Opportunistic Collaborative Beamforming with One-Bit Feedback

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***Abstract*—An energy-efficient opportunistic collaborative beamformer with one-bit feedback is proposed for ad hoc sensor networks transmitting a common message over independent Rayleigh fading channels to a relatively distant destination node. In contrast to conventional collaborative beamforming schemes in which each relay node uses channel state information (CSI) to pre-compensate for its channel phase and local carrier offset, the relay nodes in the proposed beamforming scheme do not perform any phase precompensation. Instead, the destination node broadcasts a relay node selection vector to the pool of available relay nodes to opportunistically select a subset of relay nodes whose transmitted signals combine in a quasi-coherent manner at the destination. Since the selection vector only indicates which relay nodes are to participate in the collaborative beamformer and does not convey any CSI, only one bit of feedback is required per relay node. Theoretical analysis shows that the received signal power obtained with the proposed opportunistic collaborative beamforming scheme scales *linearly* with the number of available relay nodes under a fixed total power constraint. Since computation of the optimal selection vector is exponentially complex in the number of available relays, three low-complexity sub-optimal relay node selection rules are also proposed. Simulation results confirm the effectiveness of opportunistic collaborative beamforming with the low-complexity relay node selection rules.**

***Index Terms*—Collaborative beamforming, reduced feedback, ad hoc sensor networks, noisy channel estimation.**

I. Introduction

In wireless ad hoc sensor networks with battery powered nodes, efficient use of the limited energy resources in each node is necessary in order to extend the usable lifetime of the network. In many applications, e.g. environmental monitoring, reachback communication [1] from nodes in the sensor network to a relatively distant destination node can be a significant source of power consumption in the network. Direct reachback communication from a source node in the sensor network to a relatively distant destination is usually energy-inefficient. To circumvent this obstacle, a more energy-efficient approach for reachback communication has been developed by exploiting the multiuser diversity gain inherent in the sensor network [2]–[9]: the source node first transmits the message to one or more other nodes in the sensor network before these nodes relay the message to the destination. In particular, a low-overhead example of this approach was described in [2], [3] where the best available relay node, based on end-to-end channel conditions, is selected in a distributed fashion for the reachback communication link. The appeal of the opportunistic relaying schemes proposed in [2], [3] is their simplicity: increased energy efficiency and collision avoidance is achieved in a distributed manner without requiring global channel state information (CSI) and feedback from the destination to select the best relay. Each relay in [2], [3] obtains only local CSI by observing a ready-to-send (RTS) clear-to-send (CTS) handshake between the source and destination. The simplicity of this approach, however, comes at the cost of inefficient transmission in the relay-destination link since the bandpass signal forwarded by a typical low-cost single-antenna relay is undirected. Consequently, only a fraction of the transmit energy from the single-antenna relay is useful for reachback communication while the rest is not fully utilized. In contrast, directed transmission by employing multiple antennas, i.e. beamforming, is attractive. By steering the wireless signal toward the intended destination, multi-antenna directed transmission can increase the energy efficiency of reachback communication with less transmitted energy scattered in unintended directions. In addition to improved energy efficiency, directed transmission also potentially reduces interference on other networks and/or strengthens security. While size and cost constraints usually preclude the use of conventional antenna arrays with individual sensor nodes, directed transmission can be achieved in sensor networks with single-antenna nodes by having $K \geq 2$ relays transmit a common message simultaneously as a *collaborative beamformer* [4]–[9].

In a collaborative beamformer, the relay nodes emulate a conventional beamformer: each relay node typically pre-compensates for its channel phase and local carrier phase offset so that the bandpass transmissions combine constructively at the intended destination. It is well known that, for a fixed total transmission power, the power gain of an ideal collaborative beamformer scales linearly in K whereas the power gain of selecting the single best relay scales according to $\log(K)$ [10]. The tradeoff, however, is increased complexity and the need for some amount of feedback from the destination to the relay nodes.

A major source of complexity in collaborative beamforming

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systems results from the fact that relay nodes transmitting as a collaborative beamformer must obtain accurate CSI estimates to perform local phase precompensation. Without accurate phase precompensation, collaborative beamforming tends to perform poorly due to pointing errors and mainbeam degradation [4]. A master-slave approach to phase-precompensation for collaborative beamforming was described in [11] where the destination node (the master) continuously broadcasts a common beacon to the relay nodes (the slaves). Each relay node synthesizes their local clock from this beacon and, upon a trigger signal from the destination, transmits a direct sequence code division multiple access (DS-CDMA) signal with a unique code back to the destination node with a carrier synthesized from the original master beacon. The destination separates these signals, estimates and quantizes the overall phase offset of each round-trip link, and then transmits phase/timing precompensation messages to the relay nodes via DS-CDMA. This approach allows for low-complexity relay node hardware, but it may result in significant overhead, particularly for networks composed of a large number of relay nodes. This problem was discussed in [12], where a technique was proposed in which only a subset of the available relay nodes with the largest channel gains to the destination are selected for collaborative beamforming. As a result, the total amount of CSI feedback is reduced according to the fraction of selected relay nodes. Nevertheless, the processing burden on the master node is unchanged and the amount of coordination required to implement this type of systems may be prohibitive in some scenarios.

Several other approaches to phase precompensation for collaborative beamforming have also recently been proposed. A master-slave iterative phase precompensation technique was described in [13] in which the relay nodes randomly adjust their phases and undo the adjustment if the destination indicates, via one-bit feedback to the entire pool of relays, that the signal to noise ratio (SNR) did not improve. The appeal of this technique is that the rate of feedback can be considerably less than that of [11]. An open-loop phase precompensation technique was described in [5] where the relay nodes pre-synchronize their local oscillators and wait for a pilot broadcast by the destination. Upon reception of the pilot, each relay node estimates the phase of its respective channel and then uses this CSI estimate directly for local phase precompensation during collaborative beamforming since the time-division duplex (TDD) channel is assumed to be reciprocal. A distributed phase precompensation technique for collaborative beamforming called "round-trip synchronization" was also recently described in [14].

