This tutorial provides an introduction to game-theoretic analysis of selfish behavior in wireless ad hoc networks with a focus on the packet forwarding and relaying scenarios. Noncooperative game theory has been successfully applied to many problems in wireless networks, but much of the prior work on packet forwarding has been centered around the development of extrinsic incentive mechanisms including community enforcement through reputation propagation and virtual currency exchange under the implicit assumption that the benefits of cooperation are one-sided. While these techniques can be effective at stimulating cooperation among selfish nodes, each has shortcomings that include, for example, additional network traffic and/or potential for fraud. Recent developments in this area have shown that there are scenarios in which the benefits of cooperation are not one-sided and selfish nodes can cooperate without extrinsic incentive mechanisms. This notion of natural cooperation is appealing in that it can be implemented at layers in the protocol stack below the application layer and can be analyzed using tools from noncooperative game theory. This article provides specific examples as well as general guidelines that describe the conditions under which natural cooperation can and cannot exist in wireless networks with selfish nodes. Comparisons between the efficiency of systems that rely on natural cooperation and systems that enforce cooperation under a central authority are also provided.

INTRODUCTION
Multihop transmission is often used in wireless ad hoc networks to enable the delivery of messages to destination nodes beyond the range of a source node as well as to increase the energy efficiency of the network by allowing packets to be delivered over
several short links rather than one long link. One or more intermediate nodes between the source and destination assist in the transmission by forwarding or relaying the packet along the route to the destination. In networks with a central authority, e.g., a military network, it is reasonable to assume that intermediate nodes will always forward packets for other nodes when requested to do so. This assumption may not be valid, however, in networks without a central authority: autonomous nodes acting in their own self-interest may refuse to use their limited resources to forward packets for other nodes. This can lead to inefficient use of the network resources since messages may have to be retransmitted and/or rerouted through different paths to the destination node [1].

Over the last decade, several techniques have been proposed to encourage cooperation and improve the efficiency of wireless ad hoc networks with autonomous nodes acting in their own self-interest. One approach is the development of a virtual currency [2], [3] that allows nodes to be remunerated for packet forwarding or relaying. Nodes accumulate credit through cooperative behavior and use this credit to purchase cooperative behavior from other nodes. Pricing and account balances can be managed centrally [4] or, if tamper proof hardware is available, balances can be kept locally by each node [5]. The idea of virtual currency is intuitively appealing in many scenarios but the analysis of these techniques has been mostly based on heuristics. Moreover, the use of virtual currency requires at least some increase in network traffic overhead, increased node complexity, and may also have the potential for fraud and/or collusion as discussed in [6].

Another approach to encouraging cooperation in wireless ad hoc networks is community enforcement through the dissemination of reputation information about each node. The basic idea in this case is that nodes monitor the transmissions of other nodes to see if they are forwarding packets or if they are misbehaving. In [1], two applications were proposed to facilitate identification of misbehaving nodes and routing of traffic around these nodes. This approach has been extended to include punishment of misbehaving nodes by having nodes refuse to forward packets originating from misbehaving nodes [7], [8]. The problem of coming to an agreement on the reputation of a node was discussed in [9]. A self-learning repeated-game framework for community enforcement of cooperation under different observability assumptions was also proposed recently in [10]. Although community enforcement has been shown to encourage cooperation, there are several potential drawbacks including the potential for misinterpreting the behavior of nodes, increased node complexity needed to monitor the behavior of other nodes, increased network overhead for reputation propagation messages, as well as the potential for collusion and/or spoofing [1].

A common theme of the virtual currency and community enforcement techniques is that the near-term costs and benefits of cooperative behavior are expected to be one sided. The implicit assumption is that nodes that are asked to forward packets do not receive any near-term benefit (and perhaps only a small chance of long-term benefit) for this help. Hence, they will not help unless they are remunerated with a currency (or a good reputation) that they can later use to obtain cooperation from other nodes. While this assumption may be true in some scenarios, recent developments have also shown that cooperative behavior can be jointly beneficial to the nodes in a wireless ad hoc network even on a near-term basis in some scenarios. Rather than developing extrinsic incentive mechanisms at the application layer to induce cooperation, the idea is to consider the possibility of natural cooperation at layers in the protocol stack below the application layer. The analysis of these types of systems is based on techniques from noncooperative game theory and is focused primarily on the existence and efficiency of cooperative equilibria rather than heuristics.

