Distributed Reception with Coarsely-Quantized Observation Exchanges

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***Abstract*—This paper considers the problem of jointly decoding binary phase shift keyed (BPSK) messages from a single distant transmitter to a cooperative receive cluster connected by a local area network (LAN). A distributed reception technique is proposed based on the exchange of coarsely-quantized observations among some or all of the nodes in the receive cluster. By taking into account the differences in channel quality across the receive cluster, the quantized information from other nodes in the receive cluster can be appropriately combined with locally unquantized information to form aggregate posterior likelihoods for the received bits. The LAN throughput requirements of this technique are derived as a function of the number of participating nodes in the receive cluster, the forward link code rate, and the quantization parameters. Using information theoretic analysis and simulations of an LDPC coded system in fading channels, numerical results show that the performance penalty (in terms of outage probability and block error rate with respect to ideal receive beamforming) due to coarse quantization is small in the low SNR regimes enabled by cooperative distributed reception.**

***Index Terms*—Distributed reception, receive beamforming, quantization, likelihood combining**

I. Introduction

Distributed reception is a technique where multiple receivers in a wireless network combine their observations to increase diversity and power gain and, consequently, improve the probability of successfully decoding noisy transmissions. Distributed reception has been used historically in the context of aperture synthesis for radio astronomy, e.g. the Very Large Array [1], where each antenna typically forwards observations over a high-speed optical backhaul network to a processing center for subsequent alignment and combining. The advantages of this approach are well-documented and include improved resolution as well as signal-to-noise (SNR) gains.

More recently, the idea of distributed reception has been considered for wireless networks with limited backhaul capabilities. A simple form of distributed reception, i.e. soft handoff [2], has been successfully used in cellular systems since the 1990s. Recent information theoretic studies [3]–[6] have shown that more sophisticated distributed reception techniques have potential to increase diversity, improve capacity, and improve interference rejection, even with tight backhaul constraints. Several techniques have been proposed to achieve these gains including link-layer iterative cooperation [7], [8], distributed iterative receiver message-passing [9], and most-reliable/least-reliable bit exchange iterative decoding [10]–[15]. A limitation of all of these techniques is that they are based on iterative transmissions and decoding. As such, the backhaul requirements are variable and the decoding latency can be significant if the number of iterations is large. The focus of these studies is also often on achieving diversity gains, rather than SNR gains. SNR gains through distributed receive beamforming are particularly appealing since they can be linear in the number of receivers and allow for longer-range and/or higher-data rate communication as well a reduction in the size, weight, power and cost of the transmitter.

In this paper, we consider the problem jointly decoding binary phase shift keyed (BPSK) messages from a single distant transmitter to a cooperative receive cluster with a conventional LAN comprising the backhaul. We show that exchanging quantized observations among the nodes in the receive cluster can provide a simple but powerful approach for non-iterative, fully-distributed reception over a LAN with limited capacity. Unlike most-reliable/least-reliable bit exchange techniques in which information is transmitted over the backhaul/LAN based on requests from other receivers, our approach is for receivers to quantize each demodulated bit (prior to decoding) and broadcast all of these quantized values to the other receivers in the cluster. A naïve implementation with fine-grained quantization of the observations at each receiver can generate large amounts of LAN traffic. For example, in a 10 node cluster with a rate $r = 1/2$ forward link code and $b = 16$ bits per observation, the LAN would need to support a normalized throughput of approximately 320 bits per forward link information bit. Our approach is based on *coarse* quantization and adapts to LAN throughput constraints by allowing for different quantization parameters as well as allowing a subset of the receivers in the cluster to participate in the broadcast of quantized observations.

The numerical results from information-theoretic analysis, as well as simulations of an LDPC-coded system, show that exchanging just one bit per forward-link coded bit (i.e., hard decisions based on the sign of the observation) typically results in outage probability performance within 1.5 dB of ideal receive beamforming, while two bits per coded bit (one sign bit and one amplitude bit) peforms within 0.5 dB of ideal receive beamforming. Our results lead to the intuitively pleasing observation that the low (per node) SNR regimes enabled by cooperative distributed reception limit the performance loss caused by coarse quantization. We also provide explicit estimates of backhaul throughput requirements as a function of the forward link information rate, and demonstrate the efficacy of the technique with full and limited receiver participation.

