Fundamental Bounds on the Age of Information in General Multi-Hop Interference Networks

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***Abstract*—This paper studies the "age of information" of status updates in a general multi-source multi-hop wireless network with time slotted transmissions and general interference constraints. Specifically, the scenario considered in this paper assumes that each node in the network is both a source and a monitor of information and that all nodes wish to receive fresh status updates from all other nodes in the network. Lower bounds for the instantaneous peak and average age of information are derived for three interference models using properties of each interference model's family of feasible activation sets. These bounds generalize prior results derived for the "global" interference model where only one node transmits in each time slot. Achievability results are presented through the development of explicit schedules for ring networks in three specific interference models: (i) global interference, (ii) interference free, i.e., all nodes can transmit simultaneously without interference, and (iii) topologically-dependent interference where multiple nodes transmit simultaneously if they share no one-hop neighbors. Numerical examples are presented to quantify the gap between the achieved age and the bounds.**

***Index Terms*—Age of information, multi-source, multi-hop, explicit contention, graph theory, interference models.**

I. Introduction

In many contemporary networked monitoring and control systems, e.g., intelligent vehicular systems, timely status updates are critically important to maintain safe operation and provide stable control loops. This understanding has led to a new line of research centered around an *Age of Information* (AoI) metric [1], [2] which measures the staleness of a monitor's knowledge of a time varying process measured by a separate source in the network. Much of the literature on this subject has focused on the single-source, single-monitor, single-hop setting, where the hop is typically modeled as a random delay through a queue [3]–[7]. Multi-source and/or multi-monitor extensions, also in the single-hop context, have been considered in [8]–[14].

This paper considers age of information in multi-hop networks. While the multi-hop setting was first considered in the context of vehicular networks in [15], this setting has received relatively little attention in the literature. One line of study has focused on analyzing specific multi-hop network structures, e.g., line, ring, and/or two-hop networks [15]–[21]. A general multi-hop network setting where a single-source disseminates status updates through a gateway to an interference-free network was considered in [22], [23]. A practical age control schedule to improve AoI in multi-hop IP networks was also recently proposed in [24]. The work in [25] considers a network with multiple source-monitor pairs communicating over multiple hops with interference constraints. The analysis in [25] addresses age-optimal random transmission policies where links are activated according to a fixed probability distribution.

This paper considers a general multi-source, multi-monitor, multi-hop setting with explicit interference constraints. Unlike [25], we assume that (i) every node in the network is both a source and a monitor of information, (ii) every node wishes to receive timely status updates from all other nodes in the network, and (iii) updates are disseminated with deterministic schedules. The work in this paper builds on our prior results in [26], [27] by generalizing our analysis to allow for arbitrary interference constraints and simultaneous transmissions of status updates. The main contributions of this paper are:

1) We consider AoI in a general multi-source, multi-monitor, multi-hop setting with explicit interference constraints expressed through the network's graph and a corresponding family of feasible activation sets.
2) We derive general lower bounds on the instantaneous peak and average AoI under arbitrary interference constraints. These lower bounds generalize the bounds in [26], [27] and are based on properties of each interference model's family of feasible activation sets.
3) For ring networks with any number of nodes, we develop explicit schedules and present numerical results quantifying the gap between the achieved age and the bounds.

II. System Model and Age Metric

In this section, we describe the system model and the age metric that is considered in this paper.

A. Network Model

We consider a wireless network where connectivity of nodes is modeled by a time-invariant directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where the vertex set $\mathcal{V}$ represents the wireless nodes and the edge set $\mathcal{E}$ represents the channels between the nodes in the network. Edge $e_{i,j}$ is in set $\mathcal{E}$ when transmissions can be reliably delivered from node i directly to node j in the absence of interference. We assume that nodes have equal transmission range, hence $e_{i,j} \in \mathcal{E} \Leftrightarrow e_{j,i} \in \mathcal{E}$. We denote the number of nodes as $N = |\mathcal{V}|$ and the set of one-hop neighbors of node i

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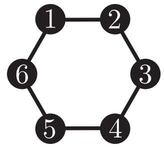

Fig. 1. Example $N = 6$ ring network.

as $\mathcal{N}_1(i)$, i.e., $j \in \mathcal{N}_1(i) \Leftrightarrow e_{i,j} \in \mathcal{E}$. Finally, we assume that the network is connected, i.e., there exists a path between any two distinct vertices $i, j \in \mathcal{V}$.