This paper proposes a new quasi-coherent collaborative beamforming technique that fills a gap between opportunistic relaying [2], [3] and fully-coherent collaborative beamforming. This approach, which we call "opportunistic collaborative beamforming", is inspired by the observation that bandpass signals with even moderate phase offsets can still combine to provide considerable beamforming gains. Opportunistic collaborative beamforming is a centralized technique similar to the master-slave approaches described in [11]–[13] in that the destination node is responsible for global CSI estimation and for providing feedback to the pool of available relay nodes. It is also similar to [13] in that no explicit CSI is fed back from the destination to the relay nodes. Instead of using an SNR objective function and an iterative procedure with one-bit feedback per iteration, however, the feedback from the destination to the pool of available relay nodes in the proposed opportunistic collaborative beamformer is a single K-bit "relay node selection vector" (one bit per available relay) indicating which relay nodes should transmit. The relays do not adjust their phases prior to or during transmission. To avoid destructive combining at the destination, only a subset of the K available relay nodes whose received signals will combine in a quasi-coherent manner at the destination are selected to participate in the collaborative beamformer. The relay node selection vector is computed by the destination with the goal of maximizing the power gain of the collaborative beamformer toward the destination.

It is worth mentioning that, despite the similarity in name, the proposed "opportunistic collaborative beamformer" in this paper differs from the notion of "opportunistic beamforming" proposed in [10]. Specifically, the central problem in opportunistic collaborative beamforming is the selection of a subset of relay nodes whose received signals combine in a quasi-coherent manner at a *given destination*. In contrast, the opportunistic beamforming considered in [10] investigates scheduling data transmission from a *given source*, i.e. a base station, to the optimum destination among multiple candidates. The multiuser diversity in opportunistic *collaborative* beamforming stems from the presence of multiple transmitters whereas the opportunistic beamformer in [10] exploits multiuser diversity inherent in multiple receivers.

A novel feature of the opportunistic collaborative beamforming scheme described in this paper is that beamforming is achieved without any phase precompensation by the relays. Also, unlike [13], the proposed opportunistic collaborative beamforming scheme does not require multiple feedback iterations. It is fair to say, however, that these advantages are obtained at the cost of increased overhead and complexity at the destination node. Rather than estimating a single parameter, e.g. SNR, the proposed opportunistic collaborative beamformer requires the estimation of $2K$ parameters (amplitude and phase for each relay) in order to compute the relay selection vector. Nevertheless, since the tasks of global CSI estimation and relay node selection are performed by the destination for coherent data detection, the proposed opportunistic collaborative beamformer incurs only marginal additional cost or complexity in the sensor network nodes.

The main contributions of this paper are an explicit description of the proposed opportunistic collaborative beamforming scheme and a theoretical analysis of the power gain attainable by this scheme. We show that the power gain scales at the same rate as ideal collaborative beamforming, i.e. linearly with K for large K, even with noisy CSI estimates at the destination. Since computation of the optimal selection vector is exponentially complex in the number of available relays, three low-complexity sub-optimal relay node selection rules are also proposed: sector-based, iterative greedy, and iterative pruning. Simulation results confirm the effectiveness of opportunistic collaborative beamforming with the optimal and low-complexity relay node selection rules.

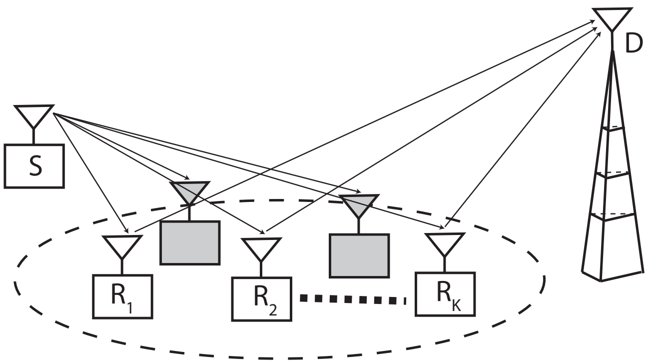

Fig. 1. A collaborative beamforming system with one source, one destination, and N available relay nodes of which $K \leq N$ nodes correctly decode the source transmission. The shaded relay nodes represent nodes that did not correctly decode the source transmission and do not participate in beamforming.

The rest of the paper is organized as follows. We first introduce the signal model in Section II. Then, a suboptimal sector-based opportunistic collaborative beamforming scheme is proposed in Section III, assuming that either perfect or imperfect CSI is available at the destination. To further improve the performance of the sector-based scheme, two low-complexity iterative selection rules are developed in Section IV. Finally, simulation results are shown in Section V while conclusions are given in Section VI.

Notation: Vectors and matrices are denoted by boldface letters. $\|\cdot\|$ represents the Euclidean norm of the enclosed vector and $|\cdot|$ denotes the amplitude of the enclosed complex-valued quantity. We use $\mathrm{E}\{\cdot\}$, $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ for expectation, complex conjugation, transposition and Hermitian transposition. Finally, for random variables x and y, var(x) and cov(x, y) represent the variance of x and the covariance of x and y, respectively.

II. Signal Model

We consider a network composed of one source, N single-antenna relay nodes, and one destination as illustrated in Figure 1.

To facilitate the development of opportunistic collaborative beamforming schemes, we condition on the event that $K \leq N$ relay nodes have correctly decoded the source transmission in an earlier interval. To simplify our analysis, we assume that any carrier frequency offset and/or symbol timing offset at each relay node is sufficiently small with respect to the symbol duration such that the resulting phase offsets are approximately constant over the transmission interval as in [15], [16]. The complex channel gain between the k-th relay node and the destination, denoted by h_k, is modeled as a $\mathcal{CN}(0,1)$ random variable with

$$h_k = a_k e^{j\phi_k}, \quad k = 1, 2, \cdots, K \tag{1}$$

where $a_k \geq 0$ and $\phi_k \in (-\pi, \pi]$ are independent and identically distributed (i.i.d.) Rayleigh-distributed channel amplitudes and i.i.d. uniformly-distributed channel phases, respectively. The amplitude a_k and phase ϕ_k are assumed to be statistically independent.

