LINEAR DETECTOR LENGTH CONDITIONS FOR DS-CDMA PERFECT SYMBOL RECOVERY

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ABSTRACT

Within the established framework of linear detection for multiuser DS-CDMA systems, we derive length conditions for perfect recovery of the transmitted symbols of all K in-cell mobile users. The analysis accounts for multiple access interference with arbitrary asynchronism as well as multipath interference with arbitrary delay spread. The results imply that linear detectors achieving perfect symbol recovery may require an observation interval much longer than a single bit. Moreover, we show that linear detectors employing oversampling and/or antenna arrays require less taps to achieve perfect symbol recovery than conventional non-oversampled, single antenna receivers. Simulation results are presented verifying the derived length conditions.

1. INTRODUCTION

Considerable research effort has focused on the problem of multiuser detection for DS-CDMA communication systems recently. Linear detectors, although suboptimal, have been extensively studied due to their good performance properties and reasonable complexity. In noise-free conditions it has been shown that the linear decorrelating detector can perfectly recover the symbols of all users in the system by completely canceling multiple access interference. Analogously, in single user equalization, it has been shown that zero-forcing fractionally-spaced equalizers (FSEs) can also perfectly recover symbols in the noise-free case by completely cancelling the effects of multipath interference. In this paper we analyze the perfect symbol recovery problem for multiuser DS-CDMA systems with arbitrary multipath interference. To perfectly recover transmitted symbols in this system, the linear detector must completely cancel both multiple access and multipath interference.

Conditions for perfect symbol recovery in the single user FSE problem are well known (see [4] for example). The necessary and sufficient conditions may be summarized as

- sufficient equalizer (linear detector) length,
- subchannel disparity (for FSEs), and
- no additive channel noise.

It is well known that satisfying the first two requirements leads to a system matrix with full column rank. As a consequence of this full column rank, a FSE can perfectly recover every symbol (i.e., every delay) in its observation interval. In this paper, we discuss the applicability of the full column rank condition to the DS-CDMA multiuser detection problem and define a weaker criterion for perfect symbol recovery of all users at only *some* delays. We derive a necessary and sufficient vector space condition for weak perfect symbol recovery (WPSR) and, in the spirit of single user equalization, we extend this idea to calculable linear detector length requirements for synchronous and asynchronous DS-CDMA systems. Just as length requirements in and of themselves are not sufficient for perfect symbol recovery in single user equalization, our proposed linear detector length requirements are also not sufficient for perfect symbol recovery and turn out to be slightly stronger than necessary due to a simplifying assumption on the structure of the interference subspace. Simulations are presented that verify the analysis and show that satisfaction of the proposed length conditions is actually often sufficient for WPSR.

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2. DISCRETE TIME SYSTEM MODEL

Consider the baseband DS-CDMA system model shown in Figure 1. In this general MIMO system model, K possibly asynchronous users transmit coherently modulated DS-CDMA signals with common spreading gain N through dispersive channels (assumed time invariant over the linear detector's observation interval) to a bank of M antennas at the receiver. The observed signal at each antenna is corrupted by AWGN independent of the users' symbols. We assume that the pulse shaping and channel filters are FIR

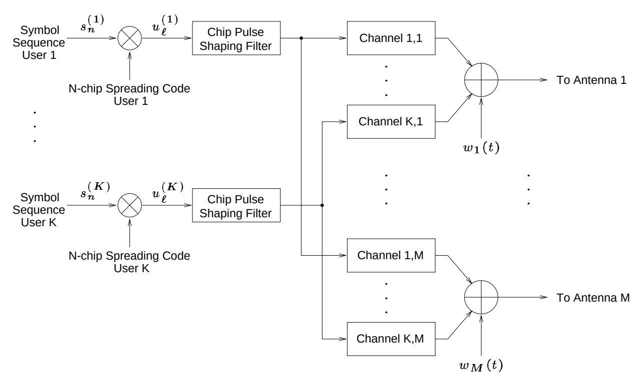

Figure 1: DS-CDMA system model.

and that the spreading codes are periodic with period equal to the spreading gain $N = T/T_c$.