We consider a setting where each node $i \in \mathcal{V}$ is associated with a local process $H_i(t)$. No assumptions are made about these processes other than they are time-varying and each is of timely interest to all nodes in the network. In addition to its local process, each node $i \in \mathcal{V}$ has a table of "statuses" of all of the non-local processes in the network. We denote a status as the tuple $(H_j^{(i)}(t), \tau_j^{(i)}(t))$, where $H_j^{(i)}(t)$ and $\tau_j^{(i)}(t)$ denote the most recent sample value and the corresponding timestamp of $H_j(t)$ known to node i at time t, respectively.

Since the processes $\{H_j(t)\}$ are time-varying and of timely interest to all nodes in the network, each node $i \in \mathcal{V}$ seeks to maintain a table of "fresh" statuses with recent timestamps. For simplicity, we assume each node can sample its own local process without delay. The remaining statuses must be updated via broadcast transmissions containing status updates from other nodes in the system. We assume:

1) Transmissions are time slotted, require one unit of time to complete, and are received at times $t = 1, 2, 3, \ldots$.
2) Each transmission contains one status, i.e., one sample and its corresponding timestamp, of a process.
3) A transmitting node may transmit the status of its own local process or its status of another node's process.
4) Transmissions from node i are received reliably by all nodes in the one-hop neighborhood of node i, denoted by $\mathcal{N}_1(i)$, while nodes outside the one-hop neighborhood receive nothing from node i.
5) At least one node can reliably transmit a status update to its neighbors in each time slot. Depending on the interference model (as discussed in Section III), more than one node may reliably transmit updates to its neighbors in each time slot subject to interference constraints.

As each node in the network is both a source and a monitor of information, there are inherent tradeoffs in how fresh the status at each node can be in this setting. For example, in the ring network in Fig. 1, node 1 can keep nodes 2 and 3 updated with fresh status updates of $H_1(t)$ by transmitting new samples of its local process in every time slot. While statuses $H_1^{(2)}(t)$ and $H_1^{(3)}(t)$ remain fresh, all of the other statuses in the network become stale since they are not refreshed.

B. Age Evolution and Schedules

The age of a status update of process $H_j(t)$ at node i is defined below.

Definition 1 (Age). *Given the status of process j at node i denoted as $(H_j^{(i)}(t), \tau_j^{(i)}(t))$, the* ***age*** *of this status at time $t \geq \tau_j^{(i)}(t)$ is defined as $\Delta_j^{(i)}(t) \triangleq t - \tau_j^{(i)}(t)$.*

Note that the age $\Delta_j^{(i)}(t)$ is non-negative and is not defined for $t < \tau_j^{(i)}(t)$ or if no status update for process $H_j(t)$ has been received at node i. We denote by $\bar{t}$ a time such that all ages $\Delta_j^{(i)}(t)$ are defined for $t \geq \bar{t}$. Given Definition 1 and the assumed time slotted nature of the status updates in the system, we can describe the dynamics of each age in the system with a simple discrete time model similar to [9], [11]. Specifically, given a status update from node i regarding process j, the age at each node $m \in \mathcal{V}$ with $m \neq j$ is updated at integer times $t = n$ according to

$$\Delta_j^{(m)}[n+1] = \begin{cases} 1 & m \in \mathcal{N}_1(i) \text{ and } i = j \\ \Delta_j^{(i)}[n] + 1 & m \in \mathcal{N}_1(i),\ i \neq j, \\ & \text{ and } \Delta_j^{(i)}[n] < \Delta_j^{(m)}[n] \\ \Delta_j^{(m)}[n] + 1 & \text{otherwise.} \end{cases} \quad (1)$$

In order for node $m \neq j$ to update its status of process j and reduce the corresponding age $\Delta_j^{(m)}(t)$, it must (i) receive the status update transmission, i.e., be within the one-hop neighborhood of a transmitting node, and (ii) the status update must be fresher than the current status at node m. Otherwise, the age simply increases by one. The first case in (1) corresponds to the case when node i transmits a status update of its local process $H_i(t)$. In this case, since transmissions require unit time to complete, the local age at the start of the transmission is $\Delta_i^{(i)}[n] = 0$ and the age when nodes $m \in \mathcal{N}_1(i)$ receive the status update is $\Delta_i^{(m)}[n+1] = 1$. The second case in (1) corresponds to the case when node i transmits a status update of a non-local process $H_j(t)$ with $j \neq i$. In this case, nodes receiving the transmission update their statuses to match that at node i if the status from node i is fresher. When no update is received or the update is staler than the current status at node m, i.e., the third case in (1), the age simply increases by one.