Denote by $\boldsymbol{s}$ the relay node selection vector of length K. The k-th entry of $\boldsymbol{s}$ is one, i.e. $s_k = 1$, if the k-th relay node is selected for transmission; otherwise $s_k = 0$. Thus, the normalized received signal can be written as

$$r = \frac{1}{\sqrt{\boldsymbol{s}^T\boldsymbol{s}}}\boldsymbol{h}^T\boldsymbol{s}d + n, \tag{2}$$

where d is the unit-power data symbol, $\boldsymbol{h} = [h_1, h_2, \cdots, h_K]^T$ and n is complex Gaussian noise modeled as a $\mathcal{CN}(0, \sigma_n^2)$ random variable. It should be emphasized that the total transmitted signal power is normalized to unity, regardless of the number of selected relay nodes. As a result, collaborative beamforming improves the energy efficiency of reachback communications only if the received signal power at the destination is increased with respect to single-relay transmission.

In the sequel, we first propose a suboptimal sector-based opportunistic collaborative beamforming scheme with either perfect or imperfect CSI at the destination in Section III. After that, we proceed to develop two opportunistic collaborative beamforming schemes employing iterative selection rules in Section IV.

III. Opportunistic Collaborative Beamforming: Performance Analysis

In the proposed opportunistic collaborative beamforming scheme, the destination feeds back one-bit selection information to turn on/off each relay node such that the transmitted signals from all selected nodes can combine in a quasi-coherent manner at the destination. Clearly, the most critical design consideration in the proposed scheme is the selection of participating relay nodes. To shed light on the optimal design of node selection criteria, we first consider the case in which two relay nodes are available for cooperative transmission with perfect CSI available at the destination to facilitate optimal node selection.

A. Two-relay network with perfect CSI

We assume without loss of generality that $a_1 \geq a_2$. Then we can say

$$P_{\{1\}} = a_1^2 \geq a_2^2 = P_{\{2\}}, \tag{3}$$

where $P_{\{i\}}$ denotes the received power at the destination when only node i transmits.

When both relay nodes transmit, the received power can be expressed as

$$\begin{aligned} P_{\{1,2\}} &= \frac{1}{2}\left|a_1 e^{j\phi_1} + a_2 e^{j\phi_2}\right|^2, & (4)\\ &= \frac{a_1^2}{2}\left|1 + \rho e^{j\Delta}\right|^2, & (5)\end{aligned}$$

where

$$\rho \stackrel{\text{def}}{=} a_2/a_1 \tag{6}$$

and

$$\Delta \stackrel{\text{def}}{=} \phi_2 - \phi_1. \tag{7}$$

Note that the factor "$\frac{1}{2}$" in (4) normalizes the total transmission power to unity.

Clearly, simultaneous transmission is a better option only if $P_{\{1,2\}} \geq P_{\{1\}}$, which corresponds to the equivalent condition

$$\cos(\Delta) \quad \geq \quad \frac{1-\rho^2}{2\rho}. \tag{8}$$

The following special cases of (8) are of interest.

- When $\rho = 1$, both relay nodes have identical channel amplitudes and the simultaneous transmission condition in (8) reduces to $|\Delta| \leq \frac{\pi}{2}$. The gain with respect to single-best-relay transmission can be expressed as

$$\Gamma = \frac{P_{\{1,2\}}}{P_{\{1\}}} = \frac{1}{2}\left|1+e^{j\Delta}\right|^2, \tag{9}$$

  which attains a maximum of two when $\Delta = 0$ and a minimum of one when $\Delta = \pm\frac{\pi}{2}$. Note that even relatively large phase offsets between the relay nodes can lead to significant gains with respect to single-best-relay transmission. For example, when $\Delta = \frac{\pi}{3}$, the resulting gain can be computed to be $\Gamma = 1.76$ dB.
- When $\Delta = 0$, the transmissions from both relay nodes arrive in perfect phase alignment at the destination. Interestingly, (8) implies that simultaneous transmission is preferred only if $\rho \geq \sqrt{2}-1 \approx 0.4142$. In other words, even though both nodes have perfect phase alignment, simultaneous transmission is better than single-best-relay transmission only if the ratio of the second node's channel amplitude to that of the first node is at least 0.4142.

While the relay selection schemes described in [2], [3] can be thought of as "single-best-relay" selection schemes, it is important to note that the notion of "single-best-relay" described here is not directly comparable to that in [2], [3]. Under our assumption that the common message has already been disseminated among the available relays, the choice of the single best relay in this paper is based only on the quality of the relay-destination link. The choice of single-best-relay in [2], [3] is based on the quality of the composite source-relay-destination link.

It should also be emphasized here that no additional information beyond the K bit relay node selection vector needs to be fed back from the destination to the relays in order to implement an opportunistic collaborative beamformer. Each relay node determines whether it should transmit or not by the presence of a one or a zero in the appropriate index of the relay node selection vector. Power control is also implicit in the Hamming weight of the relay selection vector. In the $K = 2$ scenario, relay node 1 transmits with unit power if $\boldsymbol{s} = [1, 0]^T$; both relay nodes transmit with half power if $\boldsymbol{s} = [1, 1]^T$. For general $K \geq 2$, each relay node can determine its transmit power directly from the inverse of the Hamming weight of the relay selection vector.

B. Large network with perfect CSI upper bound

We now consider the more general case with $K \geq 2$ available relay nodes. The received power of a K-node opportunistic collaborative beamformer with the optimal selection rule can be written as

$$P_{opt}^{(K)} = \max_{\boldsymbol{s}\in\{0,1\}^K} \frac{1}{\boldsymbol{s}^T\boldsymbol{s}}\left|\boldsymbol{h}^T\boldsymbol{s}\right|^2. \tag{10}$$

Optimal selection of nodes that participate in the beamformer entails an exhaustive search over all possible $2^K - 1$ possible selection vectors. Since it requires approximate $\mathcal{O}(2^{K+1})$ operations to evaluate (10), the computational complexity required to obtain the optimal selection is formidable, even for moderate K. To better understand the performance of the optimal opportunistic collaborative beamformer, this section develops lower and upper bounds on its performance for the large-network case, i.e. $K \to \infty$.