This article provides an introduction to noncooperative game theoretic analysis of wireless ad hoc networks with autonomous nodes acting in their own self-interest. Our focus is on the possibility of natural cooperation at the network layer (packet forwarding games) and the physical layer (relaying games) without the use of extrinsic incentive mechanisms at the application layer. We describe several necessary conditions for natural cooperation to emerge including the need for a repeated game framework, credible punishment for defection, and patience in the sense that nodes value long-term payoffs. While natural cooperation is appealing from several perspectives, extrinsic incentive mechanisms at the application layer are likely to still play a role in scenarios where natural cooperation is difficult or impossible to sustain. This article discusses these scenarios in more detail as well as scenarios under which natural cooperation can be inefficient.

THREE ILLUSTRATIVE PACKET FORWARDING SCENARIOS

This section describes three simple packet forwarding scenarios to provide specific examples under which natural cooperation can emerge as well as how the possibility of natural cooperation is affected by different assumptions about the structure and duration of the interaction between the nodes. Through these examples, we also illustrate three fundamental noncooperative game theoretic concepts that appear throughout the literature in this field: i) a Nash equilibrium (NE), ii) Pareto optimality, and iii) repeated games with uncertain ending. These first two concepts are basic tools of noncooperative game theory and the interested reader is referred to [11] for a more comprehensive treatment of this material.

A TWO-PLAYER PACKET FORWARDING SCENARIO

As a didactic example, consider the two-source, two-destination network shown in Figure 1. Each source wishes to send packets

![Figure 1: A two-player packet forwarding scenario.](image-url)
For example, if \( S_1 \) forwards and \( S_2 \) does not forward, i.e., the players in the game receive as a function of the chosen actions. The two players are agents in a two-relaying framework, where they need to define the set of available actions, the set of available destinations, and the cost of forwarding a packet for the other source. Both sources are assumed to have identical fixed costs to forward packets to their respective destinations, i.e., \( S_1 \) sends packets to \( D_1 \) and \( S_2 \) sends packets to \( D_2 \), and each source relies on the other source to forward its packet. Without cooperation, neither source can deliver packets to its destination. Note that this model differs from the relaying model discussed in the section “The Emergence of Cooperation in Relaying Games” as it does not include a direct link between a source and its desired destination.

### Stage Game Formulation

To put this scenario into a noncooperative game-theoretic framework, we need to define the set of players, the set of available actions, and the payoffs that each player receives as a function of the chosen actions. The two players in Figure 1 are the sources \( S_1 \) and \( S_2 \) and the available actions, assumed to be chosen simultaneously by the sources, are defined as “forward” or “do not forward” the packet of the other source. The payoff is defined as the difference between the reward of a successfully delivered packet and the cost of forwarding a packet for the other source. Both sources are assumed to have an identical fixed cost to forward a packet for the other source, denoted as \( 0 < c < 1 \), and a fixed unit reward for having a packet successfully delivered to its destination.

The payoff matrix in Table 1 shows the payoffs for both sources in the pair \( (\pi_1, \pi_2) \), where \( \pi_i(\alpha) \) is the payoff for the \( i \)th source, as function of the actions that both sources choose. For example, if \( S_1 \) forwards and \( S_2 \) does not forward, i.e., \( \alpha = (F, DNF) \), then \( S_1 \) receives a payoff of \( \pi_1(\alpha) = -c \) and \( S_2 \) receives a payoff of \( \pi_2(\alpha) = 1 \). This scenario has been described in [12] as the two-player Forwarder’s Dilemma because of its similarity to the classic Prisoner’s Dilemma [13].

The sources are assumed to be rational in the sense that they choose their actions in order to maximize their own payoffs. In the case when \( 0 < c < 1 \), the only rational action for both sources is to choose “do not forward.” To see this, for \( S_1 \), note that the payoff is always better for \( S_1 \) if \( S_1 \) plays “do not forward” irrespective of the action chosen by \( S_2 \) since
\[
0 = \pi_1(DNF, DNF) > \pi_1(F, DNF) = -c \quad \text{and} \quad 1 = \pi_1(DNF, F) > \pi_1(F, F) = 1 - c.
\]
The same can be shown for \( S_2 \) by symmetry of the payoff matrix. Hence the only rational action profile is \( \alpha = (DNF, DNF) \). This result allows us to introduce the important concept of the NE [11] below.

### Definition 1

An NE is a set of actions \( \alpha^* = (a_1^*, \ldots, a_N^*) \) such that no individual player can improve its own payoff by unilaterally changing its action, i.e.,

\[
\pi_i(a_1^*, \ldots, a_i^* - 1, a_i^* + 1, \ldots, a_N^*) > \pi_i(a_1^*, \ldots, a_i^*, \ldots, a_N^*)
\]

for all \( a_i \) in the available action set of player \( i \) and for all \( i = 1, \ldots, N \).