We define a *schedule* as a sequence of transmissions indexed by integer time n with one or more pairs (i, j) with $i \in \{1, \ldots, N\}$ corresponding to the transmitting node index and $j \in \{1, \ldots, N\}$ corresponding to the index of the process for which node i is transmitting a status update. For the trivial example discussed previously where node 1 repeatedly sends updates of its own process $H_1(t)$ to its neighbors, the schedule can be simply written as $n : \{(1, 1)\}$ for all $n = 1, 2, \ldots$. If the interference model allows for nodes 1 and 4 to transmit simultaneously (note that nodes 1 and 4 share no neighbors in Fig. 1), and both nodes just transmit updates of their own processes $H_1(t)$ and $H_4(t)$, respectively, then the schedule can be written as $n : \{(1, 1), (4, 4)\}$. To facilitate the development of non-trivial schedules, the following section formalizes the notion of interference models and feasible activation sets.

III. Interference Model Assumptions

The AoI in networks where only one node can transmit per time slot was analyzed in [26], [27]. This paper generalizes this prior work by allowing for multiple nodes to transmit in each time slot, subject to interference constraints. Similar to [25], for any directed graph $(\mathcal{V}, \mathcal{E})$, we call $f \subset \mathcal{E}$ a "feasible activation set" if all links in f can be activated simultaneously without interference. An edge $e_{i,j}$ is said to be "active" during a time slot if node i is transmitting and $j \in \mathcal{N}_1(i)$.

In this section, we present specific interference models spanning from the most pessimistic model (*global interference* [26], [27]) to the most optimistic model (*interference free* [22], [23]). Between these extremes, we also describe a *topologically-dependent interference* model in which multiple nodes transmit simultaneously if they share no one-hop neighbors. Each of these settings will be analyzed in the sequel.

A. Global Interference Model

The global interference model was considered in [26], [27]. This pessimistic interference model imposes the constraint that only one node can transmit during each time slot. In this setting, there are a total of N feasible activation sets, given by $\mathcal{F}_{\text{glob}} = \{f_1, \ldots, f_N\}$, where $f_i = \{\text{all } e_{\ell,j} \in \mathcal{E} \text{ s.t. } \ell = i\}$ is the set of directed edges exiting node i.

B. Interference Free Model

In contrast to the pessimistic global interference model, this model considers a setting in which all nodes can transmit simultaneously without interference. This optimistic "interference free" model has been considered previously in the context of AoI in [22], [23]. In this setting, each of the N nodes in the network can transmit in each time slot. The collection of all feasible activation sets $\mathcal{F}_{\text{ifree}}$ in this setting can be expressed as $\mathcal{F}_{\text{ifree}} = \bigcup_{k=1}^{N} \mathcal{F}_k$ where $\mathcal{F}_k$ is the collection of all sets of edges with k transmitting nodes. For example, $\mathcal{F}_1 = \mathcal{F}_{\text{glob}}$ is the collection of all sets of edges with one transmitting node. Similarly, the collection of all sets of edges with two transmitting nodes $\mathcal{F}_2 = \{f_{1,2}, f_{1,3}, \ldots, f_{N-1,N}\}$, with $f_{i,j} = f_i \cup f_j$ and f_i as defined previously is the union of the sets of directed edges exiting nodes i and j. Note that $\mathcal{F}_N = \mathcal{E} \subset \mathcal{F}_{\text{ifree}}$, i.e., the collection of all feasible activation sets in this setting includes the set of all directed edges.

C. Topologically-Dependent Interference Model

In some networks, the global interference model may be overly pessimistic since multiple nodes may be able to transmit simultaneously due to frequency reuse, e.g., with sufficient spacing between some nodes, the use of multiple-access strategies, or other interference avoidance approaches. Similarly, the interference free model may be overly optimistic in some settings because nodes may not be able to separate simultaneously received updates or operate in full-duplex. As such, we consider a topologically-dependent interference model that falls between the pessimistic and optimistic models above. Specfically, if $\mathcal{N}_1(i) \cap \mathcal{N}_1(j) = \emptyset$, then i and j can transmit simultaneously without interference in this model.

