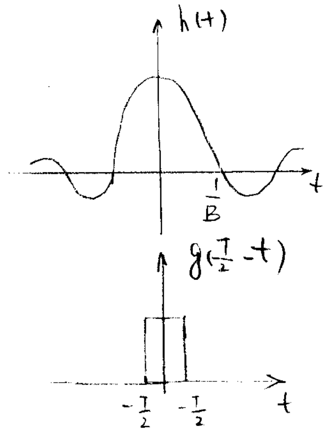
ECE 4304 HW#3

1. (a) $g(t) = \begin{cases} A & 0 \le t \le T \\ 0 & \text{else} \end{cases}$

We know for a LPF $H(f) = \begin{cases} 1 & -B \le f \le B \\ 0 & \text{else} \end{cases}$

$h(t) = 2B\,\text{sinc}(2Bt)$

$y(t) = g(t) * h(t)$



we can notice that the convolution has the maximum value when $t = \frac{T}{2}$

so $y(\frac{T}{2}) = 2B \cdot A \int_{-\frac{T}{2}}^{T/2} \sin(2Bt)$

$= (2A/\pi)\,\text{sinint}(\pi BT)$

Hence. $SNR_{LPF} = \dfrac{[(2A/\pi)\,\text{sinint}(\pi BT)]^2}{N_0 B}$

$= \dfrac{4A^2\,\text{sinint}^2(\pi BT)}{\pi^2 N_0 B}$

$SNR_{MF} = 2A^2T/N_0$

So $\dfrac{SNR_{LPF}}{SNR_{MF}} = \dfrac{2\,\text{sinint}^2(\pi BT)}{\pi^2 BT}$

the maximum is achived at $BT = 0.685 \Rightarrow B = \frac{0.685}{T}$

plug this number in, we have PPSNR of ideal LPF is 0.84 dB lower than that of matched filter.

2. a) $\lambda = \frac{\varepsilon}{2}(a_1+a_0) + \frac{N_0}{2(a_1-a_0)} \ln\left(\frac{P_0}{P_1}\right)$

$\varepsilon = A^2 T_b$

$a_1 = 1$

$a_0 = -1$

$\Rightarrow \lambda = 0$

$P_e = \frac{1}{2} P(r_1 < \lambda) + \frac{1}{2} P(r_0 > \lambda)$

$= \frac{1}{2} Q\left(\frac{\varepsilon}{\sqrt{N_0\varepsilon/2}}\right) + \frac{1}{2} Q\left(\frac{\varepsilon}{\sqrt{N_0\varepsilon/2}}\right)$

$= Q\left(\sqrt{\frac{2A^2 T_b}{N_0}}\right)$

b)

see attached page.

c)

d) Larger A or T will result in smaller BER. Because Increasing A is the same as increasing signal power thus will reduce the effect of noise power. Increasing T can also increase signal power. thus reduce BER.

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$$\left.\begin{array}{l}\lambda = \frac{\mathcal{E}}{2}(a_1 + a_0) + \frac{N_0}{2(a_1 - a_0)} \ln\left(\frac{0.5}{0.5}\right) \\ \mathcal{E} = A^2 T_b \\ a_1 = 1 \\ a_0 = 0\end{array}\right\} \Rightarrow \lambda = \frac{\mathcal{E}}{2}$$

$$P_e = \frac{1}{2} Q\left(\frac{\frac{\mathcal{E}}{2}}{\sqrt{N_0 \mathcal{E}/2}}\right) + \frac{1}{2} Q\left(\frac{\frac{\mathcal{E}}{2}}{\sqrt{N_0 \mathcal{E}/2}}\right)$$

$$= Q\left(\frac{A^2 T_b / 2}{\sqrt{N_0 A^2 T_b / 2}}\right) = Q\left(\sqrt{\frac{A^2 T_b}{2N_0}}\right)$$

BER increased compare with Non-return-to-zero signal. Because the distance between two signal is closer, thus more prone to error

