On the Convergence of Linear Parallel Interference Cancellation

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Abstract — **We consider the convergence of the multistage linear parallel interference cancellation (LPIC) multiuser detector in a synchronous CDMA system. Specifically we study and characterize the commonly observed ping-pong effect, where the probability of error oscillates and can even be greater than one-half.**

I. Introduction and Motivation

We assume the standard discrete symbol-synchronous CDMA system model [1] with K users using binary (± 1) spreading sequences of length N and binary signaling over an additive white Gaussian noise (AWGN) channel with variance σ^2. Let $\boldsymbol{R}$ be the $K \times K$ normalized spreading sequence cross-correlation matrix and $\rho(\boldsymbol{R})$ be its spectral radius, i.e., the maximum magnitude of the eigenvalues. If the spreading sequences are chosen randomly, we resort to large system techniques [2, 3] to get analytical results. By "large system", we mean that $K, N \to \infty$ but $K/N \to \beta$, for some constant $\beta \in (0, 1]$.

Interference cancellation is a natural multiuser detection scheme for CDMA based on the intuition that if one can accurately estimate the interference component of the received signal (which consists of the sum of the users' modulated signals plus channel noise), then one can get better estimates of the desired user's transmitted data by first subtracting the interference. In parallel interference cancellation (PIC), the interference component for each user is estimated simultaneously and subtracted from the observed signal to yield new decision signals, each of which presumably has lower multiple access interference. PIC also lends itself to a multistage structure where multiple concatenated PIC stages are employed to generate a set of final decision statistics.

It is the choice of interference estimator which characterizes one PIC detector from another. Using the soft output at each stage to generate the interference estimates leads to the linear parallel interference cancellation (LPIC) detector. While the LPIC detector is attractive due to its low computational complexity and relatively low decision delay, it does have limitations and can actually degrade system performance [6, 7, and the references therein].

II. Results and Conclusions

In this paper, we attempt to understand the behavior of the LPIC detector, specifically the commonly observed phenomenon where the probability of bit error for a certain user oscillates. This oscillation as a function of the number of cancellation stages is the so-called ping-pong effect and was studied in [4]. It is well known that linear interference cancellation receivers can be considered as iterative implementations of matrix-inverse based receivers [5]. It was shown in [4] that the decision statistic for the multistage LPIC undergoes ping-pong behavior which depends on the extreme eigenvalues of the iteration matrix and this leads to similar behavior in the probability of bit error. The convergence behavior of various iterative implementations of multiuser detectors in a large system scenario was studied in [5] and [5, Lemma 2] shows that the multistage LPIC converges to the decorrelator if and only if $K/N < 0.17$.

We do not consider the iterative implementation approach and instead deal directly with the uncoded probability of bit error. Let $P_{\mathsf{LPIC}}^{(k)}(M)$ be the probability of bit error for the k^{th} user after M stages of interference cancellation. We prove that [6]:

1. Given an $\boldsymbol{R}$ such that $\rho(\boldsymbol{R}) > 2$, there exists at least one k such that $P_{\mathsf{LPIC}}^{(k)}(M) > 0.5$ as M tends to infinity for odd M. We say that the k^{th} user *suffers*.

2. For the unfortunate user in item 1, $P_{\mathsf{LPIC}}^{(k)}(M)$ oscillates as a function of M with the odd and even subsequences respectively converging to a pair of fixed points symmetric about one-half.

3. In a large system scenario with randomly chosen spreading sequences, if $\beta = K/N > (\sqrt{2} - 1)^2 \approx 0.17$, at least one user, k, will suffer (with probability one) in each bit interval for large odd M. If the spreading sequences remain fixed from bit-to-bit, then $P_{\mathsf{LPIC}}^{(k)}(M)$ will exhibit ping-pong behavior. If the spreading sequences change every bit interval, we cannot prove that any one user will suffer on average. Numerical simulations, however, suggest that if $K/N > 0.17$, then *all* users do suffer on average and undergo ping-pong behavior.

We hope that the results in this paper increase our understanding of PIC receivers. Future work should certainly be aimed at characterizing the performance of the LPIC detector for all users under increasing system load.

References

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