In the deterministic model, we have a finite number of known signal vectors \( s_i = [s_{i,0}, \ldots, s_{i,n-1}]^\top, i = 0, \ldots, M - 1 \), observed in additive noise \( W = [W_0, \ldots, W_{n-1}]^\top \).

The problem is to use the \( n \)-sample observation \([Y_0, \ldots, Y_{n-1}]^\top\) to determine which of the \( M \) signals was sent.
Example 1

Suppose

\[ s_0 = [0, \ldots, 0]^\top \]
\[ s_1 = [0, \sin(\pi/4), \ldots, \sin((n - 1)\pi/4)]^\top \]

and that \( W_k \sim \mathcal{N}(0, 1) \). Example observation:

Which signal was sent, \( s_0 \) or \( s_1 \)?
Example 2

Suppose

\[ s_0 = [1, \cos(\pi/4), \ldots, \cos((n - 1)\pi/4)]^\top \]
\[ s_1 = [0, \sin(\pi/4), \ldots, \sin((n - 1)\pi/4)]^\top \]

and that \( W_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1) \). Example observation:

Which signal was sent, \( s_0 \) or \( s_1 \)?
Example 3

Suppose

\[ s_0 = [1, \cos(\pi/4), \ldots, \cos((n - 1)\pi/4)]^\top \quad s_2 = [-1, -\cos(\pi/4), \ldots, -\cos((n - 1)\pi/4)]^\top \]
\[ s_1 = [0, \sin(\pi/4), \ldots, \sin((n - 1)\pi/4)]^\top \quad s_3 = [0, -\sin(\pi/4), \ldots, -\sin((n - 1)\pi/4)]^\top \]

and that \( W_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1) \). Example observation:

Which signal was sent, \( s_0 \) or \( s_1 \)?
Detection of Deterministic Signals in Noise

Detection of deterministic signals in noise is a simple hypothesis testing problem with $N = M$ states and hypotheses.

1. Example 1: $N = M = 2$.
2. Example 2: $N = M = 2$.

The first two examples are simple binary hypothesis testing problems. Those examples are particularly easy to solve because we know that the optimum decision rule is going to be of the form

$$\rho(y) = \begin{cases} 
1 & \text{if } L(y) > v \\
\gamma & \text{if } L(y) = v \\
0 & \text{if } L(y) < v 
\end{cases}$$

where $v$ and $\gamma$ depend on the criterion (N-P or Bayes) and $L(y) = \frac{p_1(y)}{p_0(y)}$. 

Worcester Polytechnic Institute
D. Richard Brown III