutual cooperation condition (3) is satisfied.

B. Fading Channels

To analyze the two-source relaying game in the case of fading channels, we model our system as an infinitely repeated game. The payoff matrix of each stage game corresponds to one channel state realization of each transmission session. Each element in the payoff matrix of Table I is a random variable and will change with each new channel state realization in each stage game. We model the payoffs in the payoff matrix, α_i and β_i , as ergodic random processes. Denote the realizations of α_i and β_i at time t as $\alpha_i^{(t)}$ and $\beta_i^{(t)}$ respectively. Since the payoff matrix will change with each stage game, both sources are not able to know the exact value of the payoff matrix in the future. To investigate the conditions of when cooperation can be stimulated between selfish nodes, we propose a *conditional trigger strategy*.

Suppose S_i adopts the conditional trigger strategy. In each stage game, S_i 's strategy can be expressed as Figure 3. First, check the value of both sources' optimum relay energy $\beta_i^{(t)}$ and $\beta_j^{(t)}$. If either of their relay energy exceeds their corresponding ceiling value, C_i and C_j , S_i plays N. Otherwise, S_i checks the value of $\beta_i^{(t)}$. If it is equal to 0 (notice that in our definition of β_i , β_i can not be less than 0), then S_i plays N (S_j uses direct transmission as specified by optimum energy allocation). If $\beta_i^{(t)} > 0$ and neither sources has *defected* in all the previous games, S_i plays R. Otherwise, S_i plays N.

Here we do not simply consider the behavior of "not relay" as "defect". S_i is considered to "defect" in stage game t if all of the conditions are satisfied for S_i to play R but, instead, S_i plays N. For the reason that only the "not relay" behavior under certain conditions will be considered "defect" and thus trigger non-cooperation forever after, we call our strategy "conditional trigger strategy". If S_i defects, it obtains an additional payoff in the current stage game since it does not expend any relaying energy. This short-term gain, however, is obtained at the cost of the loss of future payoffs since defection triggers non-cooperation for all future games.

Proposition 2: If C_i satisfies

$$C_i = \frac{\delta}{1-\delta} \mathbb{E}\left[(\alpha_i - \beta_i) I_{\{\beta_1 \le C_1\}} I_{\{\beta_2 \le C_2\}} \right]$$
(4)

for both i = 1 and i = 2, then the *conditional trigger strategy* specified in Figure 3 is a Nash Equilibrium.



Fig. 3. S_i 's conditional trigger strategy in the t^{th} stage game.

Define

$$I_{\{x \le y\}} = \begin{cases} 1 & x \le y \\ 0 & x > y. \end{cases}$$

Proof:

For the stages that do not satisfy $\beta_i \leq C_i$, $\forall i \in \{1, 2\}$, since S_i plays N, S_j 's best response is also to play N. When $\beta_i \leq C_i$, it is trivial to show that when S_i should use direct transmission (specified by optimum energy allocation), S_j does not need to relay. For the reason that S_i plays N forever if either sources has defected previously, S_j 's best response is also to play N forever once S_i or S_j defect. The rest is to determine S_j 's best response when the current stage game satisfies the condition of $\beta_i \leq C_i$, $\forall i \in \{1, 2\}$ and neither sources has defected in the previous games.

Suppose the current stage is t_0 and S_j plays N, then S_j will get a payoff of $\alpha_j^{(t_0)}$ but will trigger (N, N) forever after. Hence the present value of the accumulated payoff is

$$\overline{\Pi}_{jN} = \alpha_j^{(t_0)} + \delta \cdot 0 + \delta^2 \cdot 0 + \dots = \alpha_j^{(t_0)}$$
(5)

On the other hand, playing R will yield a payoff of $\alpha_j^{(t_0)} - \beta_j^{(t_0)}$ and then lead to the exactly same choice in the next stage. We have the present value of the accumulated payoff as

$$\overline{\Pi}_{jR} = (\alpha_j^{(t_0)} - \beta_j^{(t_0)}) + \delta \cdot (\alpha_j^{(t_1)} - \beta_j^{(t_1)}) I_{\{\beta_j^{(t_1)} \le C_j\}} I_{\{\beta_i^{(t_1)} \le C_i\}} + \dots$$
(6)

Although S_j does not know the exact value of payoffs in the payoff matrix in the future stage games, S_i knows what can *expect*. Hence equation (6) can be rewritten as

$$\overline{\Pi}_{jR} = \alpha_j^{(t_0)} - \beta_j^{(t_0)} + \delta \cdot \mathsf{E}[(\alpha_j - \beta_j)I_{\{\beta_j \le C_j\}}I_{\{\beta_i \le C_i\}}] + \dots$$
$$= \alpha_j^{(t_0)} - \beta_j^{(t_0)} + \frac{\delta}{1-\delta}\mathsf{E}[(\alpha_j - \beta_j)I_{\{\beta_j \le C_j\}}I_{\{\beta_i \le C_i\}}]$$

