ECE503 Spring 2014 Quiz 11

Your Name: ________________________________  ECE Box Number: _______

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 40 points. Suppose $x[n]$ is a real-valued length-$N$ sequence defined for $n = 0, 1, \ldots, N - 1$ with $N$ even and

$$y[n] = (-1)^n x[n]$$

is another length-$N$ sequence. Explicitly relate $Y[k] = \text{DFT}_N\{y[n]\}$ and $X[k] = \text{DFT}_N\{x[n]\}$. Hint: It may be useful to observe that $(-1)^n = e^{-j\pi n} = W_N^n$.

2. 30 points. Suppose

$$x[n] = \begin{cases} \frac{n}{n+1} & n = 0, \ldots, 9 \\ 0 & \text{otherwise} \end{cases}$$

and let $Y[k] = X(e^{j\omega})|_{\omega=2\pi k/8}$ for $k = 0, \ldots, 7$. Compute $y[n]$ for $n = 0, \ldots, 7$.

3. 30 points. Suppose you have length-4 sequences $x[n] = \{1, 0, 2, 1\}$ and $y[n] = \{a, 0, b, 0\}$ with $a$ and $b$ both unknown. The 4-point circular convolution of these sequences results in $z[n] = \{1, -1, -1, 1\}$. Determine valid values for $a$ and $b$. Is your answer unique?
1. can use property 8 on table 8.2

\[ x_1[n] = (-1)^n \iff X_1[k] = \{0, \ldots, 0, N, 0, \ldots, 0\} \] at index \( k = \frac{N}{2} \)

\[ x_2[n] = x[n] \iff X_2[k] = X[k] \]

\[ x_1[n] x_2[n] \iff \frac{1}{N} \sum_{k=0}^{N-1} x_1[k] x_2[(k-k_0)N] \] (circular convolution of \( x_1[k] \) and \( x_2[k] \))

This circular convolution is just a shift of \( X[k] \) by \( \frac{N}{2} \).

Note that the \( N \) and \( \frac{1}{N} \) cancel. So

\[ Y[k] = X[\left(\left\lfloor \frac{k-N}{N}\right\rfloor\right)N] \]

You can also get this from property 6 by noting that

\[ W_N^{-kn} = e^{-j\pi kn/N} = (-1)^n \]

2. Since the DFT length \( N = 8 \) is less than \( N = 10 \), sampling \( X(e^{j\omega}) \) will result in time-domain aliasing.

We have

\[ y[n] = \sum_{r=-\infty}^{\infty} x[n+8r] \]

\[ y[0] = \ldots \; x[-8] + x[0] + x[8] + x[16] + \ldots = 1 + 9 = 10 \]


Hence \( y[n] = \{0, 10, 2, 12, 3, 4, 5, 6, 7, 8\} \; \text{ for } n = 0, \ldots, 7 \)

3. Set up circular conv. matrix

\[
\begin{bmatrix}
1 & 2 & 0 \\
0 & 1 & 2 \\
2 & 0 & 1 \\
1 & 2 & 0
\end{bmatrix}
\begin{bmatrix}
a \\
0 \\
1 \\
b
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
-1 \\
-1 \\
-1
\end{bmatrix}
\]

First row implies \( a = 1 \)

Third row implies \( b = -1 \)

Rows 2 and 4 consistent.

Answer is unique.