The receiver shown in Figure 2 consists of a bank of M antennas each followed by a FIR receiver input filter, a T_c/P rate sampler, and a digital linear filter with $N_f < \infty$ taps. For notational convenience, we group the bank of

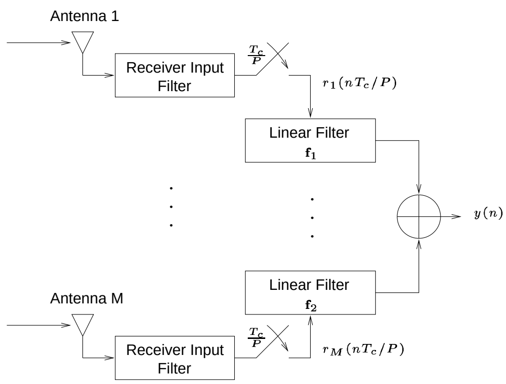

Figure 2: Multi-antenna, oversampling receiver with a linear multiuser detector.

digital linear filters into a long vector as $\mathbf{f} = \left[\mathbf{f}_1^\top \ \ldots \ \mathbf{f}_M^\top\right]^\top$ and call $\mathbf{f}$ the linear detector. The output of the digital filters is summed to form the soft decision for a desired user at symbol index n denoted by $y(n)$.

The prior assumptions imply that the combined impulse response of the pulse shaping filter, the k, m^{th} propagation channel, and the receiver input filter is FIR. For notational convenience, we will hereafter refer to these combined impulse responses simply as the "channels". We denote the common time support of all channels as $[0, LT_c/P)$.

In the single antenna case, it has been shown [1, 5] that the regressor input to the linear detector may be expressed as a linear combination of the source symbols with additive noise such that $\mathbf{r}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{w}(n)$. The system matrix

$$\mathbf{H} = \left[\begin{array}{cccc} \mathbf{H}^{(1)} & \mathbf{H}^{(2)} & \ldots & \mathbf{H}^{(K)} \end{array} \right]$$

is constructed of sub-system matrices for each user of dimension $N_f \times Q^{(k)}$ exhibiting the structure

$$\mathbf{H}^{(k)} = \left[\begin{array}{cccc} \top & & & \\ \mathbf{a}^{(k)} & \uparrow & & \\ \downarrow & \mathbf{a}^{(k)} & & \\ & \downarrow & \ddots & \uparrow \\ & & & \mathbf{a}^{(k)} \\ & & & \perp \end{array} \right]$$

where $\mathbf{a}^{(k)}$ accounts for the FIR T_c/P sampled k^{th} user's spreading code convolved with the k^{th} user's channel. The vector of user-ordered source symbols at symbol index n is given by

$$\mathbf{s}(n) = [s^{(1)}(n) \ \ldots \ s^{(1)}(n - Q^{(1)} + 1) \ \ldots \\ s^{(K)}(n) \ \ldots \ s^{(K)}(n - Q^{(K)} + 1)]^\top \quad (1)$$

where $Q^{(k)}$ denotes the number of symbols from the k^{th} user in the observation interval.

It is straightforward to generalize this single-antenna model to account for multiple antennas at the expense of slightly more complicated notation. In this case the total number of taps in the linear detector is now MN_f and the regressor input to the linear detector may be written as $\mathbf{r}(n) = \mathcal{H}\mathbf{s}(n) + \mathbf{w}(n)$ where the multi-antenna system matrix $\mathcal{H}$ is given by

$$\mathcal{H} = \left[\begin{array}{cccc} \mathbf{H}_1^{(1)} & \mathbf{H}_1^{(2)} & \ldots & \mathbf{H}_1^{(K)} \\ \mathbf{H}_2^{(1)} & \mathbf{H}_2^{(2)} & \ldots & \mathbf{H}_2^{(K)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_M^{(1)} & \mathbf{H}_M^{(2)} & \ldots & \mathbf{H}_M^{(K)} \end{array} \right] \quad (2)$$