The action profile \( \alpha = (DNF, DNF) \) is the only NE of the two-player packet forwarding game. The dilemma of this game is that both players could receive a better payoff of \( 1 - c > 0 \) if they chose the action profile \( \alpha = (F, F) \). This action profile is Pareto optimal [11], as defined below, but is not an NE of the two-player packet forwarding game.

### Definition 2

A set of actions \( \alpha = (a_1, \ldots, a_N) \) is Pareto optimal if there exists no other set of actions for which one or more players can improve their payoffs without reducing the payoffs of other players.

From this definition, it can be seen that any action profile other than \( \alpha = (DNF, DNF) \) is Pareto optimal for the two-player packet forwarding game.

While the concepts of an NE and Pareto optimality have been described here for pure strategies in which each player chooses its action deterministically from its available action set, these concepts can also be easily extended to the more general case of mixed strategies where each player specifies a probability mass function over its available action set [14]. Some games only have mixed strategy Nash equilibria, e.g., the “jamming game” described in [12]. It can be shown that the pure strategy action profile \( \alpha = (DNF, DNF) \) is the only NE of the two-player packet forwarding game, however, even when the nodes can choose mixed strategies.

### Repeated Game Formulation

The game formulation in the previous section assumed that the packet forwarding game was played just once. In many practical scenarios, it is reasonable to expect that the packet forwarding game may be played several times. We introduce the concept of a repeated game in this section and show how a repeated game formulation can allow a Pareto-optimal cooperative NE to emerge in repeated games with uncertain endings. Repeated spectrum sharing games and repeated file sharing games have also been described in [15]–[17].

First consider the scenario where the stage game is repeated \( 1 \leq L < \infty \) times with \( L \) known to all of the players. Looking at the \( L \)th stage game, since there is no possibility of gain from future cooperation, it is clear that both sources will rationally choose the action “do not forward” to maximize their payoff. Knowing this, both sources will then rationally choose “do not forward” in the \( (L - 1) \)th stage game, and so on, ensuring that the only rational strategy for both sources is to not forward in any of the stage games [13, p. 10].

Now consider the scenario where the game is repeated but the players do not know when the game will end. The expected total payoff in this case is equivalent to a game having an infinite number of stages with future payoffs discounted according to the expected duration of the
game [13]. Each player seeks to maximize its total discounted payoff
\[
P_i = \sum_{n=0}^{\infty} \delta^n \pi_i(a(n)),
\]
where \(0 < \delta < 1\) is the discount factor and \(\pi_i(a(n))\) is the \(i\)th player’s payoff given action profile \(a(n)\) in the \(n\)th stage game. If the players expect to have many opportunities for future interaction, then \(\delta\) will be close to one. The discount factor \(\delta\) can also interpreted as the patience of the players in the game; a patient player will have \(\delta\) close to one.

In repeated games, players use a strategy to specify their actions in each stage game. We define a trigger strategy in the two-player repeated packet forwarding game as follows: each source chooses the action “forward” in the first stage game and continues to play “forward” until the other player chooses “do not forward.” A player that chooses “do not forward” is said to defect. If either player defects, the other player plays “do not forward” in all future stage games. In other words, defection triggers everlasting punishment.

To see that (trigger, trigger) can be a Pareto-optimal NE of the two-player repeated packet forwarding game with uncertain ending, we can calculate the total discounted payoff when both sources faithfully play the trigger strategy as \(P_1 = P_2 = 1 - c/1 - \delta\). If \(S_1\) deviates from the trigger strategy by defecting in stage game \(n\), then its total discounted payoff is
\[
P_1 = (1 + \delta + \cdots + \delta^{n-1})(1 - c) + \delta^n \cdot 1,
\]
which is no more than the total discounted payoff from faithfully playing the trigger strategy if \(c \leq \delta\). This result implies that if \(\delta\) is sufficiently close to one, i.e., the players are sufficiently patient, then neither player has any incentive to unilaterally deviate from the trigger strategy. Note that the strategy profile (always defect, always defect) is also an NE of the two-player repeated packet forwarding game with uncertain ending since neither player stands to gain from cooperation with an opponent that always defects.

To summarize, three key factors that allow natural cooperation to emerge as an equilibrium of the two-player packet forwarding game are i) a repeated game with uncertain ending, ii) credible punishment for defection, and iii) sufficiently patient players. Punishment in this scenario is direct in the sense that each player can directly affect the payoff of the other player. As shown in the following section, punishment for defection does not need to be direct to establish cooperation without extrinsic incentive mechanisms.