Note that the collection of feasible activation sets in the topologically-dependent interference setting includes all transmissions from a single node, i.e., $\mathcal{F}_{\text{glob}} \subseteq \mathcal{F}_{\text{tdi}}$. It also contains the collection of sets of edges of multiple transmitting nodes that share no common neighbors. As an example of the collection of feasible activation sets in topologically-dependent interference setting, consider the ring network in Fig. 1. Note that $\mathcal{F}_{\text{tdi}} = \mathcal{F}_{\text{glob}} \cup \{f_{1,4}, f_{2,5}, f_{3,6}\}$ where $f_{i,j} = f_i \cup f_j$ as defined previously. Hence, for the six-node ring network in Fig. 1, there are a total of nine feasible activation sets.

In general, note that $\mathcal{F}_{\text{glob}} \subseteq \mathcal{F}_{\text{tdi}} \subset \mathcal{F}_{\text{ifree}}$ and the sets of possible schedules in each case has the same ordering. The following section analyzes the age statistics in each of these interference settings.

IV. Age of Information Analysis

In this section, we present the main results consisting of lower bounds on the peak and average age (as defined below) under general interference constraints including the three interference models described in Section III. The basic approach is to use a property of the feasible activation sets to lower bound the number of time slots required to update all statuses throughout the network. This leads directly to a lower bound on the instantaneous peak age. Additional properties of the feasible activation sets are then used to derive a lower bound on the instantaneous average age.

Before proceeding, we first review some key graph theoretic principles needed to establish a lower bound on the number of time slots required to update all statuses. Recall that a set $\mathcal{S} \subset \mathcal{V}$ of vertices in a graph is called a *dominating set* if every vertex not in $\mathcal{S}$ is adjacent to a vertex in $\mathcal{S}$ [28]. A minimum connected dominating set (MCDS) $\mathcal{S} \subset \mathcal{V}$ is a dominating set satisfying (i) the subgraph induced by $\mathcal{S}$ is connected and (ii) $\mathcal{S}$ has the smallest cardinality among all connected dominating sets of $\mathcal{G}$. Recall that for any $\mathcal{W} \subset \mathcal{V}$, the induced subgraph $\mathcal{G}[\mathcal{W}]$ consists of $\mathcal{W}$ and all edges whose endpoints are in $\mathcal{W}$. The cardinality of any MCDS is called *connected domination number* of $\mathcal{G}$ and is denoted by γ_c. In general graphs do not have a unique MCDS, yet all MCDSs of a graph have the same cardinality [29], [30]. Below, we introduce the notion of a *pseudo-leaf* vertex.

Definition 2 (Pseudo-leaf vertex)**.** *A vertex $i \in \mathcal{V}$ is a* ***pseudo-leaf vertex*** *if it is not a member of any MCDS.*

We refer to the set of all pseudo-leaf vertices of $\mathcal{G}$ as $\mathcal{L}$. Under this definition, every true leaf (i.e., every vertex with degree one) is also a pseudo-leaf.

The following lemma establishes a lower bound on the number of time slots required to update all of the statuses in the network.

Lemma 1 (Number of time slots required to update all statuses)**.** *For any schedule, updating all of the statuses throughout the network requires at least*

$$T^* \triangleq \left\lceil \frac{N\gamma_c + |\mathcal{L}|}{\nu} \right\rceil \tag{2}$$

time slots, where ν is the maximum number of simultaneously transmitting nodes over all feasible activation sets.

Proof sketch: A proof for the global interference model is provided in [26], [27] where $\nu = 1$ since all feasible activation sets correspond to a single transmitting node. For general interference models with ν corresponding to the maximum number of simultaneously transmitting nodes over all feasible activation sets, the desired result follows from considering ν simultaneous transmissions in each time slot. □

Note that the bound in Lemma 1 is tight for all network topologies when $\nu = 1$ but can be loose in some cases when $\nu > 1$. This is a consequence of the fact that the bound is simply based on assuming all time slots use the maximum number of parallel transmissions and ignoring the constraint that each node can only transmit one status update per time slot. While the bound in Lemma 1 is general, it is possible to develop tighter lower bounds for specific interference models and/or topologies by considering these additional constraints.

