Resource Allocation for Cooperative Relaying

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***Abstract*—The delay-limited capacity of the half-duplex relay channel is analyzed for several cooperative protocols under a long-term average total transmit power constraint. It is assumed that the source and the relay have access to partial channel state information in the form of channel amplitudes. Non-orthogonal amplify-and-forward (NAF), compress-and-forward (CF) and opportunistic decode-and-forward (ODF) protocols are compared with optimal resource allocation, i.e., at each channel state, the source and the relay transmit with the minimum total power allocation required to achieve the target rate. A hybrid opportunistic protocol is proposed in which CF or ODF with optimal resource allocation is chosen at each channel state. Numerical results demonstrate that, while the hybrid protocol offers the best delay-limited capacity, ODF follows the hybrid scheme closely for a wide range of relay locations and average power constraints. We also consider various low complexity protocols such as fixed time allocation and the estimate-and-forward (EF) protocol in order to analyze the trade-off between the system complexity and delay-limited capacity.**

I. Introduction

Cooperative transmission improves the performance of wireless communication systems by providing increased robustness to channel variations as well as potential energy savings [1]-[10]. For communication over fading environments, channel state feedback is yet another powerful method to provide adaptation to varying channel states and to obtain energy savings. In this paper we consider a cooperative system equipped with channel state feedback in the form of channel state amplitudes and explore how to exploit cooperation and feedback simultaneously for improved system performance.

Without channel state information at the transmitters (CSIT) only a limited improvement can be achieved by statistical channel resource and power allocation [4], [5]. However, in the case of instantaneous CSIT, it is possible to adapt to the channel state and achieve significant gains. Availability of CSIT is assumed in some recent literature on user cooperation as well. In [8], the ergodic capacity of a cooperative system is explored under both short-term and long-term average total transmit power constraints. Host-Madsen and Zhang also explores outage capacity with short-term total transmit power constraint for both synchronous and asynchronous relays. In [9], resource allocation is considered to optimize the ergodic capacity under separate power constraints at the source and the relay. In [10], outage performance with long-term average total transmit power constraint is investigated where full duplex relays cooperate irrespective of the channel state, thus no channel resource allocation needed. In [5], an opportunistic optimal energy allocation scheme for two-source amplify-and-forward protocol is proposed.

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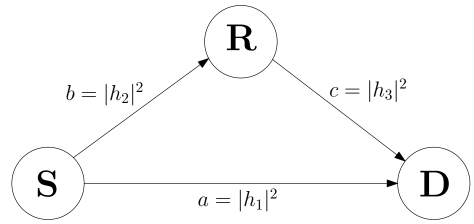

Fig. 1. The cooperative relay channel

In addition to resource allocation, when the source and the relay have access to the instantaneous channel amplitudes they also have an opportunity to select a cooperative transmission protocol. In [7], we introduced the idea of opportunistic cooperation using decode-and-forward (DF) relaying, in which the terminals choose either DF (with optimal power and time allocation) or direct transmission (DT) depending on which protocol is more power efficient in the current channel state. This hybrid protocol is called opportunistic decode-and-forward (ODF). The results in [7] show that the freedom of choosing among multiple transmission schemes improves both the delay-limited capacity and the minimum outage probability significantly. In particular, the ODF scheme is shown to achieve a nonzero delay-limited capacity, while both DF and DT individually have zero delay-limited capacities.

In this paper, we consider a network of three terminals and model the inter-terminal links (see Figure 1) as independent, frequency non-selective Rayleigh slow fading channels. We assume a delay constraint on the transmission which imposes each channel codeword to be transmitted over one fading block of the network. Since the channel is not ergodic, we consider delay-limited capacity [11] as our performance measure. Our goal is to maximize the delay-limited capacity of the system under a long-term average total transmit power constraint. Our protocols and analysis can be extended to provide minimum outage probability when the available long-term average total transmit power does not support the target delay-limited capacity [7].