A THREE-PLAYER PACKET FORWARDING SCENARIO

To demonstrate that direct punishment for defection is not necessary to establish natural cooperation, consider the three-source, three-destination network shown in Figure 2. As in the two-player packet forwarding game, each source wishes to send packets to its respective destination and relies on another source to forward its packets.

Assuming the same payoff structure as the two-player game, Table 2 shows the payoff matrices of the three-player game. The payoffs in this case are given in the form \((\pi_1, \pi_2, \pi_3)\).

It can be shown that the only NE of the three-player packet forwarding stage game is the action profile \(a = (\text{DNF}, \text{DNF}, \text{DNF})\). In the more interesting case of a repeated game with uncertain ending, we can consider the case when all of the players play the trigger strategy. We can compute the total discounted payoffs of the (trigger, trigger, trigger) strategy profile as \(P_1 = P_2 = P_3 = 1 - c/1 - \delta\). If \(S_1\) deviates from the trigger strategy by defecting in stage game \(n\), then \(S_3\) will cease forwarding packets in stage game \(n + 1\) and \(S_2\) will cease forwarding packets in stage game \(n + 2\). Hence \(S_1\) will receive a total discounted payoff of
\[
\Pi_1 = (1 + \delta + \cdots + \delta^{n-1})(1 - c) + (\delta^n + \delta^{n+1}) \cdot 1,
\]
which is no more than the total discounted payoff from playing the trigger strategy if \(c \leq \delta\). Hence, no source has any incentive to unilaterally change its strategy and (trigger, trigger, trigger) is a Pareto-optimal NE of the three-player packet forwarding game when \(c \leq \delta\).

The key difference in the three-player scenario is that it takes longer for the punishment of the trigger strategy to reach the defecting source. The defecting source enjoys two stage games with payoff of one before its packets are no longer forwarded. Nevertheless, for sufficiently patient sources, there is credible punishment for defection because \(S_1, S_2,\) and \(S_3\) form a dependency loop [18]. As discussed in the section “The


[TABLE 2] PAYOFF MATRICES FOR A SYMMETRIC THREE-PLAYER PACKET FORWARDING GAME.

<table>
<thead>
<tr>
<th>S3 DOES NOT FORWARD (DNF)</th>
<th>S2 DOES NOT FORWARD (DNF)</th>
<th>S2 FORWARDS (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 DOES NOT FORWARD (DNF)</td>
<td>(0, 0, 0)</td>
<td>(1, -c, 0)</td>
</tr>
<tr>
<td>S1 FORWARDS (F)</td>
<td>(-c, 0, 1)</td>
<td>(1-c, -c, 1)</td>
</tr>
<tr>
<td>S3 FORWARDS (F)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

IEEE SIGNAL PROCESSING MAGAZINE | SEPTEMBER 2009
Emergence of Cooperation in Packet Forwarding Games,” dependency loops are critical for establishing cooperation in networks without extrinsic incentive mechanisms.

**A FOUR-PLAYER PACKET FORWARDING SCENARIO**

As an example of a network that does possess a fully cooperative NE, consider the four-player packet forwarding game shown in Figure 3. We assume that the cost to forward each packet is \( c \), the reward for a successfully delivered packet is one, and that there is no congestion, i.e., each source can forward as many packets as requested by other sources in a stage game and all packets are forwarded instantaneously. We also assume that sources can distinguish the originating source (or, equivalently, the intended destination) of each packet and make individual forwarding decisions based on this information.

We can analyze the rational behavior of the sources in the network in Figure 3 without explicitly writing the payoff matrices. Note that sources S2 and S3 rely only on each other to forward packets to their respective destinations and do not require help from sources S1 and S4. Sources S1 and S4 rely on each other as well as intermediate forwarders S2 and S3 to forward packets to their respective destinations. Unlike the prior examples, the strategy profile (trigger, trigger, trigger, trigger) is not an NE of this repeated four-player packet forwarding game for any value of \( c > 0 \), even if the game has an uncertain ending. To see this, note that the total discounted payoff of S2 when all players faithfully play the trigger strategy is \( \Pi_2 = 1 - 3c/1 - \delta \) since S2 forwards packets for all three of the other sources under this strategy profile. Now suppose S2 does not forward any packets from S1 or S4, but continues to play the trigger strategy with packets from S3. The total discounted payoff in this case is \( \Pi_2 = 1 - c/1 - \delta \) since S1 and S4 can do nothing to punish S2 for defection. If S2 forwards any packets from S1 and/or S4, it can only decrease its total discounted payoff. These arguments can be extended to show that, when \( c \leq \delta \), an NE strategy profile of the repeated four-player packet forwarding game with uncertain ending is for S2 and S3 to each play the trigger strategy for forwarding each other’s packets and play the always defect strategy with packets originating from S1 and S4, while S1 and S4 both play the always defect strategy. Sources S2 and S3 receive a total discounted payoff in this case of \( \Pi_2 = \Pi_3 = 1 - c/1 - \delta \) and sources S1 and S4 receive a total discounted payoff \( \Pi_1 = \Pi_4 = 0 \).