Notice that C_j satisfies the function (4), the above equation can be rewritten as

$$\overline{\Pi}_{jR} = \alpha_j^{(t_0)} - \beta_j^{(t_0)} + C_j \tag{7}$$

Given the condition of $\beta_i \leq C_i$, $\forall i \in \{1, 2\}$, we know from (5) and (7) that $\overline{\Pi}_{jR}$ must be no less than $\overline{\Pi}_{jN}$. In this case, S_j has no strict incentive to defect in the stage game. Hence given that in the first game and in any stage game that all the preceding stage games' outcomes have no defective behavior, S_j 's best response is to play R. The same analysis is applied to S_i . Hence it is a Nash Equilibrium for both sources to adopt the conditional trigger strategy of the infinitely repeated game in fading channels given both C_1 and C_2 satisfy (4).

Proposition 2 implies that for the fading channel scenarios, sources decide their moves based both on the instantaneous value of the payoff matrix, β_i and the statistics of the future payoffs, C_i . Intuitively, C_i can be looked as the ceiling value of the relay energy. Its value can be decided by the function (4). If S_i 's relay energy exceeds the ceiling C_i , it means that the optimum energy allocation requires a large relay energy from S_i to help S_j . In this case, the cost in the current stage of playing R is so much that the expected future payoff is not tempting anymore. Thus a rational source will have the incentive to defect. However, the payoff matrix is also known by S_i . Knowing that S_i will not cooperate, S_i 's best response is also not to cooperate. The no-cooperate behavior here is not considered "defection" and does not trigger non-cooperation forever. In the conditional trigger strategy, trigger will only happen if one or both players do not cooperate with optimum energy allocation strategy when both players has no incentive to defect in the stage game.

Now we consider the special case of $\delta = 1$. $\delta = 1$ implies that both sources are infinitely patient, i.e. the energy they spend in the future has the same importance as the energy they spend now or both sources are relative static (none of them have the idea of when they will run out of each other's neighborhood). As long as $\min_{i \in \{1,2\}} \mathsf{E}(\alpha_i - \beta_i) \geq 0$, the ceiling value C_i goes to infinity when $\delta \to 1$ (refer to (4)). In this case, the Nash Equilibrium strategy becomes the subtree of figure 3 where we always do optimum resource allocation (since the ceilings are never exceeded).

For Rayleigh fading, the expect value of α_i goes to infinity. This implies that even if δ is very small, the long term benefit of mutual cooperation with optimum energy allocation is still very large. The conditions in proposition 2 will be satisfied. Rational sources in a Rayleigh fading environment will choose to cooperate with optimum energy allocation to achieve high payoff in the long run.

V. SIMULATION RESULTS

This section demonstrates the impact of sources' selfish behavior on energy efficiency with respect to centrally optimized energy allocation under path loss channels and lognormal fading channels. The distance between the two destinations in Figure 1 is normalized as 1. In the path loss model, the normalized channel gain is given as $H = d_H^{-\gamma}$, where d_H is the distance between source 1 and source 2 and γ is the channel attenuation exponent. G_{is} and G_{ir} are defined in the same way. For lognormal fading channels, the channel states $\sqrt{G_{is}}$, $\sqrt{G_{ir}}$ and \sqrt{H} are lognormal distributed random variables with means $\mu_{G_{is}} = 1/d_{G_{is}}^{\gamma}$, $\mu_{G_{ir}} = 1/d_{G_{ir}}^{\gamma}$ and $\mu_H = 1/d_H^{\gamma}$, respectively. For both channel models, $\rho = 1$. Figure 4 shows the region that cooperation is "naturally" encouraged in a wireless system under path-loss channels with different channel attenuation exponents. The results show that the region expands with γ , which indicates that with more sever channel attenuation, cooperation with optimum energy allocation is more preferable than direct transmission.

The model in Figure 5 is used to demonstrate the impact of sources' selfishness on energy efficiency. In this model, S_1 and S_2 is located on the line connecting the two destinations with $d_{G_{1r}} = 0.25$. Energy efficiency is investigated while S_2 moves between S_1 and D_1 . All of the following results assume $\gamma = 4$.