To facilitate the development of opportunistic protocol selection with optimal resource allocation, we analyze optimal resource allocation for the compress-and-forward (CF), estimate-and-forward (EF) in which the received signal at the relay is compressed by ignoring the destination side information (Wyner-Ziv compression is not employed), and non-orthogonal amplify-and-forward (NAF) [3] protocols. Note that, together with DF, these protocols have been extensively analyzed in terms of ergodic capacity as well as outage/error probability performance over static or fading channels. However, to our knowledge, a comparative analysis of these protocols and the cut-set upper bound in the case of instantaneous CSI feedback has not been done. In this paper, we explore the delay-limited capacity of CF and NAF under optimal power and time allocation. We compare these results with the ODF performance obtained in [7].

We then propose a hybrid opportunistic protocol that selects from all available protocols the protocol that achieves the rate target with the least total transmit power. We show that, for protocols employing optimal resource allocation under a total power constraint, the instantaneous rate of EF is at least that of NAF for any channel state. Since the instantaneous rate of CF is also at least that of EF for any channel state, the hybrid opportunistic protocol only needs to select between CF and ODF with optimal resource allocation in each channel state. Since the hybrid protocol uses the least power in each transmission interval, it also provides the best delay-limited capacity performance of all protocols considered in this study. Our numerical results show that the hybrid protocol can offer delay-limited capacity close to the cut-set upper bound.

The organization of the paper is as follows: In Section II, we introduce the system model. In Section III we first introduce the general delay-limited capacity maximization problem. Then we provide the instantaneous capacity expressions for NAF, CF, and EF cooperation protocols, as well as the cut-set upper bound. Section IV is devoted to the presentation and discussion of numerical results. Finally Section V concludes the paper.

II. System Model

We consider a wireless communication system consisting of a single source (S), single destination (D), and an available relay (R) as shown in Figure 1. The links among the terminals are modeled as having independent, quasi-static Rayleigh fading as well as path loss with channel gains h_i, $i \in \{1, 2, 3\}$. We assume zero-mean additive white Gaussian noise with unit variance at the receivers. The channel coefficients are assumed to be constant over a block of N symbols during which one codeword is transmitted, and are independent from one block to the other. We assume N is large enough to achieve instantaneous capacity. The squared channel amplitudes, denoted by $a = |h_1|^2$, $b = |h_2|^2$, and $c = |h_3|^2$ as in Figure 1, are exponentially distributed random variables with means λ_a, λ_b, and λ_c, respectively. The means capture the effect of pathloss across the corresponding link. It is also assumed that the channel amplitude vector $\mathbf{s}$ is known at the source, the relay and the destination, while the phase information for h_1, h_2 and h_3 is only available at the corresponding receivers. The lack of channel phase information at the transmitters implies that the source and the relay can not beamform.

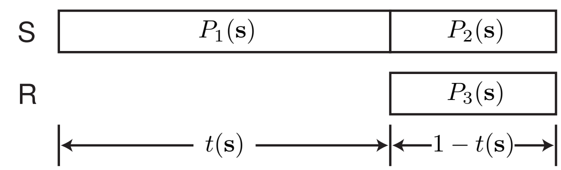

Fig. 2. Power and time allocation for cooperative relaying

We assume a half-duplex relay and normalize the time interval for each cooperative protocol (N symbols) to 1 unit as in Figure 2. Let $\mathbf{P}(\mathbf{s}) = (P_1(\mathbf{s}), P_2(\mathbf{s}), P_3(\mathbf{s}), t(\mathbf{s}))$ be a resource allocation rule defined over the set of all possible network states $\mathbf{s} = (a, b, c)$, where $P_1(\mathbf{s})$ is the source power in the first timeslot of duration $t(\mathbf{s})$, and $P_2(\mathbf{s})$, $P_3(\mathbf{s})$ are the transmission powers of the source and the relay, respectively, in the second timeslot of duration $1 - t(\mathbf{s})$, $0 < t(\mathbf{s}) \leq 1$. We define Ω as the set of all possible resource allocation functions. We have

$$\Omega = \{\mathbf{P}(\mathbf{s}) : P_1(\mathbf{s}) \geq 0, P_2(\mathbf{s}) \geq 0, P_3(\mathbf{s}) \geq 0, 0 < t(\mathbf{s}) \leq 1\}.$$

