Homework 5 Solution

Homework Problems

1. An \((n, 1)\) repetition code takes each bit and repeats it \(n\) times. What is the minimum distance \(d_{\text{min}}\) of such a code?

   Answer: Consider, for example \(n = 3\). The two codewords are 000 and 111. It is clear that \(d_{\text{min}} = 3\) in this case. In general, we have \(d_{\text{min}} = n\).

2. Suppose you have a codebook with minimum distance \(d_{\text{min}} = 5\). What is the maximum number of errors this code can correct?

   Answer: The maximum number of correctable errors is \(t = \left\lfloor \frac{d_{\text{min}} - 1}{2} \right\rfloor = 2\) in this case.

3. Which encoding scheme is better, a Hamming (7,4) code or a (3,1) repetition code? Explain why.

   Answer: Both the Hamming (7,4) code and the (3,1) repetition code have a \(d_{\text{min}}\) equal to 3. So they can both correct for single bit errors. The Hamming code is better, however, because it has less redundancy. For the Hamming code, the redundancy is \(\frac{7-4}{4} = 0.75\) whereas for the repetition code the redundancy is \(\frac{3-1}{1} = 2\).

4. For the Hamming (7,4) code, what does the bit sequence 0101 get encoded as? Show your work. You can use a computer to check your answer but make sure you understand the procedure.

   Answer: 1100101.

5. Consider the (5,2) code

   \[
   \begin{array}{c|c}
   \text{Data Bits} & \text{Codeword} \\
   \hline
   00 & 00000 \\
   01 & 00001 \\
   10 & 10000 \\
   11 & 10001 \\
   \end{array}
   \]

   Is this a good code? Explain why or why not.

   Answer: This is not a good code. It has high redundancy \(\frac{5-2}{2} = 1.5\) and it has \(d_{\text{min}} = 1\), hence it can not even correct a single bit error. A much better (5,2) code with \(d_{\text{min}} = 3\) is given in Example 6.9 of the Stallings textbook.

6. Suppose you are using a (7,4) Hamming code and receive the codeword 0101001. Confirm that this is not a valid (7,4) Hamming codeword. Determine the closest valid (7,4) Hamming codeword and identify the incorrect bit in the received codeword.

   Answer: This codeword fails the parity checks. The closest valid codeword 0111001 which has Hamming distance one from the invalid codeword. The incorrect bit is 0101001.
7. Stallings Problem 6.16.

Answer:

a)

\[ d(00000, 10101) = 3 \]
\[ d(00000, 01010) = 2 \]
\[ d(10101, 01010) = 5. \]

b)

\[ d(000000, 010101) = 3 \]
\[ d(000000, 101010) = 3 \]
\[ d(000000, 110110) = 4 \]
\[ d(010101, 101010) = 6 \]
\[ d(010101, 110110) = 3 \]
\[ d(101010, 110110) = 3. \]