II. System Model

We consider the scenario shown in Figure 1 where we have a single transmitter and a cluster of N receive nodes. The goal is to communicate common broadcast messages over the forward link from the distant transmitter to all of the receive nodes. As one example, the scenario in Figure 1 could correspond to a long-range downlink in which the receive cluster jointly processes messages from a distant base station.

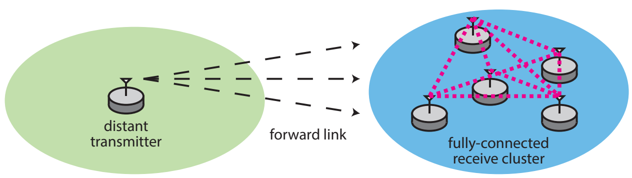

Fig. 1. Distributed reception scenario.

The forward link complex channel from the distant transmitter to receive node i is denoted as h_i for $i = 1, \dots, N$ and we denote the vector channel $\boldsymbol{h} = [h_1, \dots, h_N]^\top$. It is assumed that the receive cluster has already established a LAN backhaul, either ad-hoc or through infrastructure such as an access point, and that LAN transmissions are reliable. The LAN is also assumed to support broadcast transmission in which any single node can send a message to all other nodes simultaneously. The LAN and the forward link are assumed to operate on different frequencies so that the receive cluster can transmit/receive on the LAN while also receiving signals from the distant transmitter over the forward link. The LAN is also assumed to support a larger throughput than the coded bitrate of the forward link.

For ease of exposition, we assume the distant transmitter uses binary phase shift keying (BPSK) modulation and that messages are (n, k) block coded where n and k correspond to the block length the message length, both in bits, respectively. The forward link code rate is denoted as $r = k/n$. A mechanism for detecting a correctly decoded block, e.g. a CRC check, is assumed. The forward link channels are assumed to be block fading, where each h_i is constant over a block and is independent and identically distributed (i.i.d.) in each block. The channels are also assumed to be spatially i.i.d.

Given a channel input of $X = \pm 1$, the phase-corrected signal received at the i^{th} receive node is given as

$$Y_i = \sqrt{\rho_i} X + W_i \tag{1}$$

where $\rho_i = 2|h_i|^2 \mathcal{E}_s / N_0$, $\mathcal{E}_s$ is the energy per coded forward link bit, $N_0/2$ is the additive white Gaussian noise power spectral density, and $W_i \sim \mathcal{N}(0, 1)$. The noise realizations are assumed to be spatially and temporally i.i.d. The quantity ρ_i corresponds to the signal-to-noise ratio (SNR) of the coded forward link bits at receive node i.

III. Distributed Reception Protocol

This section first provides an overview of the main idea behind the proposed distributed reception protocol, followed by additional details pertaining to a specific implementation.

In the low per-node SNR regimes of interest for large receive clusters, individual nodes are typically unable to successfully decode messages from the distant transmitter. Thus, while receiving a block over the forward link, each node in the receive cluster locally demodulates the transmission and generates LLRs for each of the n coded bits in the current block. These LLRs are not immediately used for decoding. Rather, all of the receive nodes (or a subset of nodes with better channel quality) quantize their soft demodulator outputs and broadcast *all* of their quantized values, along with quantized SNR estimates, over the LAN to the other receive nodes in the cluster. Each receive node then combines the information received over the LAN with their locally unquantized LLRs and passes these results to their local block decoder for decoding. If any receive node successfully decodes the message, it then forwards the decoded message over the LAN to the other receive nodes in the cluster. If two or more nodes successfully decode the message and attempt to broadcast the successfully decoded block, it is assumed the LAN has a mechanism for contention resolution.

