1. 4 points. Specify the N-P decision rule and probability of detection for the following hypothesis testing problem

\[ H_0 : p_Y(y; H_0) = e^{-y}u(y) \]
\[ H_1 : p_Y(y; H_1) = 2e^{-2y}u(y) \]

as a function of the significance level \( \alpha \) where \( u(y) \) is the usual unit step function equal to one for \( y \geq 0 \) and equal to zero otherwise. Plot the probability of detection as a function of the significance level of the test.

2. 4 points. Specify the N-P decision rule and probability of detection for the following hypothesis testing problem

\[ H_0 : y \sim \mathcal{N}(0,1) \]
\[ H_1 : y \sim \mathcal{U}(-1,1) \]

as a function of the significance level \( \alpha \). Plot the probability of detection as a function of the significance level of the test.

3. 4 points. Specify the Bayes decision rule for problem 1 as a function of the prior probability \( \pi_0 = \text{Prob}\{\text{state is } x_0}\} \) assuming the uniform cost assignment (UCA). Also plot the Bayes risk as a function of \( \pi_0 \).

4. 5 points. (Revisiting the quality checking problem from HW2): Suppose you work in a microprocessor manufacturing facility and that, before boxing and shipping, each microprocessor undergoes a quality check to avoid shipping defective units. The quality checking machine has the following characteristics:

- It declares good microprocessors to be defective (D) with probability \( p = 0.15 \).
- It declares defective microprocessors to be good (G) with probability \( q = 0.03 \).

Suppose there are \( n \) such quality checking machines that give independent results with the same probabilities. Also let \( H_0 \) be the hypothesis that the microprocessor is good and let \( H_1 \) be the hypothesis that the microprocessor is defective. You receive some new information from manufacturing and customer service saying that

- It costs $200 to replace a defective microprocessor after it has been shipped to the customer.
- It costs $50 each time good microprocessors are discarded.
- It costs nothing when good microprocessors are shipped and bad microprocessors are discarded.

Assume \( n = 2 \). Determine the Bayes decision rule and the associated Bayes risk as a function of the prior probability \( \pi_0 = \text{Prob}\{\text{microprocessor is good}\} \). Comment on your decision rule in the extreme cases when \( \pi_0 \to 0 \) and \( \pi_0 \to 1 \). Plot the risk of the Bayes detector as a function of \( \pi_0 \).

5. 4 points. Kay II: 3.18.

6. 4 points. Kay II: 3.21. Be explicit about how you derive your decision regions.