We begin with a sector-based selection rule by choosing all relay nodes whose channels belong to a pre-defined selection region. Exploiting the following inequality in (10),

$$\left|\boldsymbol{h}^T\boldsymbol{s}\right|^2 = \left|\boldsymbol{a}^T\boldsymbol{\Phi}\boldsymbol{s}\right|^2 \leq \left|\boldsymbol{a}^T\boldsymbol{s}\right|^2, \tag{11}$$

where

$$\boldsymbol{a} \quad = \quad [a_1, a_2, \cdots, a_K]^T, \tag{12}$$
$$\boldsymbol{\Phi} \quad = \quad \text{diag}\left\{e^{j\phi_1}, e^{j\phi_2}, \cdots, e^{j\phi_K}\right\}^T, \tag{13}$$

an upper bound for $P_{opt}^{(K)}$ can be derived by considering the case when all of the transmissions are received coherently at the zero phase, i.e. $h_k = a_k \geq 0$ for all $k \in \{1, \ldots, K\}$.

As discussed in Section III-A, even though signals from the relay nodes combine constructively at the destination, the optimal beamforming selection rule should not select all K nodes for simultaneous transmission. Instead, only nodes with sufficiently large amplitude should be selected such that the resulting *normalized* received power is maximized. Denoting the selection threshold as γ, we can write

$$s_k \quad = \quad \begin{cases} 1 & \text{if } a_k \geq \gamma \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Recall that the $\{a_k\}$ are i.i.d. Rayleigh distributed channel amplitudes with means $\text{E}[a_k] = \frac{\sqrt{\pi}}{2}$. By the law of large numbers, we can say that

$$\lim_{K\to\infty} \frac{\boldsymbol{s}^T\boldsymbol{s}}{K} = \Pr\left(a_k \geq \gamma\right) = e^{-\gamma^2}. \tag{15}$$

As shown in Appendix A, we can express the received power upper bound normalized by K as

$$\lim_{K\to\infty} \frac{P_{ub}^{(K)}(\gamma)}{K}$$
$$= \lim_{K\to\infty} \frac{K}{\boldsymbol{s}^T\boldsymbol{s}}\left[\int_\gamma^\infty 2x^2 e^{-x^2}\,dx\right]^2, \tag{16}$$
$$= \frac{\pi}{4} f(\gamma), \tag{17}$$

where

$$f(\gamma) \quad \stackrel{\text{def}}{=} \quad e^{\gamma^2}\left[\text{erfc}(\gamma) + \frac{2\gamma}{\sqrt{\pi}}e^{-\gamma^2}\right]^2, \tag{18}$$

with $\text{erfc}(x)$ being the complementary error function defined as $\text{erfc}\,(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$.

Note that received power upper bound grows linearly with K, as would be expected of an ideal coherent beamformer. Numerical maximization of $f(\gamma)$ can be performed to show

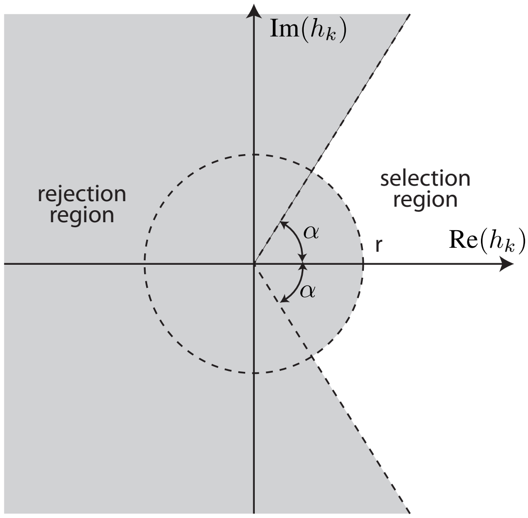

Fig. 2. Sector-based selection region for channel coefficients $\{h_k\}$ used to derive the received power lower bound (25) whereas the illustration is also applicable to (31) in which the selection is based on the estimated CSI, $\left\{\hat{h}_k\right\}$.

that $\max f(\gamma) \approx 1.0849$ and $\gamma^* = \arg\max f(\gamma) \approx 0.5316$. Hence, we can write

$$\lim_{K\to\infty} \frac{P_{opt}^{(K)}}{K} \le \lim_{K\to\infty} \frac{P_{ub}^{(K)}(\gamma^*)}{K} = 0.8521. \quad (19)$$

C. Large network with perfect CSI lower bound

To develop a lower bound on $P_{opt}^{(K)}$, we propose a sub-optimal selection rule using the sector-based selection region shown in Figure 2.

The selection region is characterized by two parameters: γ corresponding to a minimum amplitude and α corresponding to a maximum angle. Nodes must satisfy both the minimum amplitude and maximum angle requirements to be selected for transmission, i.e.,

$$s_k = \begin{cases} 1 & \text{if } a_k \ge \gamma \text{ and } |\phi_k| \le \alpha \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

Given i.i.d. channel coefficients $h_k = a_k e^{j\phi_k}$ with a_k being Rayleigh-distributed and ϕ_k uniformly distributed on $(-\pi, \pi]$, the probability that h_k falls in the selection region Ω_{lb} can be expressed as

$$\begin{aligned} \Pr(h_k \in \Omega_{lb}) &= \Pr\left(|\phi_i| \le \alpha\right) \Pr\left(a_i \ge \gamma\right), \quad (21) \\ &= \frac{\alpha}{\pi} \exp\left(-\gamma^2\right). \quad (22) \end{aligned}$$

Asymptotically in K, the lower bound can be expressed as shown in (24) following procedures similar to those employed in deriving (17):

$$\begin{aligned} &\lim_{K\to\infty} \frac{P_{lb}^{(K)}(\gamma,\alpha)}{K} \\ &= \lim_{K\to\infty} \frac{K}{\boldsymbol{s}^T\boldsymbol{s}} \left[\int_{-\alpha}^{\alpha}\int_{\gamma}^{\infty} \frac{\cos\theta}{\pi} x^2 e^{-x^2}\, dx\, d\theta\right]^2, \quad (23) \\ &= \frac{\sin^2\alpha}{4\alpha} f(\gamma), \quad (24) \end{aligned}$$

where $f(\gamma)$ is given in (18).

