ECE4703 Final Exam

Your Name: SOLUTION Your box #: ____________

December 17, 2009

Tips:

• Look over all of the questions before starting.

• Budget your time to allow yourself enough time to work on each question.

• Write neatly and show your work!

• This exam is worth a total of 200 points.

• Attach your "cheat sheet" to the exam when you hand it in.

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1. 30 points. Determine the theoretical maximum number of single-precision floating point multiply accumulates that a TMS320C6713 running at 225MHz can compute per second. Explain your answer. Are there DSP applications in which it is possible to get close to this theoretical maximum?

We saw an example in class of a fully-pipelined single-precision-floating point dot product with two multiplies and two accumulates in each clock cycle. Hence, the theoretical maximum MACs for the C6713 is 450,000,000 MACs/sec.

Any DSP application that requires a long dot product will get close to this theoretical maximum since the prologue/epilogue overhead will be small.
2. 60 points total. Suppose you are given two \( N \)-point discrete-time sequences

\[
x[0], \ldots, x[N-1] \text{ and } y[0], \ldots, y[N-1]
\]

and you want to convolve these sequences to form the \( 2N - 1 \) sample output sequence \( z[0], \ldots, z[2N - 2] \).

(a) 30 points. What is the asymptotic complexity of directly convolving these two sequences together using the discrete-time convolution formula

\[
z[n] = \sum_{\ell=0}^{N-1} x[\ell]y[n-\ell] \text{ for } n = 0, 1, \ldots, 2N - 2
\]

For each \( n \), we have \( N \) multiplies and \( N - 1 \) adds.

Hence there are

\[
(2N-1)N = 2N^2 - N \text{ multiplies}
\]

and

\[
(2N-1)(N-1) = 2N^2 - 3N + 1 \text{ adds}
\]

\( \Rightarrow \) asymptotic complexity is \( \Theta(N^2) \)

(b) 30 points. Suppose that, instead of directly convolving these sequences, you instead

- zero-pad the \( x \) and \( y \) sequences to make them each \( 2N - 1 \) samples long,
- compute the FFT of each sequence to get \( X[0], \ldots, X[2N-2] \) and \( Y[0], \ldots, Y[2N-2] \)
- compute the product of these new sequences in the frequency domain such that \( Z[k] = X[k]Y[k] \) for \( k = 0, 1, \ldots, 2N - 2 \),
- and then compute the inverse FFT of the sequence \( Z[0], \ldots, Z[2N - 2] \) to get the time-domain output sequence \( z[0], \ldots, z[2N - 2] \).

Assuming the zero-padding operation requires a constant number of operations (not dependent on \( N \)), what is the asymptotic complexity of performing the convolution using this approach? Hint: The inverse FFT has the same asymptotic complexity as the FFT.

- zero pad: \( \Theta(1) \)
- FFT(\( x \)): \( \Theta(N \log_2 N) \)
- FFT(\( y \)): \( \Theta(N \log_2 N) \)
- \( Z = X \times Y \): \( \Theta(N) \)
- \( z = \text{IFFT}(Z) \): \( \Theta(N \log_2 N) \)

\( \Rightarrow \) worst complexity is \( \Theta(N \log_2 N) \)

\( \Rightarrow \) overall asymptotic complexity is \( \Theta(N \log_2 N) \)
3. 60 points total. Consider the system identification adaptive filtering system shown below.

For the following questions, assume that

- the mean squared value of the input noise \( x[n] \) is one, i.e. \( E[x^2[n]] = 1 \),
- the unknown filter coefficients are \( h = [0.2, 0.6] \),
- the LMS adaptive filter \( b \) coefficients are initialized to zero prior to adaptation, and
- the LMS step-size is small enough to allow for convergence of the algorithm to the minimum mean squared error (MMSE) solution.

(a) 20 points. Suppose that the unknown gain \( a = 1 \) and that the adaptive FIR filter \( b \) has two coefficients. What will \( b \) be after convergence of the LMS algorithm? What will the MMSE be after convergence?

\[
\begin{align*}
 b & \rightarrow [0.2, 0.6] \\
\text{MMSE} & \rightarrow 0 \text{ because } b = h
\end{align*}
\]

(b) 20 points. Now suppose \( a = 2 \). What will \( b \) be after convergence of the LMS algorithm? What will the MMSE be after convergence?

\[
\begin{align*}
 b & \rightarrow [0.1, 0.3] \quad \text{Note that } a \cdot b = [0.2, 0.6], \\
\text{MMSE} & \rightarrow 0 
\end{align*}
\]

(c) 20 points. Now suppose \( a = 1 \) and that the adaptive FIR filter \( b \) has three coefficients. What will \( b \) be after convergence of the LMS algorithm? What will the MMSE be after convergence?

\[
\begin{align*}
 b & \rightarrow [0.2, 0.6, 0] \\
\text{MMSE} & \rightarrow 0 \text{ because } b \text{ matches } h \text{ in the first two coefficients and is zero in the third coefficient}
\end{align*}
\]
4. 50 points total. Suppose your DSP is running the assembly code given on the last page of this exam (this is the fully-pipelined dot product discussed in lecture).

(a) 10 pts. Draw a box around the instruction(s) in the fourth execute packet. Label it EP4.

(b) 10 points. Suppose the LDDW instruction on line 28 is currently in pipeline stage E3. Put a pound sign (#) next to the instruction(s) currently in pipeline stage E1.

(c) 10 points. Notice the two MPYSP instructions on lines 18-19. Put stars (*) next to the instruction(s) in the execute packet that run immediately after the results from these MPYSP commands become valid.

(d) 20 points. Suppose the register A6 contains the value 1000 at the start of this function. How many cycles does this function require to execute? You should count all cycles up to (but not including) the branch instruction at line 59.

\[
\begin{align*}
\text{Prologue} &= 9 \\
\text{Loop} &= 1 \times 500 \\
\text{Epilogue} &= 9 \\
\text{518 total cycles}
\end{align*}
\]
ADDSP .L2x A8, B8, B0
NOP
ADDSP .L1x A0, B0, A5
NOP
ADDSP .L2x A0, B0, B5
NOP

; branch back to calling function

B. B3
doin'
count

NOP
ADDSP .L1x A5, B5, A4
NOP
.end

; result returned in A4

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