For the interference models in Section III, we have

$$\nu_{\text{glob}} = 1, \qquad \nu_{\text{ifree}} = N, \qquad \nu_{\text{tdi}} = \chi, \tag{3}$$

where χ is the maximum number of vertices with the same color over all distance-2 colorings of $\mathcal{G}$.

The remaining analysis in this section develops bounds on the instantaneous peak and average ages in the network. To facilitate this analysis, we first define the instantaneous peak and average age by extending the scalar age update model in (1) to a vector age update model given by

$$\boldsymbol{\Delta}[n+1] = \boldsymbol{A}\left[n, \boldsymbol{\Delta}[n]\right] \boldsymbol{\Delta}[n] + \mathbf{1}, \tag{4}$$

where $\boldsymbol{\Delta}[n] \in \mathbb{Z}^{N^2-N}$, $\boldsymbol{A}[n, \boldsymbol{\Delta}[n]] \in \mathbb{Z}^{(N^2-N)\times(N^2-N)}$, and $\mathbf{1} \in \mathbb{Z}^{N^2-N}$ is a vector of ones. Note that the local ages $\Delta_i^{(i)}(t)$ are not included in $\boldsymbol{\Delta}[n]$ since they are always zero. From (1), it is clear that $\boldsymbol{A}[n, \boldsymbol{\Delta}[n]]$ is a matrix, dependent on both the status update and the current ages as time n, with elements equal to zero or one. It is also evident that the rows of $\boldsymbol{A}[n, \boldsymbol{\Delta}[n]]$ each have at most one element equal to one. Note that, for $t \in [n, n+1)$, since all (non-local) ages increase linearly with time, we can write $\boldsymbol{\Delta}(t) = \boldsymbol{\Delta}[n] + (t - n)$.

Given Definition 1 and $\bar{t}$, we now define the instantaneous peak age at any point in time $t \geq \bar{t}$.

Definition 3 (Instantaneous peak age)**.** *For any $t \geq \bar{t}$, the **instantaneous peak age** is defined as*

$$\Delta_{\text{peak}}(t) \triangleq \max \boldsymbol{\Delta}(t) \tag{5}$$

Note that t is fixed here and the maximum is computed over the $N^2 - N$ elements of the vector $\boldsymbol{\Delta}(t)$. Similarly, we define the instantaneous average age at any point $t \geq \bar{t}$ below.

Definition 4 (Instantaneous average age)**.** *For any $t \geq \bar{t}$, the **instantaneous average age** is defined as*

$$\Delta_{\text{avg}}(t) \triangleq (N^2 - N)^{-1} \mathbf{1}^\top \boldsymbol{\Delta}(t) \tag{6}$$

Note that the instantaneous average age represents the average of the $N^2 - N$ ages of the non-local statuses, i.e., the zero-age local statuses are not included in the average.

Given (4) and $\boldsymbol{\Delta}[n_0]$, the age vector at time $n \geq n_0$ can be written as

$$\boldsymbol{\Delta}[n] = \boldsymbol{\Phi}[n, n_0]\boldsymbol{\Delta}[n_0] + \sum_{k=n_0}^{n-1} \boldsymbol{\Phi}[n, k+1]\mathbf{1}, \tag{7}$$

where $\boldsymbol{\Phi}[n, m]$ is the usual discrete-time state transition matrix based on $\boldsymbol{A}[n-1, \boldsymbol{\Delta}[n-1]], \ldots, \boldsymbol{A}[m, \boldsymbol{\Delta}[m]]$. Note that the dynamics in (4) and (7) are not linear due to the dependence of $\boldsymbol{A}$ on $\boldsymbol{\Delta}$. Nevertheless, (7) can be used to derive lower bounds on the age statistics as shown below.

Theorem 1 (Lower bound on instantaneous peak age)**.** *The instantaneous peak age of information for any schedule at time $t \geq \bar{t}$ is lower bounded by*

$$\Delta^*_{\text{peak,inst}} \triangleq T^*. \tag{8}$$

Proof sketch: For $t \geq \bar{t}$ and $t \in [n, n+1)$, we can write

$$\Delta_{\text{peak}}(t) = \max \boldsymbol{\Delta}[n] + (t - n) \geq \max \boldsymbol{\Delta}[n] \geq \boldsymbol{e}_i^\top \boldsymbol{\Delta}[n]$$

for all $i \in \{1, \ldots, N^2 - N\}$. From (7), we can set $n_0 = 0$ and $n \geq \bar{t} \geq T^*$ to write