The reason that S1 and S4 suffer in this example is that they do not have any ability to directly or indirectly reward or punish S2 and S3, yet they both rely on S2 and S3 to forward their packets. A central authority or an extrinsic incentive mechanism is necessary to ensure packets from S1 and S4 are forwarded by sources S2 and S3.

**THE EMERGENCE OF COOPERATION IN PACKET FORWARDING GAMES**

The examples discussed in the previous section illustrate several of the key factors necessary for natural cooperation to emerge as an equilibrium in networks with a small number of selfish sources. This section summarizes the recent developments toward understanding the potential for natural cooperation in packet forwarding games in larger networks. We also discuss the different assumptions in these studies that have led to different conclusions.

The potential for natural cooperation in a large-network scenario with \( N \) heterogeneous nodes distinguished according to \( R \leq N \) energy classes was studied in [19]. In each stage game, one of the \( N \) nodes is randomly chosen as the source and \( 1 \leq R \leq M \) relays are randomly chosen with equal probability from the \( N - 1 \) remaining nodes, where \( R \) is also random and \( M \) represents the maximum number of hops permitted in the network. The packet is successfully delivered only if all of the relays choose to forward the packet.

Assuming a payoff model corresponding to the normalized packet acceptance rate, defined as the ratio of successfully delivered packets to the number of packets transmitted in the limit as time goes to infinity, an important contribution of [19] is a description of the Pareto-optimal operating points for the network and a proof that the generous tit-for-tat strategy results in a Pareto-optimal NE in this scenario. The generous tit-for-tat strategy is a reactive strategy that can be described for node \( i \) as selecting the action “do not forward” if a node’s fraction of forwarded packets exceeds the Pareto-optimal fraction or if a node’s fraction of forwarded packets exceeds the fraction of packets forwarded by other nodes in the system for node \( i \) (plus a small positive offset \( \epsilon \)), otherwise choose the action “forward.” The parameter \( \epsilon > 0 \) enables generosity by having a node forward slightly greater fraction of packets than the fraction of packets forwarded on its behalf. Numerical results in [19] showed that this parameter must be strictly positive to ensure convergence to the Pareto-optimal packet forwarding rates.

Like the two- and three-player packet forwarding scenarios of the section “Three Illustrative Packet Forwarding Scenarios” the system model considered in [19] satisfies the necessary conditions for the emergence of a fully cooperative NE. The game is repeated indefinitely, and the players are infinitely patient in the sense that a packet forwarded in a future stage game is as valuable as a packet forwarded in the current stage game. Credible punishment for defection is established through the generous tit-for-tat strategy as well as the random topology of the network assumed to be independently generated in each stage game. This is a critical assumption as it establishes, over time, the necessary dependency loops that might not exist in a static scenario like the four-player example in the section.

![Figure 3](image-url)
“A Four-Player Packet Forwarding Scenario.” Another important difference with respect to the examples in the section “Three Illustrative Packet Forwarding Scenarios” is that the nodes in [19] do not distinguish the source of a packet or track the actions of the other individual nodes; each node was assumed to have a fixed probability of playing “forward” irrespective of the source. Unlike [19], however, the payoff functions in [18] include discounted future payoffs to reflect the patience of nodes in the network. An important contribution of [18] is the notion of a dependency graph and formalization of dependency loops. A dependency graph is illustrated in the five-node network shown in Figure 4. A directed edge in a dependency graph from source \( i \) to source \( j \) implies that the packet forwarding decisions of source \( i \) affect the payoffs of source \( j \). A dependency loop of node \( i \) is defined as a sequence of directed edges including node \( i \), beginning and ending at the same node. In Figure 4, it can be seen that \( S1 \) is in two dependency loops: \( L_1 = \{(1, 5), (5, 1)\} \) and \( L_2 = \{(1, 5), (5, 3), (3, 1)\} \). Nodes \( S3 \) and \( S5 \) are each in \( L_2 \), and nodes \( S2 \) and \( S4 \) are not in any dependency loops.

