ECE503 Spring 2014 Quiz 10

Your Name: _______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 30 points total. Suppose you wish to convert the continuous-time filter

\[ H_c(s) = \frac{1}{s + 1} \]

to a discrete-time filter \( H(z) \). For the following questions, assume a sampling period \( T_d = 0.1 \) and simplify your answers as much as possible.

(a) 15 points. Compute \( H(z) \) using the impulse invariance method so that \( h[n] = T_d h_c(nT_d) \).

(b) 15 points. Compute \( H(z) \) using the bilinear transform method.

2. 40 points. Given the following specifications for a discrete-time low pass filter

\[ \frac{1}{\sqrt{2}} \leq |H(e^{j\omega})| \leq 1 \text{ for } 0 \leq \omega \leq \pi/3 \quad (\text{passband}) \]
\[ |H(e^{j\omega})| \leq 0.25 \text{ for } 2\pi/3 \leq \omega \leq \pi \quad (\text{stopband}) \]

determine the minimum order \( N \) and cutoff frequency \( \Omega_c \) of a continuous-time Butterworth lowpass filter satisfying the specifications assuming the bilinear transform will be used to convert the continuous-time lowpass filter to discrete time. Assume a sampling period \( T_d = 1 \). Design your filter to match the stop band specification exactly. You do not need to calculate \( H_c(s) \) or \( H(z) \); just specify the minimum order \( N \) and cutoff frequency \( \Omega_c \) and show your reasoning.

3. 30 points. Observe that the system \( H_c(s) \) in problem 1 is a first-order Butterworth filter with \( \Omega_c = 1 \). Suppose this lowpass filter was designed from a specification with passband frequency edge \( \Omega_p = 0.5 \) and stop band frequency edge \( \Omega_s = 2 \). Transform \( H_c(s) \) to a highpass filter \( H_{hp}(s) \) with passband frequency edge \( \hat{\Omega}_p = 20 \). Also calculate the resulting stop band frequency edge \( \hat{\Omega}_s \). If you were to then convert \( H_{hp}(s) \) to a discrete-time filter, which method(s) would be appropriate (and why): impulse invariance or bilinear transform?
1. a) We can use eq. 7.10 to write \((A_K = 1, s_K = -1, N=1)\)

\[
H(s) = \frac{T_d}{1 - e^{-n s}} = \frac{0.1}{1 - e^{-0.1 s}}
\]

b) \(H(s) = \frac{1}{s} \left( \frac{-e^{-s}}{1 + e^{-s}} \right) + 1\)

via bilinear transform

\[
= \frac{1}{2s} \left( \frac{1}{1 + e^{-s}} \right) = \frac{\frac{1}{2s} \left( 1 + e^{-s} \right)}{\frac{1}{2s} - \frac{1}{2s} e^{-s}} = \frac{1 + e^{-s}}{21 - 19 e^{-s}}
\]

2. Step 1: pre-warp frequencies: \(\Omega = 2 \tan \left( \frac{\pi}{2} \right)\)

\[\Rightarrow \Omega_p = 2 \tan \left( \frac{\pi}{6} \right) = 1.1547\]

\[\Omega_S = 2 \tan \left( \frac{\pi}{3} \right) = 3.4641\]

passband: \(\frac{1}{1 + (\frac{\Omega_p^{2N}}{\Omega_c})^N} \geq \frac{1}{2} \Rightarrow (\frac{\Omega_p}{\Omega_c})^{2N} \leq 1\)

stopband: \(\frac{1}{1 + (\frac{\Omega_S^{2N}}{\Omega_c})^N} \leq \frac{1}{16} \Rightarrow (\frac{\Omega_S}{\Omega_c})^{2N} \geq 15\)

\[2N \left[ \log \Omega_p - \log \Omega_c \right] = 0\]

\[2N \left[ \log \Omega_S - \log \Omega_c \right] = \log 15\]

\[2N \left[ \log \Omega_p - \log \Omega_S \right] = -\log 15 \Rightarrow N = 1.23; \text{ set } N=2\]

match stopband exactly: \(\left( \frac{\Omega_S}{\Omega_c} \right)^4 = 15 \Rightarrow \frac{\Omega_S}{\Omega_c} = \sqrt[4]{15} \Rightarrow \Omega_c = 1.7602\)

3. Lowpass \(\rightarrow\) highpass: \(s \rightarrow \frac{\hat{s} \Omega_p}{s} \quad \Omega_p = 0.5, \hat{s} \Omega_p = 20\)

We have \(\Omega_p \hat{s} \Omega_p = 10, \text{ so } s \rightarrow \frac{10}{s}\)

\(H_{hp}(s) = H_c \left( \frac{10}{s} \right) = \frac{4}{10 + 1} = \frac{s}{10 + S}\)

The new stopband frequency \(\hat{s} \Omega_S = \frac{s \hat{s} \Omega_p}{\Omega_S} = \frac{10}{2} = 5\)

Note that only the bilinear transform is appropriate here since this highpass filter is not bandlimited.