4. $$\left.\begin{aligned} \lambda_{opt} &= \frac{\mathcal{E}}{2}(a_1+a_0) + \frac{N_0}{2(a_1-a_0)} \ln\left(\frac{P(0)}{P(1)}\right) \\ \mathcal{E} &= A^2T_b, \quad a_1=1, \quad a_0=0 \end{aligned}\right\} \Rightarrow$$

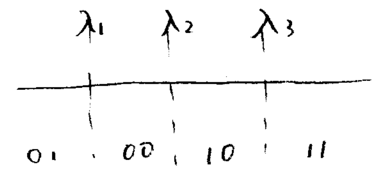
$$\lambda_{opt} = \frac{A^2T_b}{2} + \frac{N_0}{2}\ln\left(\frac{3}{7}\right)$$

so $$P_e = 0.7\,Q\left(\frac{\alpha a_1 \mathcal{E} - \lambda_{opt}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right) + 0.3\,Q\left(\frac{\lambda_{opt} - \alpha a_0 \mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right)$$

let $\alpha=1$ $$= 0.7\,Q\left(\frac{A^2T_b - \frac{A^2T_b}{2} - \frac{N_0}{2}\ln\left(\frac{3}{7}\right)}{\sqrt{\frac{N_0A^2T_b}{2}}}\right) + 0.3\,Q\left(\frac{\frac{A^2T_b}{2} + \frac{N_0}{2}\ln\left(\frac{3}{7}\right)}{\sqrt{\frac{N_0A^2T_b}{2}}}\right)$$

$$= 0.7\,Q\left(\frac{A^2T_b - N_0\ln\left(\frac{3}{7}\right)}{\sqrt{2N_0A^2T_b}}\right) + 0.3\,Q\left(\frac{N_0\ln\left(\frac{3}{7}\right) + A^2T_b}{\sqrt{2N_0A^2T_b}}\right)$$