An important constraint is that the LAN has limited capacity. If the LAN had unlimited capacity, all of the nodes in the receive cluster could effectively forward unquantized LLRs to the other receive nodes in the cluster and each node could simply sum these LLRs to realize an ideal receive beamformer, as shown in the Appendix. While this case serves as an important benchmark, this paper considers the achievable performance of distributed reception with limited LAN capacity.

As a specific example of how distributed reception can be performed with limited LAN capacity, consider the timeline shown in Figure 2. After receiving and locally demodulating a block, the following steps are performed by the receive cluster over the LAN:

1) All N nodes exchange estimates of their channel magnitudes $|h_i|$ or received SNRs ρ_i.
2) The $M \leq N$ nodes with the strongest channel magnitudes or SNRs participate[1] by forwarding all of their quantized observations over the LAN. As quantized messages are received over the LAN, each receive node (including those that do not participate) scale this quantized information (based on the previously exchanged channel magnitudes/SNRs, as discussed in Section V) and combine it with their locally unquantized LLRs.

[1] A "participating" node is a node that broadcasts its quantized observations over the LAN to the other nodes in the receive cluster. We consider the general case where, due to poor channel conditions or LAN capacity constraints, some nodes in the receive cluster may not broadcast quantized observations.

3) If any receive node successfully decodes the message, it broadcasts the decoded message over the LAN to the other receive nodes in the cluster.

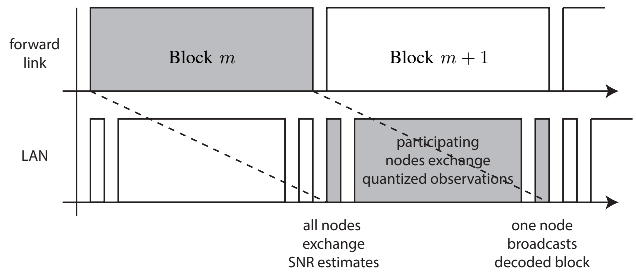

Fig. 2. Distributed reception protocol timeline example.

The number of participating nodes M can be selected to satisfy a LAN throughput constraint. To determine M, we assume the number of quantization bits per coded bit is fixed for all receive nodes and is denoted as b. The normalized LAN throughput, in units of LAN bits per forward link information bit, can be calculated as

$$\eta_{\mathsf{LAN}} = \frac{No_1 + Mbn + k + o_2}{k} \approx \frac{Mb}{r} + 1 \leq C_{\mathsf{LAN}} \qquad (2)$$

where No_1 is the overhead of exchanging SNR estimates and determining which nodes will participate, o_2 is the contention overhead in disseminating the successfully decoded block, and C_{LAN} is the maximum normalized LAN throughput. It is assumed that n and k are sufficiently large such that the overheads are negligible. Given r, b, and C_{LAN}, it follows that selecting $M \leq \min\{N, r(C_{\mathsf{LAN}} - 1)/b\}$ satisfies (2).

IV. Information Theoretic Analysis

This section develops an information theoretic framework for quantifying the performance of the proposed distributed reception scheme where each node in the receive cluster combines their local unquantized LLRs with quantized observations from other nodes in the receive cluster. Figure 3 shows an example of an information theoretic model for a three-node cluster with full participation using one-bit quantization. This model corresponds to the situation at node 3 since it combines the quantized observations from nodes 1 and 2 with the unquantized information at node 3.

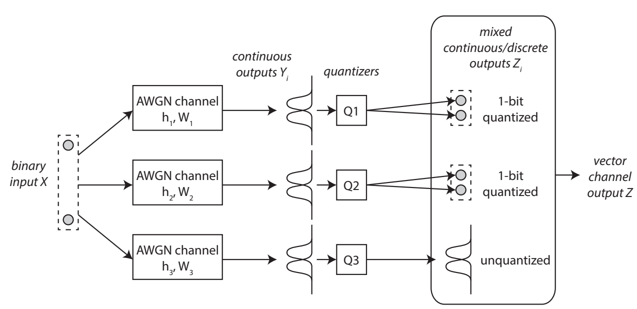

Fig. 3. $N = 3$ node information theoretic model example.