Let $F(\mathbf{s})$ be the probability distribution function of the channel states. Then the long-term average total transmit power constraint can be written as

$$\begin{aligned} \mathrm{E}[\mathbf{P}] &\triangleq \int_s [t(\mathbf{s})P_1(\mathbf{s}) + (1 - t(\mathbf{s}))(P_2(\mathbf{s}) + P_3(\mathbf{s}))]dF(\mathbf{s}) \\ &\leq P_{avg}. \end{aligned}$$

The long-term average total transmit power constraint imposes a set of feasible resource allocation functions, $\bar{\Omega} \subseteq \Omega$, that is composed of power allocation functions which satisfy the above inequality, i.e., $\bar{\Omega} = \{\mathbf{P} : \mathrm{E}[\mathbf{P}] \leq P_{avg}, \mathbf{P} \in \Omega\}$. We note here that the above power constraint is indeed equivalent to an energy constraint due to the normalization of the timeslot.

III. Delay-Limited Capacity Analysis

Delay-limited capacity is defined as the highest achievable rate that can be sustained independent of the channel state [11]. This model is especially suitable for delay sensitive applications such as real-time voice and video communications. The availability of channel state information is essential to guarantee any non-zero transmission rate with zero outage probability.

In this section, we consider different cooperation protocols and dynamically allocate the relay transmit time and power among the terminals, based on the channel states in order to maximize the delay-limited capacity. Let $\mathbf{P}(\mathbf{s})$ be the resource allocation function and $C(\mathbf{P}, \mathbf{s})$ be the instantaneous capacity of the underlying cooperation protocol with this resource allocation function at channel state $\mathbf{s}$. Then the delay-limited capacity maximization problem can be stated as follows[1].

$$\max_{\mathbf{P}(\mathbf{s})\in\bar{\Omega}} R, \tag{1}$$
$$\text{such that } C(\mathbf{P},\mathbf{s}) \geq R \text{ for all } \mathbf{s}.$$

In the following subsections, we introduce the specific cooperation protocols that will be analyzed in terms of delay-limited capacity.

A. Non-orthogonal Amplify and Forward

In the NAF protocol [3], the transmission slot is divided into two equal portions, that is $t(\mathbf{s}) = 1/2$ for all $\mathbf{s}$. During the first timeslot, the source transmits a signal to the relay and the destination while the relay is silent. In the second timeslot, the relay simply scales its received signal from the first timeslot and retransmits, and the source simultaneously transmits new symbols. For $i = 1, \ldots, N/2$, the input/output relationship for NAF can be characterized as

$$y_d[i] = h_1\sqrt{P_1}x_1[i] + n_1[i], \tag{2}$$
$$y_r[i] = h_2\sqrt{P_1}x_1[i] + n_2[i], \tag{3}$$

and, for $i = N/2+1, \ldots, N$, the input/output relationship for NAF is

$$y_d[i] = h_1\sqrt{P_2}x_1[i] + h_3\sqrt{P_3}x_2[i] + n_1[i], \tag{4}$$
$$x_2[i] = \beta y_r[i - N/2]. \tag{5}$$

Here, $x_1[i]$, $x_2[i]$ are the source and the relay symbols at time i, $y_r[i]$ and $y_d[i]$ are the received symbols at time i at the relay and the destination, respectively, and β is the scaling factor at the relay that satisfies

$$\beta \leq \sqrt{\frac{P_3}{|h_2|^2P_1 + 1}}. \tag{6}$$

Define $\mathbf{x}_1 = [x_1[1], \ldots, x_1[N]]^T$, $\mathbf{x}_2 = [x_2[N/2+1], \ldots, x_2[N]]^T$ and $\mathbf{y}_d = [y_d[1], \ldots, y_d[N]]^T$. We have $E[\mathbf{x}_1^T\mathbf{x}_1 + \mathbf{x}_2^T\mathbf{x}_2] \leq 1$. The maximum instantaneous mutual information achieved by NAF with resource allocation $\mathbf{P}(\mathbf{s}) = (P_1(\mathbf{s}), P_2(\mathbf{s}), P_3(\mathbf{s}), 1/2)$ at channel state $\mathbf{s}$ can be found as

$$I(\mathbf{x}_1;\mathbf{y}_d\,|\,\mathbf{s}) = \frac{1}{2}\log\left(1 + aP_1 + \frac{|\beta|^2bcP_1 + aP_2}{1+|\beta|^2c} + \frac{a^2P_1P_2}{1+|\beta|^2c}\right). \tag{7}$$

