ECE531 Homework Assignment Number 4

Due by 8:50pm on Wednesday 23-Feb-2011

Make sure your reasoning and work are clear to receive full credit for each problem.

1. 5 points total. Suppose you have a communication system in which two signals are transmitted to convey one bit of information. The signals are

\[ x_0 : \cos \left( \frac{\pi}{4} n \right) \]
\[ x_1 : \cos \left( \frac{\pi}{4} n + \phi \right) \]

for \( n = 0, 1, \ldots, 3 \) where \( 0 < \phi \leq \pi \) is known. The observation at the detector is corrupted by stationary zero-mean additive Gaussian noise \( w[n] \) with covariance

\[
E[w[n]w[m]] = \begin{cases} 
0.50 & n = m \\
0.25 & |n - m| = 1 \\
0 & \text{otherwise.}
\end{cases}
\]

(a) 3 points. Suppose \( \phi = \pi/2 \). Determine the minimum probability of error decision rule and its probability of error as a function of the prior probability \( \pi_0 \).

(b) 2 points. Suppose \( \pi_0 = 0.5 \) and you are allowed to design the signal in state 1 to minimize the probability of error. Determine the value of \( \phi \) that minimizes the probability of error.

2. 4 points. Kay II: 4.6

3. 4 points. Kay II: 4.7

4. 4 points. Suppose the state \( x \sim \mathcal{U}(0, 2) \) and we receive one observation \( Y = x + W \) where \( p_W(t) = e^{-t}u(t) \). Determine the Bayes decision rule to decide between the composite hypotheses

\[ \mathcal{H}_0 : 0 \leq x < 1 \]
\[ \mathcal{H}_1 : 1 \leq x \leq 2. \]

5. 4 points. Suppose you have \( n \) independent and identically distributed (i.i.d.) observations, each taking on the values 1 and 0 with probabilities \( p \) and \( 1 - p \), respectively. Find a uniformly most powerful decision rule with false positive probability \( \alpha = 2^{-n} \) for the hypothesis pair

\[ \mathcal{H}_0 : p = \frac{1}{2} \]
\[ \mathcal{H}_1 : p > \frac{1}{2}. \]

6. 4 points. Kay II: 6.21