Given equiprobable binary channel inputs X drawn from $\{x_0, x_1\}$, the channel realization $\boldsymbol{h}$, the vector channel output $\boldsymbol{Z} = [Z_1, \dots, Z_N]^\top$ with elements arbitrarily quantized or unquantized, and denoting $p(\boldsymbol{z}|k) = p_{\boldsymbol{Z}|X}(\boldsymbol{z}|X = x_k)$, the mutual information $I_{\boldsymbol{h}}(X; \boldsymbol{Z})$ can be expressed as

$$\begin{aligned} I_{\boldsymbol{h}}(X;\boldsymbol{Z}) &= H(X) - H_{\boldsymbol{h}}(X|\boldsymbol{Z}) \\ &= 1 + \frac{1}{2}\sum_{k=0}^{1}\int_{-\infty}^{\infty} p(\boldsymbol{z}|k)\log_2\left\{\frac{p(\boldsymbol{z}|k)\frac{1}{2}}{p_{\boldsymbol{Z}}(\boldsymbol{z})}\right\} d\boldsymbol{z} \\ &= 1 - \frac{1}{2}\sum_{k=0}^{1}\int_{-\infty}^{\infty} p(\boldsymbol{z}|k)\log_2\left\{\frac{\sum_{\ell=0}^{1} p(\boldsymbol{z}|\ell)}{p(\boldsymbol{z}|k)}\right\} d\boldsymbol{z} \\ &= 1 - \frac{1}{2}\sum_{k=0}^{1} \mathrm{E}\left[\log_2\left\{\frac{\sum_{\ell=0}^{1} p(\boldsymbol{z}|\ell)}{p(\boldsymbol{z}|k)}\right\} \,\Big|\, X = x_k\right] \end{aligned}$$

where all distributions are conditioned on $\boldsymbol{h}$ and the conditional expectation is over the quantized vector channel output $\boldsymbol{Z}$ given a scalar channel input $X = x_k$. Based on the symmetry of the input constellation and the noise, this conditional expectation is identical for $X = x_0$ and $X = x_1$, hence we can write

$$\begin{aligned} I_{\boldsymbol{h}}(X;\boldsymbol{Z}) &= 1 - \mathrm{E}\left[\log_2\left\{\frac{\sum_{\ell=0}^{1} p(\boldsymbol{z}|\ell)}{p(\boldsymbol{z}|0)}\right\} \Big| X = x_0\right] \\ &= 1 - \mathrm{E}\left[\log_2\{1 + L(\boldsymbol{Z})\} \,\Big|\, X = x_0\right] \qquad (3) \end{aligned}$$

where

$$L(\boldsymbol{Z}) = \frac{p(\boldsymbol{z}|1)}{p(\boldsymbol{z}|0)} = \frac{p_{\boldsymbol{Z}|X}(\boldsymbol{Z}|X = x_1)}{p_{\boldsymbol{Z}|X}(\boldsymbol{Z}|X = x_0)} = \frac{\mathrm{Prob}(X = x_1 \mid \boldsymbol{Z})}{\mathrm{Prob}(X = x_0 \mid \boldsymbol{Z})}.$$

Conditioning on $X = x_k$, the elements of $\boldsymbol{Z}$ are conditionally independent and we can write

$$p_{\boldsymbol{Z}|X}(\boldsymbol{z}|X = x_k) = \prod_{i=1}^{N} p_{Z_i|X}(z_i|X = x_k).$$

Hence

$$L(\boldsymbol{z}) = \prod_{i=1}^{N} \frac{p_{Z_i|X}(z_i|X = x_1)}{p_{Z_i|X}(z_i|X = x_0)} = \prod_{i=1}^{N} L_i(z_i)$$

and the log-likelihood $\ell(\boldsymbol{z}) = \sum_{i=1}^{N} \ell_i(z_i)$.

In the proposed distributed reception system, since one or more of the outputs in the vector channel is unquantized, the expectation in (3) must be approximated numerically, either by numerical integration or by Monte-Carlo simulation.