It can be shown that the maximum value of β also maximizes the mutual information. Then, substituting (6) in (7) we obtain

$$C_{NAF}(\mathbf{P}(\mathbf{s}),\mathbf{s}) \triangleq I(\mathbf{x}_1;\mathbf{y}_d\,|\,\mathbf{s}) = \tfrac{1}{2}\log\left(1 + aP_1 + \tfrac{bcP_1P_3 + aP_2(1+aP_1)(1+bP_1)}{1+bP_1+cP_3}\right). \tag{8}$$

The delay-limited capacity of NAF can be found by solving the optimization problem in (1), where we replace $C(\mathbf{P},\mathbf{s})$ with $C_{NAF}(\mathbf{P},\mathbf{s})$. Note that in (8) if we set $P_1 = P_2$ and $P_3 = 0$, we get DT. If we set $P_2 = 0$, we get orthogonal AF. Hence the optimization in computing the delay-limited capacity is opportunistic as in [7] and the relay is not used if DT is more power efficient.

The following lemma shows that, for the NAF protocol with CSIT and optimal power allocation, either the source transmits directly, or orthogonal AF is used in each channel state. Hence we can restrict our attention to optimal power allocation for opportunistic AF only.

Lemma 3.1: Let $P^*(\mathbf{s})$ be the optimal resource allocation function that maximize (8). Then at any channel state $\mathbf{s}$, we either have $P^*(\mathbf{s}) = (P_1^*, 0, P_3^*, 1/2)$, i.e., we use DT, or we have $P^*(\mathbf{s}) = (P_1^*, P_2^*, 0, 1/2)$, i.e., we use AF.

Proof: Let $\mathbf{s}$ be any channel state, and define $E(\mathbf{s}) \triangleq (P_2^*(\mathbf{s}) + P_3^*(\mathbf{s}))/2$ as the optimal power allocated to the second timeslot by $P^*(\mathbf{s})$. We consider the following maximization problem:

$$\begin{aligned} \max \quad & \tfrac{FP_3+BP_2}{A+cP_3} \\ \text{such that} \quad & \tfrac{P_2+P_3}{2} \leq E, \end{aligned} \tag{9}$$

where $F \triangleq bcP_1^*$, $B \triangleq a(1+aP_1^*)(1+bP_1^*)$, and $A \triangleq 1+bP_1^*$. It is easy to see that the optimal power allocation for the above problem is

$$(\bar{P}_2, \bar{P}_3) = \begin{cases} (E, 0), & \text{if} \quad FA < B(A + cE) \\ (0, E), & \text{if} \quad \text{else} \end{cases} \tag{10}$$

Combining (10) with (8), we can argue that there exists an optimal power allocation for which either the source or the relay is silent in the second timeslot. When $P_3 = 0$, we let $P_1 = P_2 = (P_1^* + \bar{P}_2)/2$ without changing the achievable rate, which is equivalent to DT with constant power over the whole timeslot. ■

For both the AF and DT protocols, the optimal power allocation at each channel state can be found analytically [12]. Hence, an analytical solution for NAF can also be found by choosing between AF and DT at each channel state.

B. Compress and Forward Relaying

In this section we consider compress and forward (CF) relaying [6]. In CF the relay compresses the signal it received in the first timeslot, and transmits the compressed version to the destination in the second timeslot, while the source continues sending independent information. The compression is done in Wyner-Ziv sense by utilizing the destination's own correlated observation about the source signal of the first slot. Note that in the CF protocol it is not necessary to have $t(\mathbf{s}) = 1/2$, resulting in more flexibility compared to AF.