5. Since ones & zeros are equally likely, we know the threshholds should be



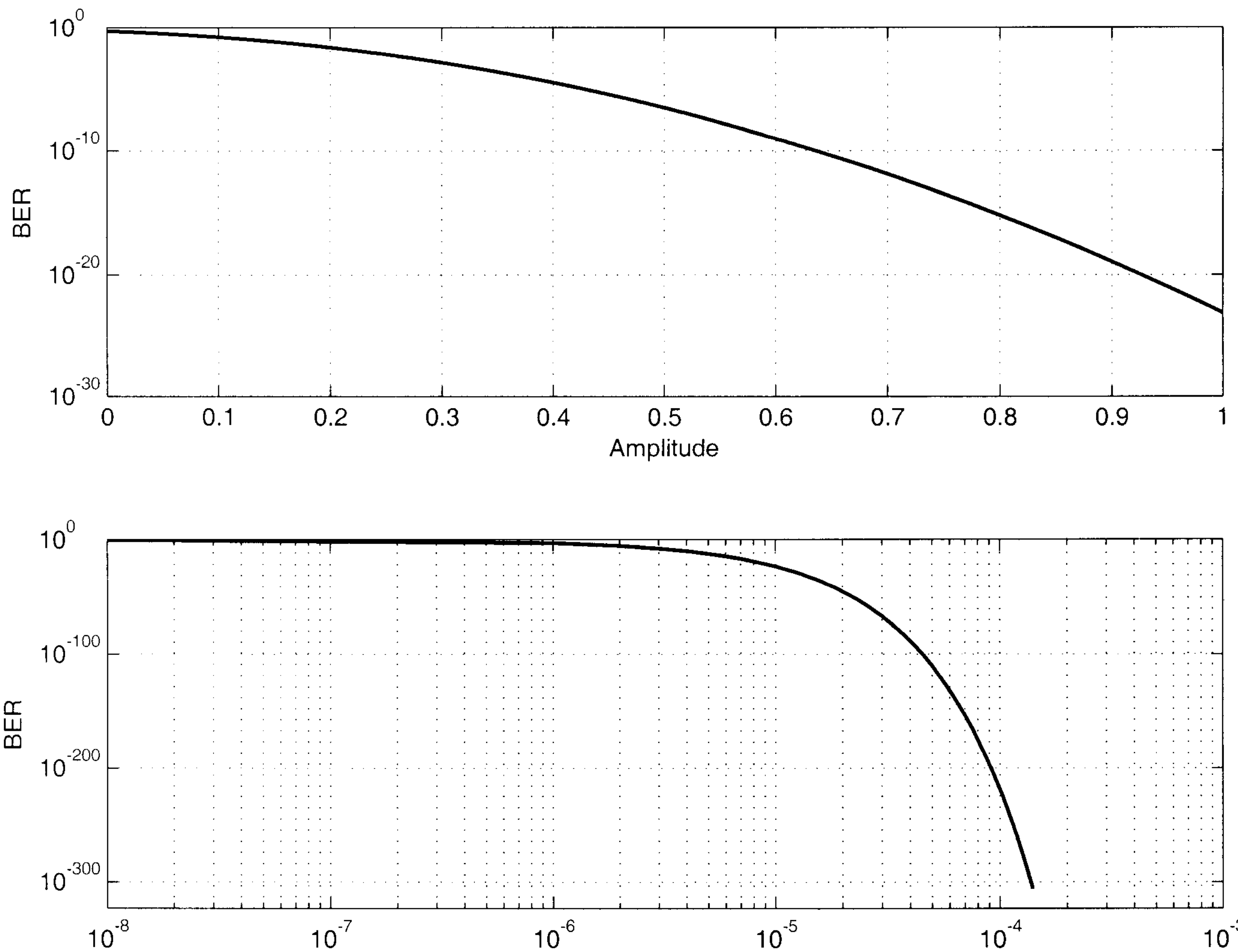
$\lambda_1 = -2\mathcal{E}$, $\lambda_2 = 0$, $\lambda_3 = 2\mathcal{E}$

$$P_e = P(Y < -2\mathcal{E}, \text{'00' sent}) + P(Y<0, \text{'10' sent}) + P(Y < 2\mathcal{E}, \text{'11' sent})$$
$$+ P(Y > 2\mathcal{E}, \text{'10' sent}) + P(Y > 0, \text{'00' sent}) + P(Y > -2\mathcal{E}, \text{'01' sent})$$

$$= \frac{1}{4} Q\left(\frac{-\mathcal{E}+2\mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right) + \frac{1}{4}\left(\frac{\mathcal{E}-0}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right) + \frac{1}{4}\left(\frac{3\mathcal{E}-2\mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right)$$
$$+ \frac{1}{4} Q\left(\frac{2\mathcal{E}-\mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right) + \frac{1}{4}\left(\frac{0+\mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right) + \frac{1}{4}\left(\frac{-2\mathcal{E}+3\mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right)$$
$$= \frac{6}{4} Q\left(\frac{\mathcal{E}}{\sqrt{\frac{N_0\mathcal{E}}{2}}}\right)$$
$$= \frac{3}{2} Q\left(\sqrt{\frac{2\mathcal{E}}{N_0}}\right) = \frac{3}{2} Q\left(\sqrt{\frac{2A^2T_b}{N_0}}\right)$$

So BER of a 4-PAM system is $P_{eb} = \frac{3}{2} Q\left(\sqrt{\frac{2A^2T_b}{N_0}}\right) / \log_2(4)$

4-PAM is more error prone than 2-PAM, but it can achieve larger bit rate. To achieve the same BER as 2-PAM, 4-PAM needs more transmit power.



10^0
10^{-10}
10^{-20}
10^{-30}
BER
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Amplitude
10^0
10^{-100}
10^{-200}
10^{-300}
BER
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}