A. Unquantized Channel Outputs

For an equiprobable binary input and an unquantized i^{th} output, we have $Z_i = Y_i = \sqrt{\rho_i} X + W_i$, hence

$$L_i(z_i) = \frac{p_{Z_i|X}(z_i|X = x_1)}{p_{Z_i|X}(z_i|X = x_0)} = \exp\{2 z_i \sqrt{\rho_i}\}.$$

The log-likelihood ratio in this case is then $\ell_i(z_i) = 2 z_i \sqrt{\rho_i}$.

B. Quantized Channel Outputs

Quantization of the soft demodulator outputs at receive node i induces a discrete memoryless channel from the distant transmitter to that receiver, as shown in Figure 3. In general, for a quantized i^{th} output, the quantizer partition at the i^{th} receive node specifies a mapping from continuous observations $Y_i = \sqrt{\rho_i}X + W_i$ to a codebook index $Z_i \in \{0, \dots, K_i - 1\}$. The conditional distribution $p_{Z_i|X}(z_i|X = x_k)$ in this case is a probability mass function with probabilities

$$\text{Prob}(Z_i = z_i \,|\, X = x_k) = p^{(i)}_{z_i|k}$$

for $z_i = 0, \dots, K_i - 1$. Hence, for equiprobable binary inputs and arbitrarily quantized outputs, we have

$$L_i(z_i) = \frac{p_{Z_i|X}(z_i|X = x_1)}{p_{Z_i|X}(z_i|X = x_0)} = \frac{p^{(i)}_{z_i|1}}{p^{(i)}_{z_i|0}}.$$

The quantity $p^{(i)}_{z_i|k}$ can be thought of as the probability of observing quantizer output $Z_i = z_i$ at node i given a channel input $X = x_k$, i.e., $p^{(i)}_{z_i|k}$ is the discrete memoryless channel transition probability from input k to output z_i.

For the specific case of one-bit quantized channels, since the symbols and noise are symmetric, we will assume the one-bit quantizer partition is based on the sign of the observation at receiver i. Hence, at receiver i we have

$$z_i = \begin{cases} 0 & y_i < 0 \\ 1 & y_i \geq 0. \end{cases}$$

Observe that one-bit quantization induces a binary symmetric channel (BSC) at the i^{th} receiver. The transition probability for the resulting BSC is the error probability

$$p = p^{(i)}_{0|1} = p^{(i)}_{1|0} = Q\left(\sqrt{\rho_i}\right). \tag{4}$$

The likelihood ratio is then

$$L_i(z_i) = \frac{p^{(i)}_{z_i|1}}{p^{(i)}_{z_i|0}} = \begin{cases} \frac{p}{1-p} & z_i = 0 \\ \frac{1-p}{p} & z_i = 1 \end{cases}$$

and the LLR is given as

$$\ell_i(z_i) = \begin{cases} \ln \frac{p}{1-p} & z_i = 0 \\ \ln \frac{1-p}{p} & z_i = 1. \end{cases} \tag{5}$$

C. Numerical Example

Figure 4 shows an example of the mutual information for distributed reception with $N = 10$ receive nodes and fixed channels $\boldsymbol{h} = [1, \dots, 1]^\top$. All receive nodes are assumed to participate in the distributed reception protocol. The binary-input, all unquantized outputs result corresponds to the capacity of ideal receive beamforming. Since the forward link channels to each receive node are the same in this example, the performance when one output is unquantized and $N-1$ outputs are one-bit quantized is the same for all receive nodes (this is not the case for general $\boldsymbol{h}$, however). These results show that distributed reception can provide significant capacity gains with respect to single-receiver processing and that receiving just one bit of information from each of the other nodes in the receive cluster can result in performance within approximately 2 dB of ideal receive beamforming for fixed, equal-gain channels.

