ECE 4304 HW2

1. $R_X(\tau) = \exp(-2\nu|\tau|)$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)\exp(-j2\pi f t)\,d\tau$$

$$= \int_{-\infty}^{0} \exp(+2\nu\tau)\exp(-j2\pi f\tau)\,d\tau$$

$$+ \int_{0}^{\infty} \exp(-2\nu\tau)\exp(-j2\pi f\tau)\,d\tau$$

$$= \frac{1}{2(\nu-j\pi f)}\left[\exp(2\nu\tau-j2\pi f\tau)\right]\Big|_{-\infty}^{0} - \frac{\exp(-2\nu\tau-j2\pi f\tau)}{2(\nu+j\pi f)}\Big|_{0}^{\infty}$$

$$= \frac{\nu}{\nu^2+\pi^2 f^2}$$

From the plot we know the transfer function of the filter is

$$H(f) = \frac{1}{1+j2\pi fRC}$$

Hence

$$S_Y(f) = |H(f)|^2 S_X(f) = \frac{\nu}{[1+(2\pi fRC)^2][\nu^2+\pi^2 f^2]}$$

$$= \frac{\nu}{1-4R^2C^2\nu^2}\left[-\frac{1}{(1/2RC)^2+\pi^2 f^2} + \frac{1}{\nu^2+\pi^2 f^2}\right]$$

$$\frac{1/2RC}{(1/2RC)^2+\pi^2 f^2} \Leftrightarrow \exp(-|t|/RC)$$

$$\frac{\nu}{\nu^2+\pi^2 f^2} \Leftrightarrow \exp(-2\nu|t|)$$

so $$R_Y(\tau) = \frac{\nu}{1-4R^2C^2\nu^2}\left[\frac{1}{\nu}\exp(-2\nu|\tau|) - 2RC\exp\left(-\frac{|\tau|}{RC}\right)\right]$$

$$\text{Power} = R_Y(0) = \frac{v}{1-4R^2C^2v^2}\left[\frac{1}{v}-2RC\right] = \frac{1}{1+2RCv}$$

2. a) $S_Y(f) = |H(f)|^2 S_X(f) = \begin{cases} 10^{-6} & -W<f<W \\ 0 & \text{otherwise} \end{cases}$

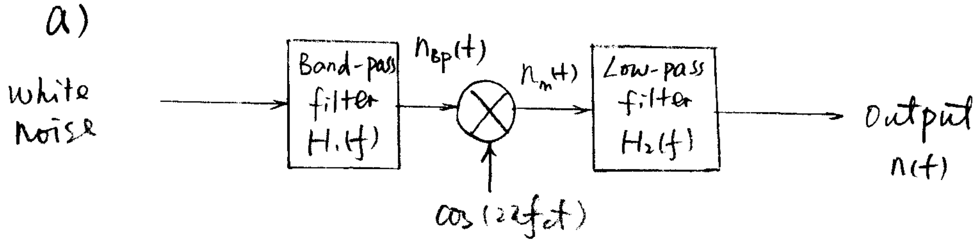
b) $R_Y(\tau) = 2\times10^{-6} W \operatorname{sinc}(2W\tau)$

c) $P_{PSD} = \int_{-W}^{W} S_Y(f)\,df = 2\times10^{-6} W$

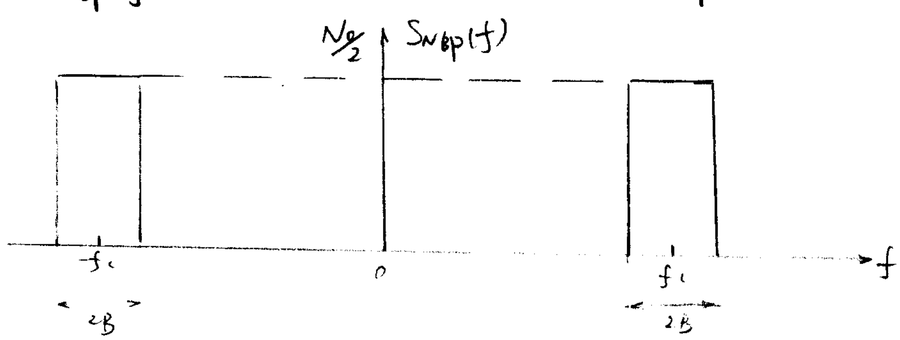
$P_R = R_Y(0) = 2\times10^{-6} W \operatorname{sinc}(0) = 2\times10^{-6} W$

$P_{PSD} = P_R$

3. a)



Let $S_{N_{BP}}(f)$ denote the PSD of $n_{BP}(t)$, we have



let $S_{Nm}(f)$ denote the PSD of $n_m(t)$, we have

$$S_{Nm}(f) = \frac{1}{4}\left(S_{NBP}(f-f_c) + S_{NBP}(f+f_c)\right)$$

After the LPF, only the part between $-B$ and B left.

So $S_N(f) = \begin{cases} \frac{N_0}{4} & -B \leq f \leq B \\ 0 & \text{otherwise} \end{cases}$

$$R_n(\tau) = 2\frac{N_0}{4}B\,\text{sinc}(2B\tau) = \frac{N_0}{2}B\,\text{sinc}(2B\tau)$$

b) $E[n(t)] = 0$

$$Var[n(t)] = E[n^2(t)] = \int_{-B}^{B} \frac{N_0}{4}\,df = \frac{N_0 B}{2}$$

Problem 4 is on the last page...