The notion of dependency loops was used in [18] to develop precise conditions under which cooperation is excluded (the only NE is for all nodes to play the \textit{always defect} strategy) as well as conditions under which natural global cooperation is possible (an NE exists in which all nodes play the \textit{tit-for-tat} strategy without extrinsic incentive mechanisms. For the former case, it was shown in [18] that if node \( i \) is a forwarder on at least one route and node \( i \) has no dependency loops, then its best strategy is to play \textit{always defect}. This is intuitive since no other node can punish node \( i \) if node \( i \) has no dependency loops.

A node that plays \textit{always defect} can cause an avalanche effect where other nodes in a route with the defecting node must also play \textit{always defect}. Moreover, if all other nodes \( j \neq i \) play \textit{always defect} then node \( i \) should also play \textit{always defect}. Hence, in certain network topologies, it was shown in [18] that the only NE is for all nodes in the network to play \textit{always defect}.

For a fully cooperative NE to exist in which all nodes play the \textit{tit-for-tat} strategy, one of the necessary conditions is that each node must have a dependency loop with all of the source nodes for which it forwards packets. This condition is clearly not satisfied in Figure 4 since \( S3 \) is not in a dependency loop with \( S2 \) or \( S4 \). If the required dependency loop does not exist, then there is no credible punishment for defection and nodes have no incentive to forward packets. Additional necessary conditions on the steepness of the nodes’ payoff functions and the length of the dependency loops necessary to establish the fully cooperative NE are given in [18, Theorem 3], but the existence of the necessary dependency loops for all of the nodes in the system was identified as the critical factor in establishing global cooperation. In fact, it was shown in 1,000 randomly generated network scenarios with 100–200 nodes and shortest path routing that there was no scenario in which this condition held for all of the nodes in the network.

While [18] establishes precise conditions under which full defection is the only NE and full cooperation is a possible NE, numerical results for random network topologies show that neither of these cases occur with significant probability. Hence, an important open problem is the characterization of conditions where full cooperation is not possible but pockets of natural cooperation can emerge in static networks. Along similar lines, it was noted in [18] that it is difficult for global natural cooperation to emerge in the static network scenario because the asymmetric relationships between nodes never change and nodes have no incentive to forward packets for nodes that can never reciprocate (or punish). This motivates an investigation of the potential for natural cooperation in mobility models that lie somewhere between the highly dynamic topology case considered in [19] and the static topology case considered in [18]. Mobility is likely to be a key factor in enabling natural cooperation since it allows different dependency loops to be established over time.

THE EMERGENCE OF COOPERATION IN RELAYING GAMES

The packet forwarding games described in the previous sections are all posed from a network-layer perspective: the transmit power required to forward a packet is a fixed constant for all nodes in the network, and a packet is considered lost if any node in a route rejects a forwarding request. This section describes recent developments on relaying games that consider the problem of delivering packets in an ad hoc network from a physical-layer perspective. Nodes in a relaying game are assumed to be able to control their transmit power according to the current channel conditions and quality-of-service requirements. Payoffs are based on the net amount of energy saved in delivering a packet to the intended destination with respect to direct (single-hop) transmission.

A two-player relaying game based on the simple orthogonal amplify-and-forward (OAF) cooperative transmission protocol [20] was analyzed in a noncooperative game-theoretic framework in [21]. The network is shown in Figure 5 where, like the packet forwarding scenario in Figure 1, each source sends packets to its respective destination and can also assist the other...
source by relaying packets. The main difference in this model with respect to the packet forwarding scenario in Figure 1 is that each source also has a direct channel to its destination. The destination combines observations from the source and the relay.

A key assumption in [21] is that both sources know the current channel state and can calculate the optimum energy allocation for OAF transmission given a target signal-to-noise ratio (SNR) at the destination as described in [22]. Given two available actions, “relay with the optimum OAF energy allocation” or “do not relay,” the payoff matrix of the stage game can be expressed as shown in Table 3, where $\beta_i \equiv 0$ is the optimum OAF relaying energy for source $i$, and $\alpha_i \equiv 0$ is the transmission energy saved by source $i$ when the other source relays with the optimum OAF relaying energy. Unlike packet forwarding games in the section “The Emergence of Cooperation in Packet Forwarding Games,” the payoff matrix of a relaying game does not include any reward for a successfully delivered packet because packets are assumed to always be delivered successfully, i.e., a minimum SNR for reliable transmission can always be achieved either by relaying or by direct transmission.