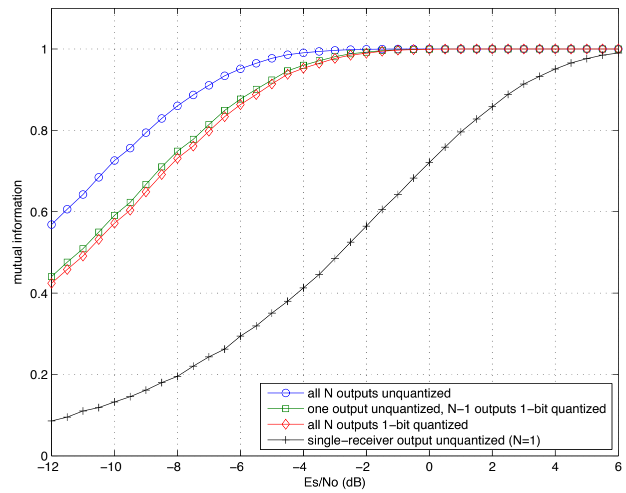

Fig. 4. Mutual information for a binary-input distributed reception system with $N = 10$ receive nodes, full participation, and $\boldsymbol{h} = [1, \dots, 1]^\top$.

V. Combining Quantized Observations

During the distributed reception protocol, each node receives quantized observations from all of the participating nodes in the receive cluster. These quantized observations are then scaled and combined with each other as well as with the locally unquantized LLRs to generate aggregate LLRs for input to the local block decoder.

To compute the aggregate likelihoods, it is sufficient for each node to use its knowledge of the participating nodes' SNRs (exchanged prior to the quantized observations as shown in Figure 2) and quantizer partitions. For example, for one-bit quantization, knowledge of the SNR allows for calculation of the BSC error probability in (4) and subsequent reconstruction of the marginal BSC output LLRs via (5). Denoting the set of participating nodes as $\mathcal{M}$, once the quantized observations received over the LAN have been converted to LLRs, they can be combined directly with the locally unquantized LLR at node j by computing $\ell(\boldsymbol{z}) = \ell_j(z_j) + \sum_{i \in \mathcal{M} \setminus j} \ell_i(z_i)$.

Note that, in general, the log-likelihood sum $\ell(\boldsymbol{z})$ will be different at each node in the receive cluster since the unquantized element in $\boldsymbol{z}$ is different at each receive node. Also, if node j does not participate ($j \notin \mathcal{M}$), it will have one more element in the log-likelihood sum than if it does participate ($j \in \mathcal{M}$). Hence, unlike ideal receive beamforming where the decision statistic is identical at all of the receive nodes, the different decision statistics in a distributed reception system with quantized observation exchanges makes it possible that some nodes will be able to decode the received message while others will not. This motivates the broadcast of successfully decoded blocks as discussed in Section III.

VI. Numerical Results

This section provides numerical results demonstrating the efficacy of distributed reception with coarse quantization. All of the results in this section assume spatially and temporally i.i.d. block fading channels with $h_i \sim \mathcal{CN}(0,1)$.

Figure 5 shows the outage probability of distributed reception versus $\mathcal{E}_s/N_0$ for $N = 1,2,5,10$ and full participation ($M = N$). These results are obtained from the information theoretic analysis in Section IV with 10000 channel realizations per receive node and 10000 noise realizations for each channel realization. An outage event occurs when $I_{\boldsymbol{h}}(X;\boldsymbol{Z}) < r_{out} = \frac{1}{2}$ at all of the receive nodes. The two-bit quantizer results used the partition[2]

$$z_i = \begin{cases} 0 & y_i < -a_i \\ 1 & -a_i \leq y_i < 0 \\ 2 & 0 \leq y_i < a_i \\ 3 & y_i \geq a_i \end{cases}$$

where a_i is the quantizer amplitude threshold selected to maximize the marginal mutual information $I(X;Z_i)$. These results show that significant improvements in outage probability can be obtained through combining locally unquantized LLRs with quantized observations from other nodes in the receive cluster and that the gap between exchanging ideal receive beamforming (unquantized LLRs) and exchanging just one bit per coded bit is less than 1.5 dB in the cases tested. Two bits per coded bit reduces this gap to better than 0.5 dB at the expense of approximately doubling the LAN throughput requirements.