5. a) $\text{Power} = \int_{-10k}^{10k} S_M(f)\,df = \int_{-10k}^{10k} 4\times10^{-5}\times\left(1-\left(\frac{f}{10^4}\right)^2\right)df$

$= \frac{8}{15}W$

b) using my notes in class:

$$\left.\begin{aligned} &SNR = \frac{\alpha^2 A_c^2 P_M \cos^2(\phi_c-\phi)}{2BN_0} = 10^3 \\ &\alpha^2 = 10^{-7} \\ &N_0 = 2\times10^{-12}\ W/Hz \\ &B = 10k\ HZ \\ &\phi = \phi_c \end{aligned}\right\} \Rightarrow A_c^2 P_M = 4\times10^{2}$$

transmit power is $P_t = E[(M(t)A_c\cos(2\pi f_c t+\phi_c))^2]$

$= \frac{1}{2}A_c^2 P_M\, E[1 + \cos(4\pi f_c t + 2\phi_c)]$

$= \frac{1}{2}A_c^2 P_M = 2\times10^{2}\ W$

(c) $u(t) = A_c[1 + 0.9M(t)]\cos(2\pi f_c t + \phi_c)$

$$r(t) = \alpha A_c[1 + 0.9M(t)]\cos(2\pi f_c t + \phi_c) + N_{BP}(t)$$

$$y(t) = \frac{1}{2}\alpha A_c[\ 0.9M(t)]\cos(\phi_c - \phi) + LPF\{\cos(2\pi f_c t + \phi)N_{BP}(t)\}$$

Power of the signal part $= E\left[\left(\frac{1}{2}\alpha A_c \quad 0.9M(t)\cos(\phi_c - \phi)\right)^2\right]$

$$= \frac{1}{4}\alpha^2 A_c^2 \cos^2(\phi_c - \phi)\, 0.81 P_M$$

Power of the noise part is the same as the notes

$$\text{so} \quad SNR = \frac{\alpha^2 A_c^2 \cos^2(\phi_c - \phi) \quad 0.81 P_M}{2BN_0}$$

$$\Rightarrow \left.\begin{array}{l} \dfrac{10^{-7} A_c^2(\ 0.81 P_M)}{2 \times 10k \times 10^{-12}} = 10^3 \\ P_M = \dfrac{8}{15} w \end{array}\right\} \Rightarrow A_c^2 = 925.93$$

$$P_t = E\left[\left[A_c[1 + 0.9M(t)]\cos(2\pi f_c t + \phi_c)\right]^2\right]$$

$$= \frac{1}{2}A_c^2 E\left[(1 + 0.9M(t))^2\right]\left(1 + E[\cos(4\pi f_c t + 2\phi_c)]\right)$$

$$= \frac{1}{2}A_c^2(1 + 0.81 P_M)$$

$$= 663 \text{ w}$$

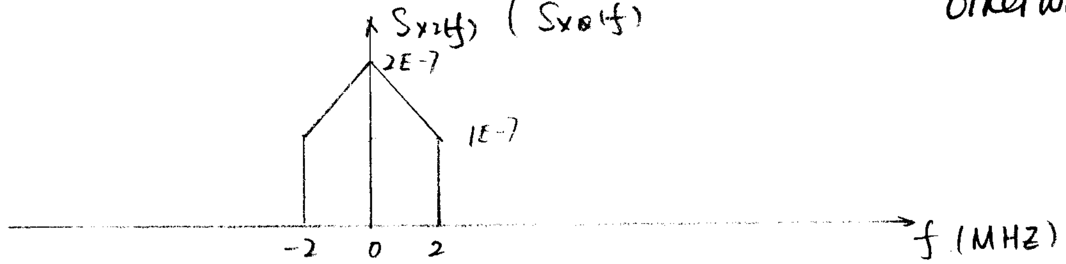
(d) From the resuts of (b) & (c), we notice that conventional DSB AM needs more transmission power. Because it needs power to transmit carrier. However, carrier does not carry any message. So we can use DSB-SC to transmit the same info with less Power

4. $P_{total} = \int_{-\infty}^{\infty} S_X(f)\, df$

$= 2\times\left(\frac{2\times10^6\times10^{-7}}{2} + 2\times10^6\times10^{-7}\right)$

$= 0.6\,W$

$$S_{XI}(f) = S_{XQ}(f) = \begin{cases} S_X(f-f_c)+S_X(f+f_c), & -2\le f\le 2 \\ 0 & \text{otherwise} \end{cases}$$



$$S_{XI}(f) = S_{XQ}(f) = \begin{cases} 2E\text{-}7\left(1-\frac{|f|}{4}\right) & -2\le f\le 2. \\ 0 \end{cases}$$

$$S_{IQ}(f) = \begin{cases} j\,[S_N(f+f_c)-S_N(f-f_c)] & -2\le f\le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow S_{IQ}(f) = \begin{cases} -j\,0.5\times10^{-7} f & -2\le f\le 2 \\ 0 & \text{otherwise} \end{cases}$$