The instantaneous capacity for CF using resource allocation function $P(\mathbf{s})$ can be written as [8]:

$$\begin{aligned} C_{CF}(\mathbf{P},\mathbf{s}) =& t(\mathbf{s})\log\left(1 + aP_1 + \frac{bP_1}{1+\sigma_w^2}\right) \\ &+ (1 - t(\mathbf{s}))\log(1 + aP_2), \end{aligned} \tag{11}$$

where

$$\sigma_w^2 = \frac{1 + aP_1 + bP_1}{\left(\left(1 + \frac{cP_3}{1+aP_2}\right)^{\frac{1-t(\mathbf{s})}{t(\mathbf{s})}} - 1\right)(1 + aP_1)}. \tag{12}$$

[1]In the following analysis, with abuse of notation, we sometimes omit the dependence on $\mathbf{s}$ and use P_1, P_2, P_3 and t for the resource allocation functions.

The delay-limited capacity of CF protocol is found by solving (1) where $C(\mathbf{P}, \mathbf{s})$ is replaced with $C_{CF}(\mathbf{P}, \mathbf{s})$.

While using Wyner-Ziv compression at the relay improves the performance, it also increases the complexity of the relay encoder and the destination decoder. We also consider a simpler scheme in which the relay compresses its received signal ignoring the side information at the destination. This scheme is called estimate-and-forward (EF). The instantaneous capacity of EF with power allocation $P(\mathbf{s})$ at state $\mathbf{s}$ is

$$C_{EF}(\mathbf{P}(\mathbf{s}), \mathbf{s}) = t(\mathbf{s}) \log\left(1 + aP_1 + \frac{bP_1}{1 + \hat{\sigma}_w^2}\right) + (1 - t(\mathbf{s})) \log(1 + aP_2), \tag{13}$$

where

$$\hat{\sigma}_w^2 = \frac{1 + bP_1}{\left(1 + \frac{cP_3}{1+aP_2}\right)^{\frac{1-t(\mathbf{s})}{t(\mathbf{s})}} - 1}. \tag{14}$$

As expected, EF has a larger quantization noise than CF, i.e., $\hat{\sigma}_w^2 \geq \sigma_w^2$. When we provide delay-limited capacity comparisons of different protocols, we will also consider simpler version of CF and EF with fixed and equal time allocation, that is, $\mathbf{P}(\mathbf{s}) = (P_1, P_2, P_3, 1/2)$ for all $\mathbf{s}$. The instantaneous capacities for these schemes are denoted as $C_{CF}^{t=1/2}(\mathbf{P}(\mathbf{s}), \mathbf{s})$ and $C_{EF}^{t=1/2}(\mathbf{P}(\mathbf{s}), \mathbf{s})$. Their expressions can be found by setting $t = 1/2$ in equations (11)-(14). Note that both CF and EF protocols encompass DT as a special case, hence they are inherently opportunistic in the sense of [7].

Lemma 3.2: For any given power allocation and channel states, the instantaneous capacity of EF with fixed $t = 1/2$ is greater than or equal to the instantaneous capacity of NAF.

Proof: The capacity of EF with fixed time allocation can be written as

$$C_{EF}^{t=1/2}(\mathbf{P}(\mathbf{s}), \mathbf{s}) = \frac{1}{2}\log\left\{1 + aP_1 + aP_2(1 + aP_1) + \frac{bcP_1P_3(1+aP_2)}{(1+aP_2)(1+bP_1)+cP_3}\right\}.$$

Using $P_3 \geq 0$ and

$$\frac{1 + aP_2}{(1 + aP_2)(1 + bP_1) + cP_3} \geq \frac{1}{1 + bP_1 + cP_3},$$

we get

$$C_{EF}^{t=1/2}(\mathbf{P}(\mathbf{s}), \mathbf{s}) \geq \frac{1}{2}\log\left\{1 + aP_1 + \frac{aP_2(1+aP_1)(1+bP_1)}{1+bP_1+cP_3} + \frac{bcP_1P_3}{1+bP_1+cP_3}\right\} = C_{NAF}.$$