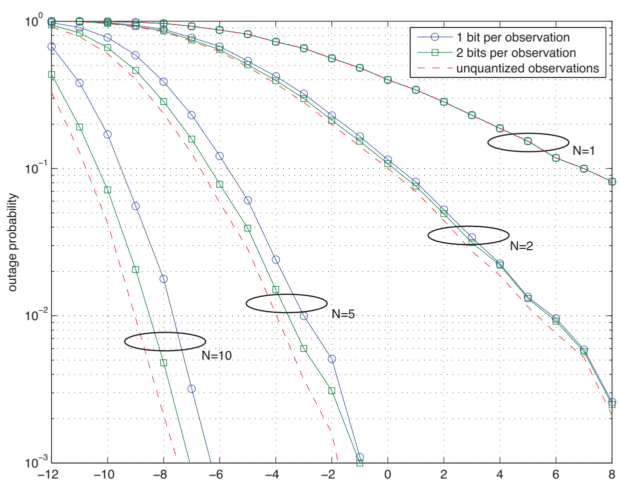

Fig. 5. Outage probability versus $\mathcal{E}_s/N_0$ for distributed reception with quantized observations, outage rate $r_{out} = 1/2$, and full participation ($M = N$).

Figure 6 shows outage probability and normalized LAN throughput η_{LAN} from (2) versus the number of participating nodes M for $N = 10$ and $\mathcal{E}_s/N_0$ set to -8 dB. The set of participating nodes was selected by choosing the M receive nodes with the strongest channel magnitudes/SNRs. The simulation parameters in Figure 6 were otherwise identical to those in Figure 5. Even with $M = 0$, distributed reception provides a diversity gain since the marginal mutual informations must all be less than r_{out} for an outage event to occur. This diversity gain can be seen by the fact that the outage probability when $M = 0$ and $N = 10$ (corresponding to no exchange of quantized observations over the LAN) is approximately 0.7 in Figure 6, whereas the outage probability at $\mathcal{E}_s/N_0 = -8$ dB and $N = 1$ in Figure 5 is close to one. The results in Figure 6 show the tradeoff between improved performance and increased LAN throughput for a fixed cluster size N, since the normalized LAN throughput scales linearly with M and b. In this example, the performance gain obtained by doubling the number of participating nodes tends to be better than the performance gain obtained by doubling the number of bits per observation when M is small. For larger values of M, e.g. $M = 5$, using two bits per observation gives a slightly better performance improvement than doubling M.

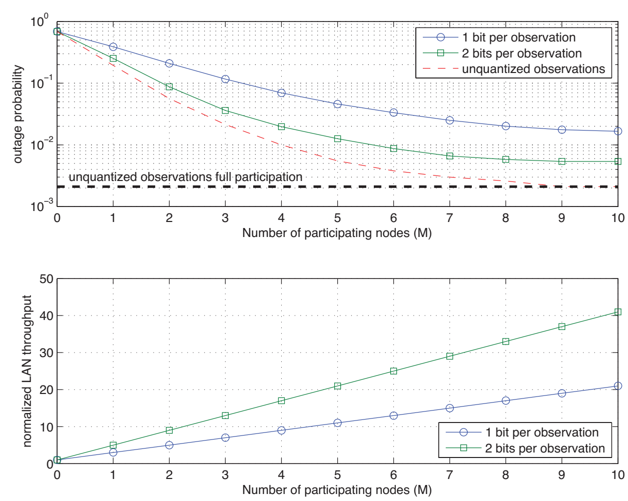

Fig. 6. Outage probability and normalized LAN throughput (in LAN bits per forward link information bit) versus number of participating nodes M for distributed reception with quantized observations, outage rate $r_{out} = 1/2$, $\mathcal{E}_s/N_0 = -8$ dB, and $N = 10$.

