ECE503 Spring 2014 Quiz 1

Your Name: Solution

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 40 points. For each of the following systems defined by difference equations, determine if the system is (1) linear, (2) time-invariant, (3) stable, (4) causal, and/or (5) memoryless. Be sure to show your reasoning for full credit.
   (a) $y[n] = x[n]x[n-1]$.
   (b) $y[n] = x[n] + a$ with a as a finite constant. Hint: some of your answers may depend on the value of a.

2. 30 points. Determine the impulse response and frequency response of the system defined by the difference equation
$$y[n] = ay[n-1] + x[n-1]$$
assuming $|a| < 1$. Your answers will be a function of a.

3. 30 points. Suppose you have an LTI system with impulse response
$$h[n] = \begin{cases} 1 & n = -1, 0, 1 \\ 0 & \text{otherwise.} \end{cases}$$
   (a) Determine whether this LTI system is causal and/or stable.
   (b) Determine the output of this system to the input $x[n] = \cos(2\pi n/3 + \pi/6)$ for all n.

1. a) nonlinear : suppose $x_1[n] = u[n] \Rightarrow y_1[n] = u[n-1]$
$x_2[n] = 2u[n] \Rightarrow y_2[n] = 4u[n-1] \neq 2y_1[n]$

time-invariant : suppose $y_1[n]$ is the output from the input $x[n]$
let $x_2[n] = x_1[n-n_d]$
then $y_2[n] = x_2[n]x_2[n-1] = x_1[n-n_d]x_1[n-n_d-1]$
$= y_1[n-n_d]$ ✓

stable : if $|x[n]| \leq B_x < \infty \ \forall n$ then
$y[n] \leq B_x^2 < \infty \ \forall n$.

causal : output at time n only depends on current and past inputs.

not memoryless : $y[n]$ depends on $x[n-1]$

b) if $a=0$; then $x[n]=y[n] \Rightarrow$ linear, time-invariant, stable, causal, & memoryless
(all obvious since system is trivial)

if $a \neq 0$ then:

non-linear : suppose $x_1[n] = 3$ then $y_1[n] = 3+a$
if we apply $x_2[n] = 2x_1[n]$ then $y_2[n] = 6+a \neq 2y_1[n]$

time-invariant : $x_1[n] \to y_1[n] = x_1[n] + a$
$x_2[n] = x_1[n-n_d]$
$x_2[n] \to y_2[n] = x_2[n] + a = x_1[n-n_d] + a = y_1[n-n_d]$ ✓

stable : if $|x[n]| \leq B_x \ \forall n$ then
$|y[n]| \leq |x[n]| + |a| \leq B_x + |a| < \infty$
since a is a finite constant.

causal : obvious since $y[n]$ only depends on present input and no future inputs

memoryless : also obvious since $y[n] = f(x[n])$ and no other input samples.

2. Compute frequency response

$y[n] - ay[n-1] \Leftrightarrow (1 - ae^{-j\omega})Y(e^{j\omega})$

$x[n-1] \Leftrightarrow e^{-j\omega}X(e^{j\omega})$

$$\Rightarrow \boxed{H(e^{j\omega}) = \frac{e^{-j\omega}}{1 - ae^{-j\omega}}}$$

From DTFT table, we have $\frac{1}{1-ae^{-j\omega}} \longleftrightarrow a^n u[n]$

Time-shifting property then implies $\boxed{h[n] = a^{n-1}u[n-1]}$

3. Since we have a sinusoidal input, the easiest approach is to compute the frequency response $H(e^{j\omega})$ at $\omega = \frac{2\pi}{3}$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = e^{j\omega} + 1 + e^{-j\omega}$$
$$= 2\cos(\omega) + 1$$

now substitute
$\omega = \frac{2\pi}{3}$

$H(e^{j\frac{2\pi}{3}}) = 0$

Hence $\boxed{y[n] = 0}$ $\forall n$.

This system is clearly stable since $\sum_{n=-\infty}^{\infty} |h[n]| = 3 < \infty$

This system is not causal since $y[n]$ depends on $x[n+1]$.