ECE504 Final Exam

9-Dec-2008

Notes:

- This exam is worth 500 points and is to be completed in 150 minutes.

- Look over all the questions before starting.

- Budget your time to allow enough time to work on each question.

- To receive maximum credit, you must show your reasoning and/or work.

1. 25 points. Give an example of an invertible matrix that is not diagonalizable.

2. 25 points. Give an example of a diagonalizable matrix that is not invertible.

3. 50 points. Given

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

   find a general expression for $\exp\{At\}$. Please simplify your answer as much as possible for full credit.

4. 100 points total. Suppose you are given a CT-LTI state-space system described by

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = 0,$$

   (a) 50 points. Discuss the internal and external stability properties of this system.

   (b) 50 points. Find a minimal realization of this system and discuss its internal and external stability properties.

5. 100 pts. Suppose you are given a CT-LTI state-space system described by

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad D = 1$$

   Show that this is not a minimal realization and then find the McMillan degree of this system.

6. 100 points total. Suppose you are given a DT-LTI system with

$$x[k+1] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} x[k] + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u[k]$$

   (a) 50 points. Find the set of reachable states, parameterized by b_1 and b_2. Under what conditions on b_1 and b_2 is this a reachable system?

(b) 50 points. Are there any choices for b_1 and b_2 so that the set of reachable states is not equivalent to the set of controllable states in this system? Explain.

7. 100 points total. You are given the CT-LTI state-space system described by

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad D = 0,$$

(a) 40 points. Find the the state feedback control rule F such that the eigenvalues of $A - BF$ are equal to -1 and -2.

(b) 10 points. Is F unique?

(c) 10 points. Is the system with state feedback asymptotically stable? Is it BIBO stable?

(d) 15 points. Is the system with state feedback reachable?

(e) 15 points. Is the system with state feedback observable?

(f) 10 points. Is the system with state feedback minimal?