■

C. Hybrid Relaying

To maximize the delay-limited capacity for each protocols, we find the optimal resource allocation at each channel state so that the target rate is supported. However, there is no reason to be limited to a single cooperation protocol. Instead, at each channel realization, we can choose the optimal cooperation protocol along with its corresponding optimal resource allocation. This is similar to the ODF protocol in [7] where the choice is among DT and DF. Here, we include CF in the possible set of cooperation protocols. Note that, once we can choose among DF and CF we do not need to consider DT, NAF or EF, since DT is already a special case of CF, EF is inferior to CF, and NAF is inferior compared to EF by lemma 3.2.

The delay-limited capacity of the hybrid protocol can be found as

$$\max_{\mathbf{P}(\mathbf{s}) \in \bar{\Omega}} R, \tag{15}$$
$$\text{such that } \max\{C_{CF}(\mathbf{P}, \mathbf{s}), C_{DF}(\mathbf{P}, \mathbf{s})\} \geq R, \text{ for all } \mathbf{s}.$$

D. Upper Bound to the Delay-Limited Capacity

Using the usual cut-set bounds for the half-duplex relay we find an upper bound (SCB) to the delay-limited capacity. For any power and time allocation scheme, the instantaneous capacity can be upper bounded by

$$C_{SCB}(\mathbf{P}, \mathbf{s}) = \min\left\{t\log(1 + (a + b)P_1) + (1 - t)\log(1 + aP_2), t\log(1 + aP_1) + (1 - t)\log(1 + aP_2 + cP_3)\right\}.$$

Solving

$$\max_{\mathbf{P}(\mathbf{s}) \in \bar{\Omega}} R, \tag{16}$$
$$\text{such that } C_{CSB}(\mathbf{P}, \mathbf{s}) \geq R, \text{ for all } \mathbf{s}.$$

yields an upper bound to the delay-limited capacity since C_{CSB} is an upper bound to the instantaneous capacity at each channel realization.

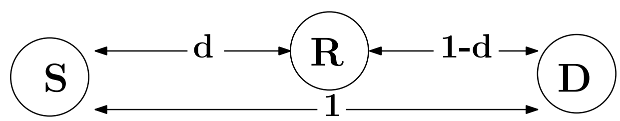

Fig. 3. The model for the source, the relay and the destination locations.

IV. Numerical Results

In this section, to consider the effect of the relay location on the performance of the network, we follow the model in Figure 3. We normalize the distance between the source and the destination, and assume that the relay is located between the source and the destination. For a fixed pathloss exponent α, the effect of this normalization is scaling the long-term average total transmit power. We denote the source-relay distance as d, where $0 < d < 1$, and the relay-destination distance as $1 - d$. Then the overall network channel state is denoted by $\mathbf{s} = (a, b, c)$, where a, b and c are independent exponential random variables with means $\lambda_a = 1$, $\lambda_b = \frac{1}{d^\alpha}$, and $\lambda_c = \frac{1}{(1-d)^\alpha}$, respectively. All of the results in this section assume $\alpha = 4$.

Figure 4 demonstrates the delay-limited capacity as a function of the long-term average total transmit power constraint for various relaying protocols for a relay location of $d = 0.5$. The cut-set bound (CSB) is also included for comparison. ODF with optimized time allocation performs closest to the CSB in this case. CF with fixed time allocation achieves almost the same performance as CF with optimized time

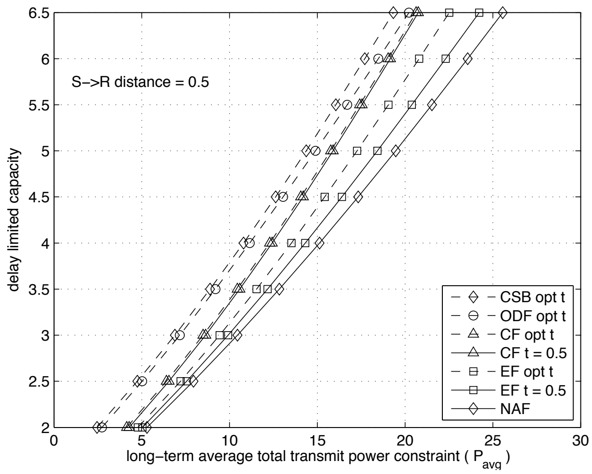

Fig. 4. Delay-limited capacity versus the long-term average total transmit power constraint P_{avg} when $d = 0.5$.