and $\mathbf{w}(n)$ is an $MN_f \times 1$ vector of additive channel noise. The $\mathbf{H}_m^{(k)}$ elements of $\mathcal{H}$ represent the k^{th} user to m^{th} antenna system matrices and the dimensions of $\mathcal{H}$ are $MN_f \times \sum_{k=1}^{K} Q^{(k)}$. The soft decisions at the output of the linear detector are then $y(n) = \mathbf{f}^\top \mathbf{r}(n) = \mathbf{f}^\top \left[\mathcal{H}\mathbf{s}(n) + \mathbf{w}(n)\right]$.

3. PERFECT SYMBOL RECOVERY

The following analysis of the discrete time system model examines the effects of linear detector length (MN_f) on the existence of linear detectors that achieve perfect symbol recovery in the absence of additive channel noise. The classical definition of perfect symbol recovery is given below.

Definition 1. *For each $k \in \{1, 2, \ldots, K\}$, strong perfect symbol recovery (SPSR) requires that the linear detector can achieve $y(n) = s^{(k)}(n - \delta)$ for all $\delta = 0, 1, 2, ..., Q^{(k)} - 1$.*

It is well known that requiring SPSR is equivalent to requiring $\mathcal{H}$ to have full column rank. Although several papers have analyzed linear detection under the assumption of a full column rank system matrix (see for example [2, 3]), simulations have shown there exist system matrices with unfortunate combinations of asynchronism and multipath channels which do not have full column rank for any value of N_f. In this case, even though a linear detector can't perfectly recover all the symbol delays of all K users, it may still be possible to perfectly recover one or more symbol delays for all K users. This motivates the definition of weak perfect symbol recovery (note that SPSR $\Rightarrow$ WPSR but not conversely).

Definition 2. *For each $k \in \{1, 2, \ldots, K\}$, weak perfect symbol recovery (WPSR) requires that the linear detector can achieve $y(n) = s^{(k)}(n - \delta)$ for at least one $\delta \in \{0, 1, 2, ..., Q^{(k)} - 1\}$.*

Given the previous development of the discrete time model, we seek to derive an expression for MN_f such that WPSR is achievable. Let $\mathcal{H}_S$ be a matrix of dimension $MN_f \times K$ formed by taking one column from each user's portion of the system matrix. These columns correspond to a desired symbol delay for each user. Let $\mathcal{H}_I$ be a matrix of dimension $MN_f \times \sum_{k=1}^{K} Q^{(k)} - K$ composed of the remaining columns of $\mathcal{H}$. Define subspaces $V_S = \text{range}(\mathcal{H}_S)$ and $V_I = \text{range}(\mathcal{H}_I)$.

Proposition 1. *A linear detector can achieve WPSR* $\Leftrightarrow$ $\exists \mathcal{H}_S$ *such that* $dim(V_S) = K$ *and* $dim(V_S \cap V_I) = 0$.

The proof of this proposition is straightforward and omitted for space. Proposition 1 implies that a necessary condition for WPSR is that $MN_f \geq \dim(V_I) + K$.

Since it is difficult to develop a simple characterization of the exact dimension of the subspace V_I, we make a worst-case simplifying assumption in the following analysis. Specifically, in order to develop calculable linear detector length conditions we assume that $\mathcal{H}_I$ has full column rank and consequently that $\dim(V_I)$ is equal to the number of columns in $\mathcal{H}_I$. Although it is easy to develop examples where $\mathcal{H}_I$ does not have full column rank implying that there are cases where this assumption leads to length conditions that are somewhat stronger than necessary, we justify this assumption by simulation in Section 4. Under our simplifying assumption, the length condition may be written as

$$MN_f \geq \sum_{k=1}^{K} Q^{(k)}. \tag{3}$$

We remind the reader that this length condition is not sufficient since it says nothing about the intersection of the subspaces V_I and V_S. The simulations in Section 4 show that satisfying (3) tends to also satisfy the WPSR conditions in Proposition 1, but we also provide an example in Section 4 where the system matrix does not satisfy the requirements for WPSR when (3) is satisfied.

