ECE2311D10 Exam 6

Your Name: SOLUTION Your box #: ____________

April 30, 2010

- Open book, open notes.
- Calculators permitted.
- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work/reasoning!
- This exam is worth a total of 100 points.

problem 1	problem 2	problem 3	TOTAL
40 points	40 points	20 points	100 points

1. 40 points. Given the signal $x(t)$ shown in Figure 1 below, compute the Laplace transform $X(s) = \mathcal{L}\{x(t)\}$ and the region of convergence.

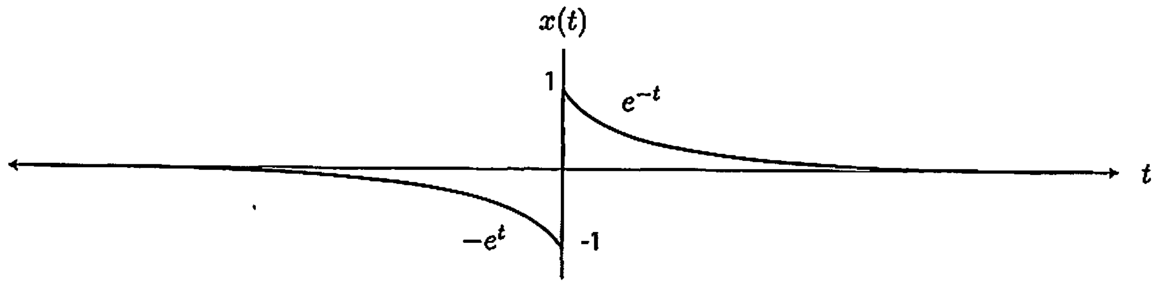


Figure 1: A signal.

Must use bilateral laplace transform here

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = -\int_{-\infty}^{0} e^{t} e^{-st}\, dt + \int_{0}^{\infty} e^{-t} e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{(1-s)t}\, dt + \int_{0}^{\infty} e^{(-1-s)t}\, dt$$

$$= \frac{-1}{1-s}\, e^{(1-s)t}\Big|_{t=-\infty}^{t=0} + \frac{1}{(-1-s)}\, e^{(-1-s)t}\Big|_{t=0}^{t=\infty}$$

$$= \frac{-1}{s-1}\Big[1 - \underbrace{\lim_{t\to-\infty} e^{(1-s)t}}_{\substack{\text{this limit only}\\ \text{converges if}\\ 1-\text{Re}(s)>0\\ \text{or } \text{Re}(s)<1}}\Big] - \frac{1}{s+1}\Big[\underbrace{\lim_{t\to\infty} e^{(-1-s)t}}_{\substack{\text{This limit only}\\ \text{converges if}\\ -1-\text{Re}(s)<0\\ \text{or } \text{Re}(s)>-1}} - 1\Big]$$

so, for $-1 < \text{Re}(s) < 1$, we have

$$X(s) = \frac{1}{s-1}\left[1-0\right] - \frac{1}{s+1}\left[0-1\right]$$

$$= \frac{1}{s-1} + \frac{1}{s+1}$$

Hence

$$\boxed{X(s) = \frac{2s}{s^2-1} \quad \text{with RoC} \quad -1 < \text{Re}(s) < 1}$$

2. 40 points total. Given the circuit in Figure 2 below, answer the following questions.

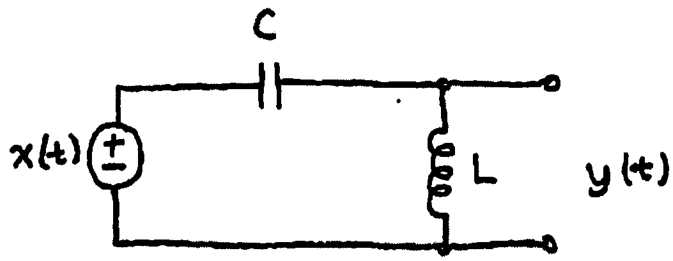
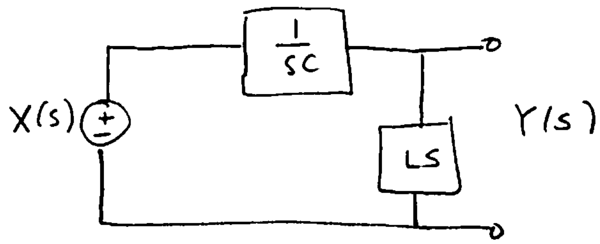


Figure 2: An LC circuit.

(a) 20 points. Compute the transfer function $H(s)$. Put your final answer in the form where the highest power of s in the denominator of $H(s)$ is multiplied by one.



$$Y(s) = \frac{Ls}{Ls + \frac{1}{sC}} X(s)$$

$$Y(s) = \frac{LCs^2}{LCs^2 + 1}$$

$$\boxed{Y(s) = \underbrace{\frac{s^2}{s^2 + \frac{1}{LC}}}_{H(s)} X(s)}$$

(b) 20 points. Using your answer from part (a), compute the step response of this circuit, i.e. compute $y(t)$ given $x(t) = u(t)$, $L = 1/9$ and $C = 1/4$. simplify...

$$u(t) \longleftrightarrow \frac{1}{s} \quad \text{(from table)}$$

$$\text{Hence } Y(s) = H(s)\cdot\frac{1}{s} = \frac{s}{s^2 + \frac{1}{LC}} = \frac{s}{s^2+36}$$

$$= \frac{s}{(s+j6)(s-j6)}$$

partial fraction expansion...

$$Y(s) = \frac{k_1}{s+j6} + \frac{k_2}{s-j6}$$

$$k_1(s-j6) + k_2(s+j6) = s + 0$$

$$k_1 + k_2 = 1$$
$$-k_1 + k_2 = 0$$

hence $k_2 = \frac{1}{2}$ and $k_1 = \frac{1}{2}$.

$$Y(s) = \frac{\frac{1}{2}}{s+j6} + \frac{\frac{1}{2}}{s-j6}$$

$$\Rightarrow y(t) = \left[\frac{1}{2}e^{-j6t} + \frac{1}{2}e^{+j6t}\right]u(t) \quad \text{(from table)}$$

$$\boxed{y(t) = \cos(6t)\,u(t)}$$

3. 20 points. Suppose you have a linear time invariant system with transfer function

$$H(s) = \frac{s+2}{s(s+1)}.$$

Note that this transfer function corresponds to an impulse response

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = 2u(t) - e^{-t}u(t)$$

Prove that this system is not BIBO stable and give a bounded input that causes the output to "blow up", i.e. grow without bound.

The easiest way to prove this system is not BIBO stable is to look at the poles of $H(s)$. The poles are at $s=0$ and $s=-1$. They do not have strictly negative real parts, so the system is not BIBO stable.

You could also prove this by showing $\int_{-\infty}^{\infty} |h(t)|\,dt = \infty$

A bounded input that causes the output to blow up is $x(t) = u(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau = \int_0^t 2 - e^{-\tau}\,d\tau$$

$$= \int_0^t 2\,d\tau - \int_0^t e^{-\tau}\,d\tau$$

$$= 2t + \left[e^{-t} - 1\right]$$

clearly, as $t \to \infty$, $y(t)$ also $\to \infty$.