From (24), it is important to observe that the term $\frac{\sin^2\alpha}{4\alpha}$ is not a function of γ and attains its maximum when $\cos\alpha = \frac{\sin\alpha}{2\alpha}$. The optimum value $\alpha^* \approx 1.1656$ radians can be found numerically. Since $f(\gamma)$ achieves its maximum at $\gamma^* \approx 0.5316$, the received power lower bound can be written as

$$\lim_{K\to\infty} \frac{P_{lb}^{(K)}(\gamma^*,\alpha^*)}{K} = 0.1965 \le \lim_{K\to\infty} \frac{P_{opt}^{(K)}}{K} \quad (25)$$

asymptotically in K. In the sequel, the selection rule employing the optimal thresholds $\{\gamma^*, \alpha^*\}$ is referred to as the "sector-based selection rule".

Summarizing (19) and (25), the upper and lower bounds on the normalized received power of opportunistic collaborative beamforming with the optimum selection rule can be written as

$$0.1965 \le \lim_{K\to\infty} \frac{P_{opt}^{(K)}}{K} \le 0.8521. \quad (26)$$

Two implications of this result merit further discussion:

1) When K is large, the ratio of the upper and lower bounds implies that $P_{opt}^{(K)}$ is no worse than 6.37 dB below the power of an ideal coherent phase-aligned beamformer.
2) When K is large, even simple sub-optimal selection rules for opportunistic collaborative beamforming can result in a normalized received power that scales linearly with K. This is the same scaling rule by which the received power of the ideal beamformer is governed. Since both the upper and lower power bounds are linear in K, the normalized received power of the optimum opportunistic collaborative beamformer must also scale linearly with K. This represents a significant improvement over the single-best-relay selection rule whose received power scales as $\log(K)$ [10].

Finally, it is worth noting that the asymptotic result derived in (26) is, in general, only accurate for large K. Computer simulations are provided in Section V to confirm the performance of the proposed scheme for small K.

D. Large network with imperfect CSI lower bound

In the previous analysis, it has been assumed that the destination has access to perfect CSI, i.e. $\{h_k\}$, in order to facilitate node selection for the proposed collaborative beamformer. However, CSI must be estimated by some means in practice, which is inevitably susceptible to channel estimation errors. Clearly, the upper bound derived in the previous section is a valid upper bound for the case with imperfect CSI. In this section, we develop a lower bound on the normalized received power of the proposed collaborative beamformer in the presence of channel estimation errors and subsequently derive the optimal thresholds for the sector-based selection rule.

The noisy channel estimate of h_k, denoted by $\hat{h}_k$, can be modeled as

$$\hat{h}_k = h_k + \xi_k, \quad (27)$$

where ξ_k is the estimation error modeled as circularly symmetric complex Gaussian noise with zero-mean and variance σ_ξ^2.

Exploiting the fact that h_k and ξ_k in (27) are statistically independent and Gaussian-distributed, we can then show $\hat{h}_k \sim \mathcal{CN}\left(0, \sigma^2_{\hat{h}}\right)$ with

$$\sigma^2_{\hat{h}} = 1 + \sigma^2_{\xi}. \quad (28)$$

In practical systems, only the noisy channel estimates $\hat{\boldsymbol{h}} = \left[\hat{h}_1, \hat{h}_2, \cdots, \hat{h}_K\right]^T$, rather than $\boldsymbol{h}$, are available to the destination. Thus, for given noisy $\hat{\boldsymbol{h}}$, a robust selection rule selects relay nodes whose signals actually distorted by $\boldsymbol{h}$ can combine in a quasi-coherent manner at the destination.

We consider a suboptimal selection characterized by two parameters: γ' corresponding to a minimum amplitude and α' corresponding to a maximum angle. Note that the superscript $(\cdot)'$ is employed to differentiate the mathematical notations with imperfect CSI from their counterparts with perfect CSI discussed in the previous section. Then, only nodes whose estimated channel gains $\hat{h}_k = \hat{a}_k e^{j\hat{\phi}_k}$ satisfy both the minimum amplitude and maximum angle requirements are selected for transmission, i.e.,

$$s_k = \begin{cases} 1 & \text{if } \hat{a}_k \geq \gamma' \text{ and } |\hat{\phi}_k| \leq \alpha' \\ 0 & \text{otherwise.} \end{cases} \quad (29)$$

It is shown in Appendix B that the normalized received power lower bound is given by

$$\lim_{K\to\infty} \frac{P'^{(K)}_{lb}(\alpha', \gamma')}{K} = \frac{1}{\sigma^2_{\hat{h}}} \frac{\sin^2 \alpha'}{4\alpha'} f\left(\frac{\gamma'}{\sigma_{\hat{h}}}\right). \quad (30)$$

For the perfect CSI case where $\sigma^2_{\hat{h}} = 1$, (30) is equivalent to (24). Similar to (24), the optimal γ'^* and α'^* can be found separately. Interestingly, the optimal $\alpha'^* \approx 1.1656$ that maximizes $\frac{\sin^2 \alpha'}{4\alpha'}$ is identical to α^*, i.e. the optimal phase threshold for the perfect CSI case, regardless of the channel estimation error variance. Furthermore, exploiting the result of $\gamma^* = 0.5316$, we have $\gamma'^* = 0.5316 \times \sigma_{\hat{h}}$. Recall that $\sigma_{\hat{h}} \geq 1$, we have $\gamma'^* \geq \gamma^*$. Thus, if perfect CSI is not available to the destination, the optimal selection rule should increase the amplitude threshold according to the variance of the channel estimation errors. Finally, summarizing (17) and (30), we have

$$\frac{1}{\sigma^2_{\hat{h}}} \frac{\sin^2 \alpha'}{4\alpha'} f\left(\frac{\gamma'}{\sigma_{\hat{h}}}\right) \leq \lim_{K\to\infty} \frac{P'^{(K)}}{K} \leq \frac{\pi}{4} f(\gamma). \quad (31)$$

For any given $\sigma_{\hat{h}}$, the lower and upper bounds of the inequality above are constant. As a result, even in the presence of imperfect CSI, the received power of the proposed collaborative beamformer still scales linearly with K.