allocation when $d = 0.5$. This observation is confirmed again in Figure 5, where the results show that time allocation is more important for CF when the relay is close to the source or the destination. The EF protocol, on the other hand, benefits more from optimal time allocation. The simplest protocol, NAF, although inferior to the other protocols still achieves a nonzero delay-limited capacity. This shows that even a simple cooperation strategy can improve the performance of delay-limited systems. Furthermore, in the low power regime, NAF still can be a viable alternative as the gains of higher complexity protocols become smaller.

Figures 5-7 show the variation of the delay-limited capacity with respect to relay location with the long-term average total power constraint of 10 dB. Figure 5 illustrates the delay-limited capacity of CF, EF and NAF with respect to different relay locations with and without optimal time allocation. The results show that EF with optimal time allocation can achieve higher delay-limited capacity than CF with fixed time allocation when the relay is very close to the source or to the destination. When the relay is close to the destination, EF benefits less from optimal time allocation. When the relay is close to the source, NAF performs almost as well as EF with fixed time allocation. Note also that the gap between NAF and CF with optimal time allocation is almost independent of the relay location.

Figures 6 and 7 show the delay-limited capacity of CF, ODF and the hybrid protocol with and without optimal time allocation, respectively. The CSB is also included for comparison. For the case with optimal time allocation, when the relay is close to the source, ODF and the hybrid protocol almost coincide with the CSB. As the relay moves towards the destination, the gap becomes larger. For CF, the gap between CSB becomes larger at first when relay moves towards the destination, then become smaller as the relay is very close to the destination. This is in accordance with the relative performances of these protocols in terms of their ergodic capacities [6]. From Figure 6, we note that CF outperforms

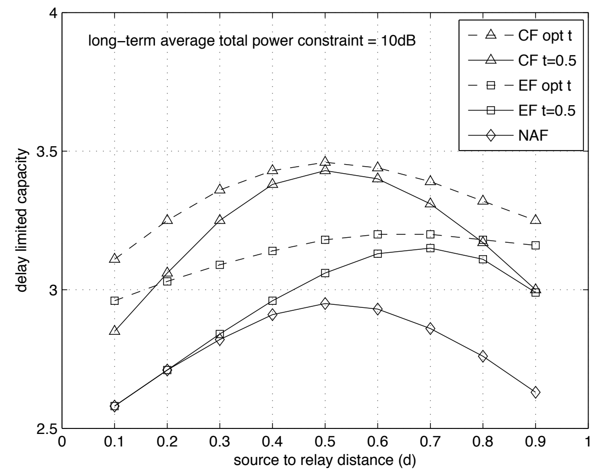

Fig. 5. Delay-limited capacity of CF, EF and NAF protocol with and without optimal time allocation versus source to relay distance d.

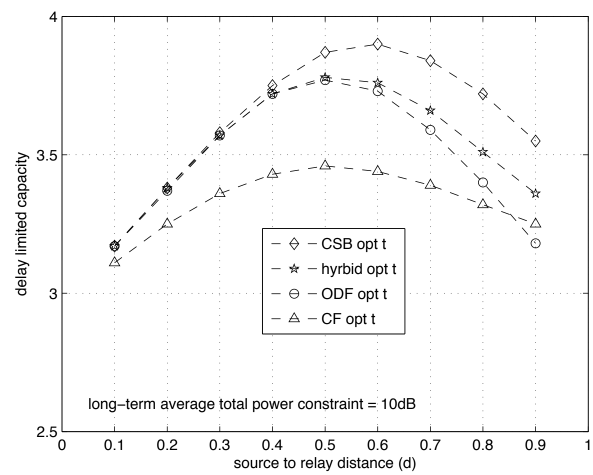

Fig. 6. Delay-limited capacity of ODF, CF, the hybrid protocol, and the CSB with optimal time allocation versus source to relay distance d.