$$\boldsymbol{\Delta}[n] \geq \sum_{k=n-T^*}^{n-1} \boldsymbol{\Phi}[n, k+1]\mathbf{1}, \tag{9}$$

where the inequality follows from the fact that each term in the sum is non-negative. Observe that there are T^* terms in the sum and that that all $\boldsymbol{\Phi}[n, k+1]$ in the sum are non-zero. Hence, there must exist at least one i such that $\boldsymbol{e}_i^\top \boldsymbol{\Phi}[n, n - T^* + 1]\mathbf{1} = 1$. Moreover, $\boldsymbol{e}_i^\top \boldsymbol{\Phi}[n, n - T^* + 1]\mathbf{1} = 1$ implies $\boldsymbol{e}_i^\top \boldsymbol{\Phi}[n, k+1]\mathbf{1} = 1$ for all $k \in \{n - T^*, \ldots, n-1\}$. Hence, given i such that $\boldsymbol{e}_i^\top \boldsymbol{\Phi}[n, n - T^* + 1]\mathbf{1} = 1$, we can write

$$\Delta_{\text{peak}}(t) \geq \boldsymbol{e}_i^\top \sum_{k=n-T^*}^{n-1} \boldsymbol{\Phi}[n, k+1]\mathbf{1} = T^*,$$

which shows the desired result. □

Theorem 2 (Lower bound on instantaneous average age)**.** *The instantaneous average age of information for any schedule is lower bounded by*

$$\Delta^*_{\text{avg,inst}} \triangleq (N^2 - N)^{-1}\mathbf{1}^\top \boldsymbol{s}, \tag{10}$$

where

$$\boldsymbol{s} \triangleq [\max(\boldsymbol{x}_1, \boldsymbol{y}_1), \max(\boldsymbol{x}_2, \boldsymbol{y}_2), \ldots, \max(\boldsymbol{x}_{T^*}, \boldsymbol{y}_{T^*})], \tag{11a}$$
$$\boldsymbol{x} \triangleq [N^2 - N, N^2 - N - \epsilon, \ldots, N^2 - N - (T^* - 1)\epsilon], \tag{11b}$$
$$\boldsymbol{y} \triangleq [T^*, T^* - 1, \ldots, 1] \tag{11c}$$

with ϵ corresponding to the maximum number of active edges over all feasible activation sets.

Proof sketch: Along the same lines as Theorem 1, for $t \geq \bar{t}$ and $t \in [n, n+1)$, we can write

$$\Delta_{\text{avg}}(t) \geq (N^2 - N)^{-1}\mathbf{1}^\top \sum_{k=n-T^*}^{n-1} \boldsymbol{\Phi}[n, k+1]\mathbf{1}.$$

Let $s[n,k+1] = \mathbf{1}^\top \mathbf{\Phi}[n,k+1]\mathbf{1}$ and observe that $s[n,k+1]$ corresponds to the number of non-zero elements, i.e., the number of statuses not updated, in $\mathbf{\Phi}[n,k+1]$. Observe that

(P1) $s[n,n] = N^2 - N$ from the fact that $\mathbf{\Phi}[n,n] = \boldsymbol{I}_{N^2-N}$.

(P2) $s[n,k+1] - s[n,k] \le \epsilon$ since at most ϵ statuses can be updated in a time slot.

(P3) $s[n, n-T^*+K] \ge K$ for all $K \in \{1,\ldots,T^*\}$.

The minimal sequence $\boldsymbol{s}$ satisfying these properties is shown in (11a), (11b), and (11c) above. □

For the interference models in Section III, we have

$$\epsilon_{\text{glob}} = \delta_{\max}, \qquad \epsilon_{\text{ifree}} = |\mathcal{E}|, \qquad \epsilon_{\text{tdi}} = \omega, \tag{12}$$

where $\delta_{\max}$ is the maximum degree of the graph and ω is the maximum number of active edges over all feasible activation sets in the topologically-dependent interference model.

V. Numerical Results

This section presents numerical examples for the specific case of ring networks (i.e., cycle graphs) that serve to illustrate the bounds on peak and average age in Section IV. Due to space constraints and the combinatorics of schedule design in the topologically-deterministic interference setting for general graphs, we do not present general schedule constructions algorithms here. Instead, we restrict our attention to status update schedules in the specific case of ring networks to illustrate the main points. For an N-node ring network, note that $|\mathcal{E}|=2N$, $\gamma_c=N-2$, $\mathcal{L}=\emptyset$, $\chi=\left\lfloor\frac{N}{3}\right\rfloor$, $\delta_{\max}=2$, and $\epsilon=2\nu$.