IV. Low-Complexity Iterative Selection Rules

Despite its simplicity and insightful analytical results, the sector-based selection rule does not fully exploit the CSI available to the destination. In this section, two iterative selection rules are proposed to select a sub-optimal subset of relay nodes for collaborative beamforming with affordable computational complexity. Clearly, the success of these selection rules hinges on effectively determining the number of selected relay nodes and identifying the suitable nodes. The proposed iterative selection rules successfully address these two issues by capitalizing on our previous analysis on the two-node case. In each iteration, the proposed selection rules either add one new or remove one existing relay node to/from the selection subset based on well-defined cost functions until no further beamforming gain can be achieved with further iterations.

A. Iterative greedy selection rule

We denote by $p^{(N)} \in \{1, 2, \cdots, K\}$ the node index chosen in the N-th iteration, $1 \leq N \leq K$. To facilitate our subsequent derivation, we first define the following two quantities:

$$z^{(N)} = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_{p^{(n)}} e^{j\phi_{p^{(n)}}}, \quad (32)$$

and

$$P^{(N)} = \left|z^{(N)}\right|^2, \quad (33)$$

where $z^{(N)}$ is the composite channel gain between the N selected relay nodes and the destination, while $P^{(N)}$ is the corresponding received signal power.

Now, we consider $P^{(N+1)}$ by adding one new relay node into the subset of selected relay nodes:

$$\begin{aligned} P^{(N+1)} &= \frac{1}{N+1} \left|\sum_{n=1}^{N+1} a_{p^{(n)}} e^{j\phi_{p^{(n)}}}\right|^2, \quad (34) \\ &= \frac{1}{N+1} \left|\sqrt{NP^{(N)}} + a_{p^{(N+1)}} e^{j\Delta_{N+1}}\right|^2, \quad (35) \end{aligned}$$

where Δ_{N+1} is the *relative* phase offset between the newly added channel gain and $z^{(N)}$.

Next, we can rewrite (35) as

$$\begin{aligned} P^{(N+1)} = \frac{1}{N+1} \Big[& NP^{(N)} + a^2_{p^{(N+1)}} + \\ & 2a_{p^{(N+1)}} \sqrt{NP^{(N)}} \cos(\Delta_{N+1}) \Big], \quad (36) \end{aligned}$$

Clearly, the condition $P^{(N+1)} > P^{(N)}$ has to hold in order to incorporate the $p^{(N+1)}$-th relay node into the collaborative transmission. After straightforward mathematical manipulation, the condition can be equivalently rewritten as

$$\cos(\Delta_{N+1}) > \frac{P^{(N)} - a^2_{p^{(N+1)}}}{2a_{p^{(N+1)}} \sqrt{NP^{(N)}}}. \quad (37)$$

Finally, we are ready to propose the following iterative greedy selection rule. Denote by $\mathcal{I}$ the node index set containing relay nodes selected for collaborative beamforming. Furthermore, let $\bar{\mathcal{I}}$ be the complementary set of $\mathcal{I}$ over $\{1, 2, \cdots, K\}$. The proposed greedy selection rule is initialized with $\mathcal{I} = \{1\}$, i.e. the best relay node, which is justified by the high likelihood of the best relay node being included in the optimal selection. The proposed greedy selection rule is summarized in Algorithm 1.

Algorithm 1 Iterative greedy selection rule

States: Initialize $N = 1$, $\mathcal{I} = \{1\}$, $\bar{\mathcal{I}} = \{2, 3, \cdots, K\}$, $z^{(1)} = a_1 e^{j\phi_1}$ and $P^{(1)} = a_1^2$;

Procedure:

for $N = 1$ to K **do**

Find

$$i^* = \arg\max_{i \in \bar{\mathcal{I}}} \left[\cos(\Delta_i) - \frac{P^{(N)} - a_i^2}{2a_i\sqrt{NP^{(N)}}} \right],$$

where Δ_i is the relative phase between h_i and $z^{(N)}$;

if $\cos(\Delta_{i^*}) > \frac{P^{(N)} - a_{i^*}^2}{2a_{i^*}\sqrt{NP^{(N)}}}$ **then**

1. Update

$$z^{(N+1)} = \frac{1}{\sqrt{N+1}} \left(\sqrt{N} z^{(N)} + a_{i^*} e^{j\phi_{i^*}} \right)$$

and

$$P^{(N+1)} = \left| z^{(N+1)} \right|^2;$$

2. Set $\mathcal{I} = \mathcal{I} \cup i^*$ while excluding i^* from $\bar{\mathcal{I}}$;

else

Terminate the algorithm;

end if

end for

Algorithm 2 Iterative pruning selection rule

States: Initialize $N = 1$, $\mathcal{I} = \{1, 2, 3, \cdots, K\}$, $\bar{\mathcal{I}} = \emptyset$, $z^{(1)} = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} a_k e^{j\phi_k}$ and $P^{(1)} = \left| z^{(1)} \right|^2$;

Procedure:

for $N = 1$ to K **do**

Find

$$i^* = \arg\max_{i \in \mathcal{I}} \left[\frac{P^{(N)} + a_i^2}{2a_i\sqrt{(K-N+1)P^{(N)}}} - \cos(\Delta_i) \right],$$

where Δ_i is the relative phase between h_i and $z^{(N)}$;

if $\cos(\Delta_{i^*}) < \frac{P^{(N)} + a_{i^*}^2}{2a_{i^*}\sqrt{(K-N+1)P^{(N)}}}$ **then**