Figure 7 shows the outage probability and block error rate (BLER) performance of an LDPC code implementation of the distributed reception protocol with one-bit quantization. The rate $r = 1/2$ LDPC code was selected from proposed codes for DVB-S2 in [16], [17] with $n = 8100$ and $k = 4050$. These results demonstrate that the achievable performance with real block codes can be close to the information theoretic predictions.

VII. Conclusion

We have shown that, in the low SNR regimes enabled by receiver cooperation, coarse quantization of observations followed by LLR reconstruction and combining across receivers results in little loss of performance relative to ideal

[2]Due to the symmetry of the input constellation and noise, this quantizer is intuitively reasonable but we make no claim as to its optimality.

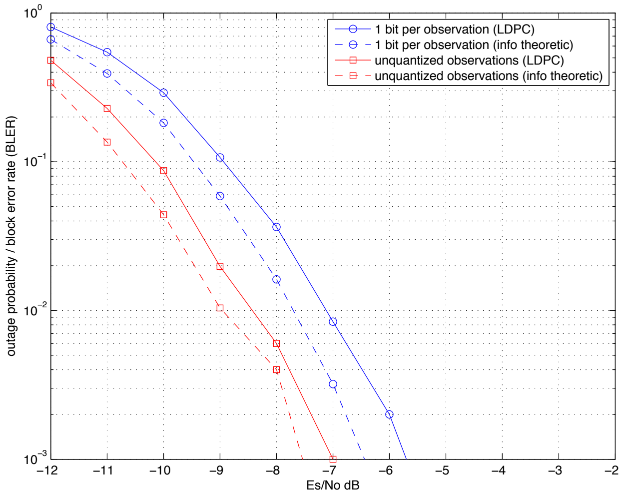

Fig. 7. Outage probability and block error rate versus $\mathcal{E}_s/N_0$ for for distributed reception with quantized observations, outage rate $r_{out} = 1/2$, and $M = N = 10$.

beamforming, which is equivalent to summing unquantized LLRs for BPSK. Thus, good performance can be achieved with significant reduction in LAN throughput requirements relative to sharing conventionally quantized LLRs. Our information-theoretic framework provides quick performance estimates that agree with that of LDPC-coded systems.

While the results in this paper extend immediately to Gray-coded QPSK, we are currently investigating extension of this approach to systems with forward links with higher-order, more spectrally efficient, constellations. Another important topic for future work is to investigate the effect of channel estimation errors, which may become a significant bottleneck at the low per-node receive SNRs of interest. It is also of interest to extend the simple frequency nonselective fading model here to more complex propagation environments. Finally, it is of interest to explore the requirements on quantizer precision for distributed reception of spatially multiplexed data streams, which is a key concept in hierarchical cooperation for scaling *ad hoc* networks [18].

APPENDIX

In this appendix, we show that the log-likelihood ratio of the ideal receive beamformer decision statistic is equivalent to the sum of the log-likelihood ratios of the unquantized decision statistics at each node in the receive cluster. Given the individual unquantized decision statistics Z_i for $i = 1, \ldots, N$, the ideal receive beamformer decision statistic can be written as

$$Z_{bf} = \frac{1}{\|\boldsymbol{h}\|} \sum_{i=1}^{N} |h_i| Z_i = \|\boldsymbol{h}\| \sqrt{2\mathcal{E}_s/N_0} X + W'$$

where $W' \sim \mathcal{N}(0, 1)$. Hence, given the realization $Z_{bf} = z$, the LLR is $\ell(z) = 2z\|\boldsymbol{h}\|\sqrt{2\mathcal{E}_s/N_0}$. But since $z = \frac{1}{\|\boldsymbol{h}\|} \sum_{i=1}^{N} |h_i| z_i$, this can be written as

$$\ell(z) = 2\sqrt{2\mathcal{E}_s/N_0} \sum_{i=1}^{N} |h_i| z_i = \sum_{i=1}^{N} 2 z_i \sqrt{\rho_i} = \sum_{i=1}^{N} \ell_i(z_i).$$

Hence, the LLR of the ideal receive beamformer decision statistic is equivalent to the sum of the individual LLRs of the unquantized decision statistics at each receive node.

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