We first present a schedule for the global interference model below. The modulus operator

$$\sigma_{p,q}(\{(i_1,j_1),\ldots,(i_r,j_r)\}) \triangleq$$
$$\{(i_1{+}p \mod N, j_1{+}q \mod N),\ldots,(i_r{+}p \mod N, j_r{+}q \mod N)\},$$

is used to simplify the notation. Also, for any x if $x+p \mod N = 0$ ($x+q \mod N = 0$), set $x+p \mod N = N$ ($x+q \mod N = N$). Schedule A below is a special case of the general minimum length periodic schedules developed for the global interference model in [26], [27].

Schedule A: ring with global interference model.
Let $1:\{(1,1)\}$ be the schedule during time slot 1. For the next time slots $n=2,3,\ldots,N(N-2)$, the schedule is obtained by $n:\sigma_{p,q}(\{(1,1)\})$, where $p = n-1-\left\lfloor\frac{n-1}{N-2}\right\rfloor(N-3)$ and $q = \left\lfloor\frac{n-1}{N-2}\right\rfloor$.

In schedule A, node 1's status update is first disseminated clockwise around the ring (requiring $N-2$ transmissions), then node 2's status update is disseminated around the ring, and so on, until node N's status update has been disseminated around the ring at which point the process repeats with node 1. It is straightforward to confirm that Schedule A is a minimum length periodic schedule with period $T = N(N-2) = T^*_{\text{glob}}$. For the $N=6$ ring network in Fig. 1, Schedule A generates

$$\begin{array}{llllll}
1{:}\{(1,1)\}, & 2{:}\{(2,1)\}, & 3{:}\{(3,1)\}, & 4{:}\{(4,1)\}, & 5{:}\{(2,2)\}, & 6{:}\{(3,2)\}, \\
7{:}\{(4,2)\}, & 8{:}\{(5,2)\}, & 9{:}\{(3,3)\}, & 10{:}\{(4,3)\}, & 11{:}\{(5,3)\}, & 12{:}\{(6,3)\}, \\
13{:}\{(4,4)\}, & 14{:}\{(5,4)\}, & 15{:}\{(6,4)\}, & 16{:}\{(1,4)\}, & 17{:}\{(5,5)\}, & 18{:}\{(6,5)\}, \\
19{:}\{(1,5)\}, & 20{:}\{(2,5)\}, & 21{:}\{(6,6)\}, & 22{:}\{(1,6)\}, & 23{:}\{(2,6)\}, & 24{:}\{(3,6)\}.
\end{array}$$

Schedule B: ring with interference free model.
Let $1{:}\{(1,1),(2,2),\ldots,(N,N)\}$ be the schedule during time slot 1. For the next time slots $n{=}2,3,\ldots,N{-}2$, the schedule is obtained by $n{:}\sigma_{p,q}(\{(1,1),(2,2),\ldots,(N,N)\})$, where $p=0$ and $q=1-n$.

In schedule B, all N nodes begin by transmitting their local status updates in parallel. In the next $N{-}3$ time slots, each node continually relays the status update sent by their counterclockwise neighbor, after which point the process repeats again with each node transmitting a fresh update of its local process. It is straightforward to confirm that Schedule B is a minimum length periodic schedule with period $T{=}N-2{=}T^*_{\text{ifree}}$.

For the $N=6$ ring network in Fig. 1, Schedule B generates

$$\begin{array}{l}
1:\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}, \\
2:\{(1,6),(2,1),(3,2),(4,3),(5,4),(6,5)\}, \\
3:\{(1,5),(2,6),(3,1),(4,2),(5,3),(6,4)\}, \\
4:\{(1,4),(2,5),(3,6),(4,1),(5,2),(6,3)\}.
\end{array}$$

Schedule C: ring with topologically-dependent interference model.
Let $1:\{(1,1),(4,4),\ldots,(3\chi-2,3\chi-2)\}$ be the schedule during time slot 1. For the next time slots $n = 2,3,\ldots,(3+\text{mod}(N,3))(N-2)$, the schedule is obtained by $n : \sigma_{p,q}(\{(1,1),(4,4),\ldots,(3\chi-2,3\chi-2)\})$, where $p{=}n{-}1{-}\left\lfloor\frac{n-1}{N-2}\right\rfloor(N{-}3)$ and $q{=}\left\lfloor\frac{n-1}{N-2}\right\rfloor$.