1. Update

$$z^{(N+1)} = \frac{1}{\sqrt{K-N}} \left(\sqrt{K-N+1} z^{(N)} - a_{i^*} e^{j\phi_{i^*}} \right)$$

and

$$P^{(N+1)} = \left| z^{(N+1)} \right|^2;$$

2. Set $\bar{\mathcal{I}} = \bar{\mathcal{I}} \cup i^*$ while excluding i^* from $\mathcal{I}$;

else

Terminate the algorithm;

end if

end for

B. Iterative pruning selection rule

Clearly, the initialization step plays an important role in determining the performance of the proposed iterative greedy selection rule summarized in Algorithm 1. However, initializing with the best relay node is not necessarily always optimal. This can be easily understood by an extreme example in which the best relay node with the largest amplitude has a zero channel phase whereas all other relay nodes have a 180° channel phase. For the iterative greedy selection rule, the initialization of the best relay node results in the single-best-relay selection while the optimal selection may reside in a combination of the phase-aligning nodes.

To circumvent the initialization obstacle, an iterative pruning selection rule is developed in this section. Rather than adding one new node in each iteration, the pruning selection rule first includes all relay nodes in its initial selection. Then, one existing node is removed from the current selection in each iteration such that the remaining nodes provide a stronger received signal power than that of the current selection. This pruning process continues until no further improvement can be achieved by removing any one of the existing nodes. Furthermore, it is straightforward to show the selection criterion for the proposed iterative pruning selection rule is given by

$$\cos(\Delta_{N+1}) < \frac{P^{(N)} + a_{p^{(N+1)}}^2}{2a_{p^{(N+1)}}\sqrt{(K-N+1)P^{(N)}}}. \tag{38}$$

Finally, the proposed iterative pruning selection rule is summarized in Algorithm 2.

C. Remarks

In Algorithms 1 and 2, perfect CSI is assumed to be available to the destination. For the imperfect CSI case, we can simply replace $\{a_k\}$ and $\{\phi_k\}$ with $\{\hat{a}_k\}$ and $\left\{\hat{\phi}_k\right\}$, respectively. Upon the completion of the iterative selection process, $\{h_k\}$ and $\mathcal{I}$ are employed to evaluate the corresponding actual received signal power.

V. Numerical Results

This section presents numerical examples of the achievable performance of the proposed opportunistic collaborative beamforming with respect to the bounds developed in Section III and single-best-relay selection. Unless otherwise specified, all of the results in this section assume i.i.d. channel coefficients $h_k = a_k e^{j\phi_k}$, $k \in \{1, \dots, K\}$, with amplitudes a_k Rayleigh distributed with mean $\mathrm{E}[a_k] = \frac{\sqrt{\pi}}{2}$ and phases ϕ_k uniformly distributed on $(-\pi, \pi]$.

To obtain numerical results for finite values of K, minor modifications were made to the ideal coherent upper bound and sector-based lower bound selection rules. These selection rules were developed for the case when $K \to \infty$ and are based on the statistics of the channel coefficients, not the current channel realization. Hence, when K is finite, it is possible that no nodes meet the selection criteria. It is also possible that one or more nodes meet the selection criteria but the resulting power is less than that of the single best relay. The modified ideal coherent upper bound and sector-based lower bound selection rules check for these cases and select the single best relay if either case occurs.

A. Performance with perfect CSI

Figure 3 shows the average received power as a function of the total number of nodes K. The optimum opportunistic collaborative beamformer performance is plotted only for

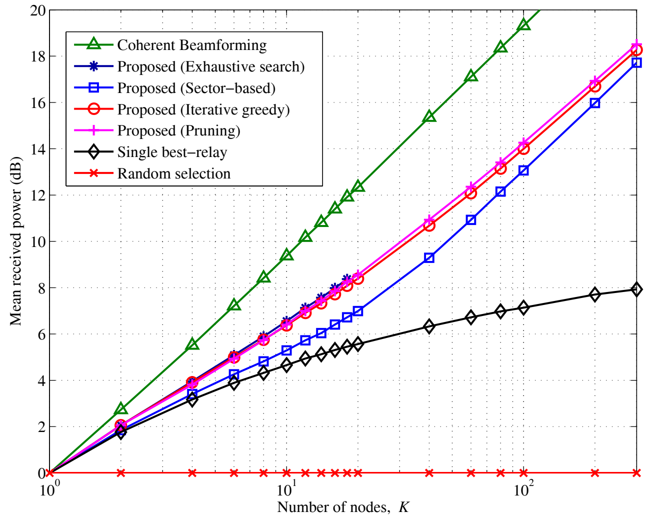

Fig. 3. Mean received power versus the total number of nodes K with perfect CSI available to the destination.

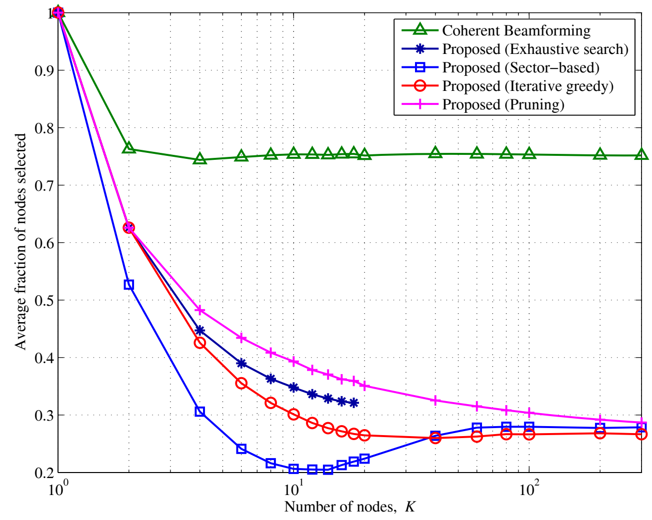

Fig. 4. Average fraction of nodes selected for participation in the collaborative beamformer versus the total number of nodes K with perfect CSI available to the destination.