Figure 6 shows the discounted total saved energy of grim trigger strategy [11] and centrally optimized energy allocation in the path loss channels. When d_H is small, $d_{G_{2r}}$ is large. S_2 's relay energy required by optimum energy allocation exceeds the energy S_2 saved by help from S_1 . In this case, S_2 refuses to relay. Thus no cooperation is stimulated by grim trigger strategy. When S_2 moving toward D_1 , the required relay energy of S_2 becomes smaller and finally S_2 falls into the region where mutual benefit occurs. Cooperation with optimum energy allocation is stimulated and the discounted total saved energy of grim trigger strategy merges to the centrally optimized case.

Figure 7 shows the average discounted total saved energy of conditional trigger strategy in lognormal fading channels with $d_H = 0.4$. Centrally optimized energy allocation strategy is also included for comparison. Here "the average discounted total saved energy" denotes the discounted total saved energy averaged over channel distributions. The results show that the energy gap between the conditional trigger strategy and centrally optimized strategy becomes smaller as δ becomes larger. Both strategies merge as $\delta \to 1$.

Figure 8 shows the fraction of the stage games in which cooperation with optimum energy allocation (OEA) is stimulated with conditional trigger strategy. The fraction of the stage games using cooperation with optimum energy allocation increases with δ . As $\delta \to 1$, sources choose to cooperate with optimum energy allocation in almost all the stage games. In this case, the conditional trigger strategy is almost as energy efficient as the centrally optimized case.

VI. CONCLUSION

This paper considers the problem of under what conditions cooperation can be stimulated between rational and selfinterest sources. A two-source relaying game is formulated for both non-fading and fading scenarios. We show that cooperative transmission with optimum resource allocation is a Nash Equilibrium in non-fading channels when the sources are sufficiently patient. In fading channels, cooperative transmission with optimum resource allocation is also a Nash Equilibrium when a ceiling is applied to the relay energy of each source. Simulation results show that sources acting in their own selfinterest can achieve an energy efficiency close to that of centrally optimized energy allocation in many cases.

References

- A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. 1998 IEEE International Symp. on Information Theory*, p. 156, August 16-21 1998.
- [2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.



Fig. 4. Region that cooperation with optimum energy allocation can emerge as a Nash Equilibrium between selfish sources for different channel attenuation exponent γ in path loss channels.



Fig. 5. Simulation Model

- [3] F. Meshkati, H. Poor, and S. Schwartz, "Energy-efficient resource allocation in wireless networks," in *Signal Processing Magazine*, *IEEE*, vol. 24, pp. 58–68, 2007.
- [4] M. B. A. Urpi and S. Giordano, "Modelling cooperation in adhoc networks: a formal description of selfishness," in WiOpt'03 Workshop: Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks, pp. 303–312, Mar 2003.
- [5] V. Srinivasan, P.Nuggehalli, C.Chiasserini, and R.R.Rao, "An analytical approach to the study of cooperation in wireless as hoc networks," in *IEEE Trans. Mobile Computing, Vol. 5, May.* 2006, pp. 1044–1049, Mar 2005.
- [6] M. Felegyhazi, J. Hubaux, and L. Buttyan, "Nash equilibria of packet forwarding strategies in wireless ad hoc networks," in *IEEE Trans. Mobile Computing, Vol. 5, May. 2006*, pp. 1044– 1049, May 2006.
- [7] W. Guang-xing and Q. Yan-hong, "Incentive mechanism for team cooperation based on implicit side contract," in Management Science and Engineering, 2006. ICMSE '06. 2006 International Conference on, pp. 1344–1349, 2006.
- [8] Y. Chen and S. Kishore, "A game-theoretic analysis of decodeand-forward cooperation in rayleigh fading channels," in *Information Sciences and Systems*, 2007. CISS '07. 41st Annual Conference on, pp. 294–299, 2007.
- [9] L. Lai and H. E. Gamal, "On cooperation in energy efficient wireless networks: The role of altruistic nodes," in *submitted* to the IEEE Transactions on Wireless Communications, Aug 2006.
- [10] J. Yang and D. R. B. III., "The effect of channel state information on optimum energy allocation and energy efficiency of cooperative wireless transmission systems," in *Proceedings* of the Conference of Information Sciences and Systems (CISS 2006), (Princeton, NJ), pp. 1044–1049, Mar 2006.
- [11] R. Gibbons, "Game theory for applied economists," in *Prince*ton University Press, 1992.



Fig. 6. Discounted total saved energy versus d_H (distance between source 1 and source 2) in path loss channels with $\gamma = 4$.



Fig. 7. Average discounted total saved transmission energy versus δ in lognormal fading channels with $\gamma=4$



Fig. 8. The fraction of stage games using cooperation with optimum energy allocation versus δ .