In schedule C, every third node around the ring transmits simultaneously to avoid interference; in other words, given a distance-2 coloring of the graph, all nodes of the same color transmit simultaneously. For example, for the $N{=}6$ ring network in Fig. 1 there are 3 colors. All nodes of a given color begin by transmitting their local status updates simultaneously, and over the next $N-3$ time slots these updates are disseminated in simultaneously, clockwise around the ring. Then, the next group of nodes of a given color take their turn, followed by the third and final group of same-colored nodes. After this, the process repeats by returning to the first color. It is straightforward to confirm that Schedule C is a minimum length periodic schedule with period $T = 3(N-2) = T^*_{\text{tdi}}$ for $N = 3k$ and $k \in \{1,2,3,\ldots\}$.

For the $N=6$ ring network in Fig. 1, Schedule C generates

$$\begin{array}{llll}
1{:}\{(1,1),(4,4)\}, & 2{:}\{(2,1),(5,4)\}, & 3{:}\{(3,1),(6,4)\}, & 4{:}\{(4,1),(1,4)\}, \\
5{:}\{(2,2),(5,5)\}, & 6{:}\{(3,2),(6,5)\}, & 7{:}\{(4,2),(1,5)\}, & 8{:}\{(5,2),(2,5)\}, \\
9{:}\{(3,3),(6,6)\}, & 10{:}\{(4,3),(1,6)\}, & 11{:}\{(5,3),(2,6)\}, & 12{:}\{(6,3),(3,6)\}.
\end{array}$$

Figure 2 compares the instantaneous peak and average age lower bounds presented in Theorems 1 and 2 with the minimum achieved instantaneous peak and average age of Schedules A, B, and C for ring networks with $N \in \{3,4,\ldots,15\}$. To compute the minimum achieved instantaneous peak and average ages, since the schedules are periodic, it is sufficient to consider only one period of the schedule after all ages are defined at all nodes in the network. The results confirm that the instantaneous peak and average age for the interference free

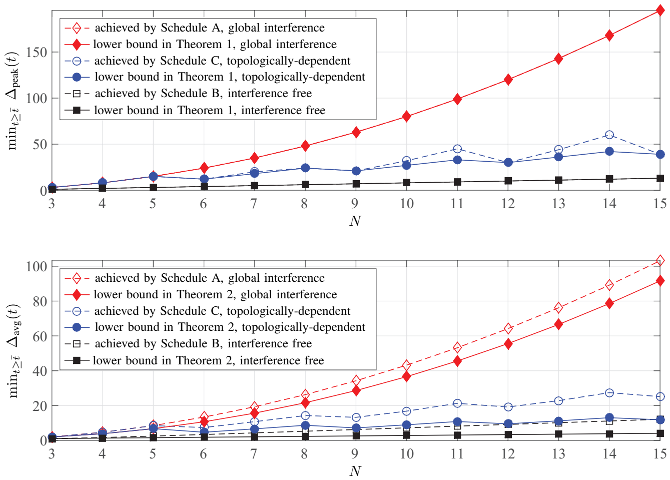

Fig. 2. Lower bounds and achievable peak and average age for ring networks with $N = \{3, 4, \ldots, 15\}$.

model and the topologically-dependent interference model are of order N, consistent with the bounds in Theorems 1 and 2.

VI. Conclusion

This paper studied the age of information problem in a general multi-source multi-hop wireless network with nodes communicating over time slotted transmissions. We presented fundamental lower bounds on the performance of any status update dissemination schedule in terms of the peak and average age metrics for general interference models. Explicit schedules and achievability results are also presented for ring networks with three interference models spanning from the most pessimistic setting (global interference) to the most optimistic setting (interference free). In contrast to the prior results for ring networks in the global interference model [27] showing ages scaling with N^2, the peak and averages ages of ring networks under topologically-dependent interference or interference free models scale with N. Future directions of this work include further generalizations of the interference models to allow for collisions at some nodes during simultaneous updates and the development of schedules for the general interference models for any connected network topology.

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