Two cases were analyzed in [21] assuming a repeated game with uncertain duration: nonfading channels and fading channels. In the first case, the channels were assumed to be approximately constant for a duration much longer than the typical duration of the repeated game. The payoff matrix in this case remains the same for each stage game. It was shown that the strategy profile (trigger, trigger) is NE if and only if

$$\max_{i \in \{1, 2\}} \left( \frac{h_i}{d} \right) = \delta,$$

where $0 < \delta < 1$ is the discount factor corresponding to the likelihood that the game will not terminate in the current stage. When $\delta \to 1$, i.e., both sources become infinitely patient, this condition reduces to $\alpha_i \geq \beta_i$ for $i = 1, 2$. In other words, the saved energy is at least as large as the required relaying energy for both sources in the current stage game. For smaller values of $\delta$, the saved energy must exceed the required relaying energy in the current stage game by enough margin to make the total discounted future payoffs more attractive than the immediate payoff from defection. Hence, natural cooperation is an NE of the two-player relaying game only if there is potential for mutual benefit and if the nodes are sufficiently patient. Note that the benefits of mutual cooperation do not need to be symmetric.

The second case analyzed in [21] assumed the channels to be constant in each stage game but to change independently between stage games according to a fading distribution. Since the optimum energy allocation is a function of the current channel state, the payoff matrix in Table 3 changes for each stage game. Moreover, only the payoffs of the current stage game are known to both sources; exact future payoffs are unknown since the future channel states are unknown. The sources decide their actions in the current stage game based only on the current payoff matrix and the statistics of the total discounted future payoffs.

A conditional trigger strategy that chooses the action “relay” only when mutual benefit is possible through mutual cooperation and neither node is asked to expend “too much” relaying energy was developed for this scenario. A failure to relay when these conditions are satisfied triggers noncooperation for all future stage games. The conditional trigger strategy required the development of a relaying energy ceiling for each source node to prevent defection in stage games where the immediate payoff from defection exceeded the expected total discounted future payoffs from cooperation. The relaying energy ceiling is computed using only the channel statistics and is applied in the conditional trigger strategy so that sources are not expected to relay if either source’s cost of relaying in the current stage exceeds the expected future payoff from continued cooperation. While this noncooperative action in the current stage game leads to some reduction in the expected total discounted payoff with respect to fully cooperative behavior, it is a necessary modification of the conventional trigger strategy to prevent rational defection and subsequent punishment by the other source. It was shown in [21] that both sources playing conditional trigger constituted an NE of the system and that this strategy achieved total payoffs very close to an optimum fully cooperative system for values of $\delta > 0.6$ in a lognormal fading channel scenario. Hence, natural cooperation was shown to be possible (and efficient) even in the scenario when sources have uncertain future payoffs, as long as each source can recognize when the other source node has a temptation to defect and can adjust its action in the current stage game to avoid triggering punishment.

One limitation of the results developed in [21] is that only a small-scale network scenario was analyzed. Relaying games for both small-scale and large-scale networks were recently considered in [23] where the effect of altruistic nodes on the existence of a cooperative NE was studied in a stage-game framework. The approach in [23] differs from [21] as well as the packet forwarding literature by partitioning the nodes in the network into two sets: selfish nodes and altruistic nodes.

<table>
<thead>
<tr>
<th>TABLE 3</th>
<th>PAYOFF MATRIX FOR A TWO-PLAYER RELAYING GAME.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S2 DOES NOT RELAY (DNR)</td>
</tr>
<tr>
<td>S1 DOES NOT RELAY (DNR)</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>S1 RELAYS (R)</td>
<td>(-b1, a2)</td>
</tr>
</tbody>
</table>
Selfish nodes are assumed to be interested in minimizing their own total energy consumption and altruistic nodes, which only serve the function of relaying packets in the network, are assumed to be interested in minimizing the overall energy consumption of the network.

The selfish nodes in the network are assumed to not be able to distinguish between sources of packets and, as such, adopt a mixed strategy corresponding to a fixed probability of accepting relay requests, denoted as \( p_i \) for the \( i \)th selfish node. The altruistic nodes in the network are assumed to have additional processing capabilities that allow them to distinguish the source of a packet as well as to allow tracking of relaying packets originated by the \( i \)th selfish node. The altruistic nodes, however, are such that mutual benefit is possible and the altruistic node can be considered to be the enforcer in this scenario: if the \( i \)th selfish node does not relay a packet, the altruistic node will punish the \( i \)th selfish node for noncooperative behavior. Setting \( p_a = 1 \) corresponds to conventional relaying where an altruistic node accepts all relaying requests.