3.1. Synchronous Case

In the synchronous case, inspection of $\mathcal{H}$ allows us to express the exact number of columns per user as $Q^{(k)} = \left\lceil \frac{N_f+L-1}{NP} \right\rceil$ for $k = 1, 2, \ldots, K$. Thus, in order to satisfy (3), the linear detector must satisfy the length condition

$$MN_f \geq K \left\lceil \frac{N_f + L - 1}{NP} \right\rceil. \tag{4}$$

To solve for N_f in (4) we will use the substitution

$$N_f + L - 1 = aNP - b \tag{5}$$

for $a \in \mathbb{N}$ and $b \in \{0, 1, \ldots, NP-1\}$. This allows us to rewrite (4) as

$$M(aNP - b - L + 1) \quad \geq \quad K \left\lceil \frac{aNP - b}{NP} \right\rceil = Ka$$

since $N_f = aNP - b - L + 1$. Solving for the minimum integer a satisfying this expression, we find that

$$a = \left\lceil \frac{L - 1 + b}{NP - K/M} \right\rceil \tag{6}$$

for $K < MNP$. The value of $b \in \{0, 1, \ldots, NP-1\}$ that minimizes N_f is

$$b = \left\lceil \frac{L-1}{NP - K/M} \right\rceil (NP - K/M) - L + 1.$$

Substituting these results for a and b into (5) and solving for N_f yields the desired linear detector length condition under synchronous transmission as

$$MN_f \geq K \left\lceil \frac{L-1}{NP - K/M} \right\rceil \tag{7}$$

for $K < MNP$ and $L > 1$.

3.2. Asynchronous Case

In the asynchronous case, the baud timing is different for each user and consequently the number of symbols in the observation interval may differ between users. Inspection of $\mathcal{H}$ shows that each user may contribute one additional symbol to the observation interval depending on their delay as well as the spreading gain, oversampling factor, and channel length. In order to develop a length condition valid for any possible combination of user delays, our approach is to select MN_f such that $MN_f \geq \max \sum_{k=1}^{K} Q^{(k)}$ where the maximum is taken over all possible delays. Since each user contributes at most one additional column over the synchronous case, we can use (4) to write the desired condition as

$$MN_f \geq K \left(\left\lceil \frac{N_f + L - 1}{NP} \right\rceil + 1 \right).$$

Following the same analysis steps as the synchronous case, for $K < MNP$ and $L > 1$, we can write a closed form expression for the desired length condition under asynchronous transmission as

$$MN_f \geq K \left(\left\lceil \frac{L - 1 + K/M}{NP - K/M} \right\rceil + 1 \right). \tag{8}$$

4. SIMULATION RESULTS

The simulations shown in Figures 3 and 4 plot the theoretical minimum linear detector length versus the number of synchronous/asynchronous users for a DS-CDMA system with 8 chip Hadamard spreading codes and channels with support $[0, 6T_c)$ Also plotted are the results of 200 Monte Carlo simulations for systems with random (time invariant over the observation interval) propagation channels and asynchronism. In these simulations, we incrementally increased the linear detector length until the necessary and sufficient WPSR conditions in Proposition 1 were satisfied. One synchronous simulation was run with the simulation results exactly matching the synchronous length condition in (7).

Note that the derived length conditions and the simulation results above imply that the linear detector observation interval may need to be much longer than a single bit in order to achieve WPSR. Furthermore, these results imply that oversampling and/or antenna arrays reduce the total number of taps required by the linear detector for WPSR when compared to the chip-rate, single antenna case.