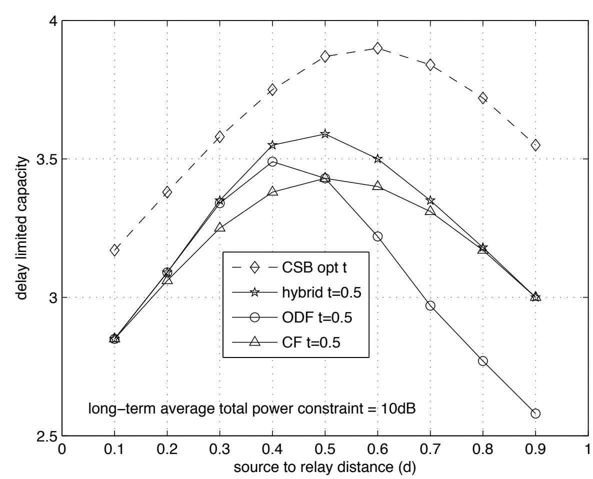

Fig. 7. Delay-limited capacity of ODF, CF, and the hybrid protocol with $t = 0.5$, and the CSB with optimal t versus source to relay distance d.

ODF when $d > 0.83$. Thus by adaptively choosing between CF and ODF, the hybrid protocol is superior to both CF and ODF. The trend is the same in Figure 7. We notice that without optimal time allocation, the performance gaps between the CSB and the other protocols become larger compared with the case with optimal time allocation. Among all the protocols, ODF is affected most by the absence of optimal time allocation. We note that CF begins to outperform ODF when $d = 0.5$ in Figure 7. In this case the advantage of the hybrid protocol is even more obvious.

V. Conclusion

In this paper, we analyze and compare the delay-limited capacity of several cooperative protocols including CF, EF, ODF, and NAF under a long-term average total transmit power constraint and under the assumption that the instantaneous channel amplitudes are available at both the source and the relay prior to transmission. Given a particular cooperative protocol and an expression for its instantaneous mutual information in terms of the channel state and transmit powers, knowledge of the instantaneous channel amplitudes allows the source and the relay to minimize their instantaneous total power allocation while guaranteeing that the rate does not fall below a desired threshold in each channel realization. This knowledge also facilitates opportunistic transmission in the sense that the source and the relay can select a cooperative protocol from the family of available protocols that requires the minimum total transmit power in order to achieve the desired rate for the given channel state. This concept of opportunistic protocol selection has been explored on a smaller scale in prior studies, e.g. opportunistic decode and forward where the choice is between DF and DT, but this paper is the first to consider opportunistic transmission over a large family of cooperative protocols with optimal resource allocation.

Our results show that, for protocols employing optimal resource allocation under a total power constraint, the instantaneous rate of EF is at least as good as that of NAF for any channel state. Since the instantaneous rate of CF is also at least as good as that of EF for any channel state, we propose a hybrid opportunistic protocol in which the source and the relay choose between CF and ODF with optimal resource allocation in each channel state. The proposed hybrid opportunistic protocol offers the best delay-limited capacity performance of all of the protocols considered since it always selects the protocol with the minimum total transmit power in each channel state. Our numerical results show that the hybrid opportunistic protocol tends to offer the most gain with respect to ODF when the mean of the relay-destination channel is better than that of the relay-source channel. The hybrid opportunistic protocol tends to offer the most gain with respect to CF when the mean of the relay-destination channel is similar to the mean of the relay-source channel.

While our results show that the delay-limited capacity of NAF is not as good as any of the other cooperative protocols considered in this study, it is the only protocol that we considered in which the optimal resource allocation can be computed analytically. Hence, NAF may still have a role in practical cooperative transmission systems since its complexity, both in terms of resource allocation and relay implementation, can be much lower than that of the other protocols considered in this paper.

VI. Acknowledgements

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