In a small-network scenario with two selfish source nodes, two destination nodes, and one altruistic relay node, it was shown in [23] that natural full cooperation, i.e., \( p_i = 1 \) for all \( i \), emerges as the only NE of the repeated game when the channels are such that mutual benefit is possible and the altruistic node selects a relaying function with \( p_a < 1 \). Intuitively, the altruistic node can be considered the enforcer in this scenario: if the \( i \)th selfish node employs a strategy \( p_i < 1 \), the altruistic node will reject a fraction of relaying requests made by node \( i \) and the resulting energy cost of these unfulfilled relaying requests will outweigh the energy saved from rejecting relaying requests from the other source node. Since the existence of the fully cooperative NE relies on the network geometry under the path-loss channel assumption, an important consequence of this result is that a noncooperative network of selfish nodes could be transformed into a cooperative network by the insertion of an altruistic node in the network in an appropriate location.

The authors of [23] also considered a large-network scenario with \( N \) total nodes, any of which could be the source or destination for a packet and a fraction of which are altruistic nodes. An asymptotic analysis was presented showing that it is sufficient to have a vanishingly small fraction of altruistic nodes in the network to ensure that natural full cooperation emerges as an NE of the repeated game. Moreover, this NE was shown to be efficient in the sense that the average energy per packet follows the optimal scaling law \( \Theta(\sqrt{N \log^3 1/N}) \) for a network with unit node density [24], where \( y \) denotes the path loss exponent. Intuitively, this result was obtained by showing that, as long as the fraction of altruistic nodes scales at an appropriate rate [which is less than \( \Theta(N) \)], then almost all of the routes from any selfish source to a sufficiently distant destination will contain at least one altruistic node. The enforcement role of the altruistic nodes ensures that selfish nodes that reject relaying requests will be punished in future stage games. When this occurs, the source node must transmit a packet to the destination directly. Hence, cooperation is established through the credible threat of punishment for defection; a selfish node's expected payoff can only be diminished by any deviation from full cooperation.

The addition of altruistic nodes in wireless ad hoc networks is potentially a more appealing solution to the problem of stimulating cooperation in ad hoc networks than the development of extrinsic incentive mechanisms. The results in [23], however, are based on an analysis of the payoffs of a single stage-game with mixed strategies and not on repeated games with uncertain ending. Cooperative NE were shown to exist in small-network relaying games without altruistic nodes in [21], but large networks were not considered. Hence, an important open problem is the development of a conditions under which natural cooperation can emerge in large-network relaying games in a repeated-game framework.

**OTHER RELATED WORK**

Game theory has been applied to a variety of problems in communication networks at various layers. We briefly mention a few of the different sorts of games that have been analyzed at different layers of the protocol stack here for the purpose of context.

At the application layer, game theory has been used to analyze peer-to-peer file sharing games [25]. At the network layer, routing games [26] and packet forwarding games as described in the sections “Three Illustrative Packet Forwarding Scenarios” and “The Emergence of Cooperation in Packet Forwarding Games” have been studied. At the media access control layer, random access games [27] have been studied. And finally, several different games have been studied at the physical layer including the relaying games described in the section “The Emergence of Cooperation in Relaying Games,” jamming games [28], waveform adaptation games [29], interference avoidance games [30], spectrum sharing games [31], and power control games [32]. Many of these physical layer games are also discussed in the context of “cognitive radio games.” An excellent overview the application of game theory at different network layers can be found in [33].

**CONCLUSIONS**

This article provides an introduction to noncooperative game-theoretic analysis of selfish behavior in wireless ad hoc networks with a focus on the packet forwarding and relaying scenarios. While extrinsic incentive mechanisms like virtual currency and community enforcement have been previously proposed to stimulate cooperation among selfish nodes, it is now understood that natural cooperation can exist in some scenarios without any influence from extrinsic incentive mechanisms. Specific packet forwarding examples are provided that demonstrate several of the conditions that allow natural cooperation to emerge: i) a repeated game with uncertain ending, ii) credible punishment for defection, and iii) sufficiently patient players. Recent literature on packet forwarding and relaying games that has begun to clarify the necessary and sufficient conditions under which natural cooperation is possible was also summarized. These conditions may not hold in all networks, hence there is...
likely to always be a role for extrinsic incentive mechanisms in some applications. Nevertheless, the recent results in this area show that the added complexity of extrinsic incentive mechanisms may not be necessary to achieve near-optimum network efficiency in some scenarios.

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