Although these simulations suggest that satisfying the proposed length condition always implies WPSR, it is easy to construct a simple example demonstrating the insufficiency of (3). Consider the classical scenario with identity, non-dispersive channels ($L = 1$). Under this assumption, the columns of the system matrix $\mathcal{H}$ are simply the

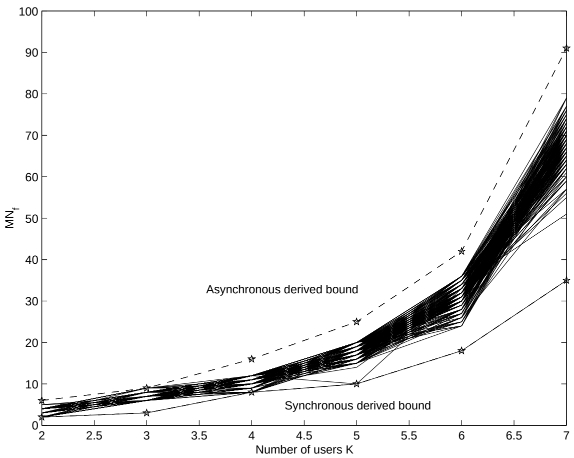

Figure 3: Linear detector length versus number of users for delay spread $L = 6$, one antenna ($M = 1$), spreading gain $N = 8$, and chip rate sampling ($P = 1$). Upper starred line is from (8) and lower starred line is from (7).

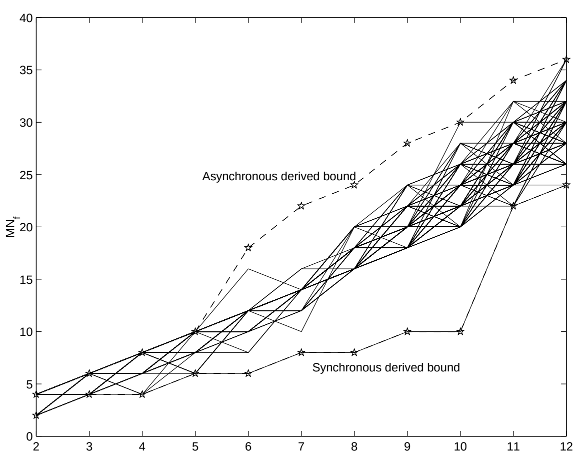

Figure 4: Linear detector length versus number of users for delay spread $L = 12$, two antennas ($M = 2$), spreading gain $N = 8$, and fractional chip rate sampling ($P = 2$). Upper starred line is from (8) and lower starred line is from (7).

spreading codes of the users. Now suppose there are two synchronous users with spreading gain $N = 4$, and the receiver has a single antenna, sampled at the chip rate. We can use (4) to write $N_f \geq 2\lceil N_f/4 \rceil$ which is satisfied for $N_f = 2$. Suppose that the first user's spreading code is given by $[1, 1, 1, 1]^\top$ and the second user's spreading code is $[1, 1, -1, -1]^\top$. Observe that these codes are orthogonal and that the conditions for WPSR would be satisfied for $N_f \geq 3$ but with $N_f = 2$ the system matrix

$$\mathcal{H} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

clearly does not satisfy the requirements for WPSR. This behavior is loosely analogous to a loss of subchannel disparity in single user equalization but simulations suggest that this sort of problem only tends to occur with systems having a small number of users.

5. CONCLUSIONS

In this paper we analyzed the relationship between linear detector length and the ability to perfectly recover transmitted symbols. Using a general system model accounting for nontrivial propogation channels and arbitrary asynchronism, we proposed an appropriate weaker definition for perfect symbol recovery in multiple access communication systems and derived closed form linear detector length conditions in both synchronous and asynchronous cases. Simulation results suggested that even though the length conditions are not sufficient, their satisfaction often creates a system matrix which meets all the requirements for WPSR. Topics for future research include the development of necessary and sufficient conditions for WPSR in multiple access communication systems under a framework similar to single user equalization.

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