$K \leq 18$ due to the computational complexity of the exhaustive search over $2^K - 1$ possible selection vectors. The upper and lower bounds confirm that the received power scaling of opportunistic collaborative beamforming is linear in K and, as predicted in (26), their performance gap is approximately 6.37 dB for large K. These results also demonstrate that the iterative greedy and pruning selection rules outperform the sector-based selection rule and exhibit an average received power performance very close to the optimum exhaustive search, at least for $K \leq 18$, with much lower computational complexity. Furthermore, Figure 3 indicates that the iterative pruning selection rule provides some marginal performance improvement compared to the iterative greedy selection rule. For comparison purposes, the performance of *incoherent* collaborative beamforming with *randomly* selected relay nodes is also depicted in Figure 3. Since the effective channel gain generated by the normalized sum of Gaussian distributed i.i.d. $\{h_k\}$ remains $\mathcal{CN}(0,1)$, such incoherent collaborative beamforming cannot render any beamforming gain [17] and only results in unity average received power, regardless of K. Finally, all four proposed opportunistic collaborative beamformers with different selection schemes substantially outperform the single-best-relay scheme, particularly for large K. For smaller K, the coherent beamformer exhibits significant power gains with respect to the proposed schemes. For instance, at $K = 10$, the coherent beamformer outperforms the proposed beamformer with exhaustive search by 2.5 dB and the single-best-relay scheme by 4.5 dB at the price of increased synchronization and feedback overhead.

Figure 4 shows the average fraction of nodes selected for participation in the opportunistic collaborative beamformer versus the total number of nodes K. In the case of the ideal coherent upper bound, the fraction of nodes selected converges to about 75%, which agrees well with our analytical result

$$\Pr(a_k \geq \gamma^*) = e^{-0.5316^2} \approx 0.7538.$$

This can be further explained by the fact that the nodes all have identical phase and only nodes with insufficient amplitude are rejected. Inspection of Figure 4 suggests that the pruning selection rule is more inclusive than the optimum exhaustive search selection rule, the iterative greedy selection rule and the sector-based selection rule. For large K, the iterative greedy selection rule, the iterative pruning selection rule and the sector-based selection rule tend to select similar fractions of nodes for beamforming, i.e.

$$\Pr(|\phi_k| \leq \alpha^*) \Pr(a_k \geq \gamma^*) = \frac{1.1656}{\pi} \cdot e^{-0.5316^2} \approx 0.2797,$$

with the iterative pruning selection rule being slightly more inclusive in this scenario.

B. Performance with imperfect CSI

In this section, we evaluate the performance of the proposed opportunistic collaborative beamformer under noisy channel estimation. Unless otherwise specified, we set $K = 100$ in the following experiments. Furthermore, we define the estimation SNR as

$$SNR = \frac{1}{\sigma_\xi^2}. \tag{39}$$

We first investigate the optimal thresholds for the sector-based selection rule with noisy channel estimates. Figure 5 shows the mean received power as a function of phase threshold α' for a few SNR values.

Inspection of Figure 5 suggests that the optimal phase threshold α'^* remains approximately 1.1656, irrespective of the SNR values, which agrees well with our analytical results shown in (30). Therefore, we set $\alpha'^* = 1.1656$ in the remaining experiments.

Figure 6 depicts the mean received power as a function of amplitude threshold γ' parameterized by SNR. Clearly, the optimal amplitude threshold, γ'^* increases as SNR decreases, which is in accord with our analysis. Furthermore, it is interesting to observe that the mean received power curve not only degrades significantly but also becomes "flatter" as SNR decreases. In other words, as SNR decreases, the performance

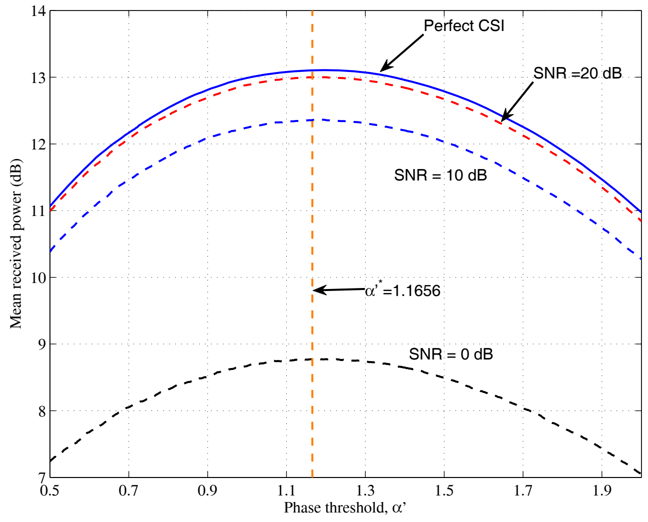

Fig. 5. Mean received power versus the phase threshold, α', in the presence of imperfect CSI with $K = 100$.

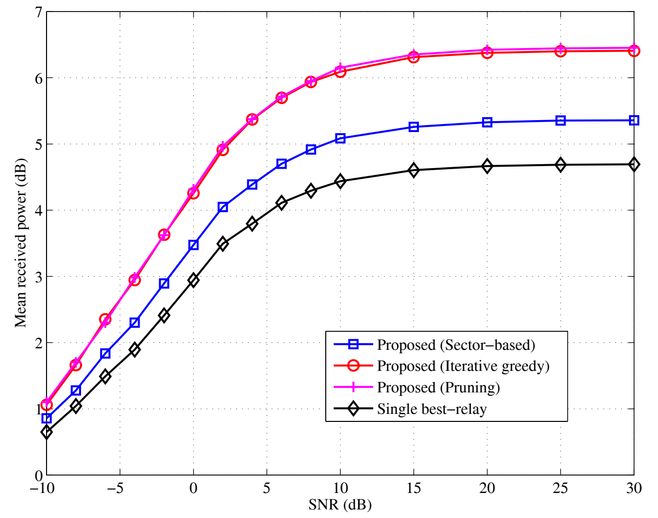

Fig. 7. Mean received power as a function of SNR in the presence of imperfect CSI with $K = 10$.

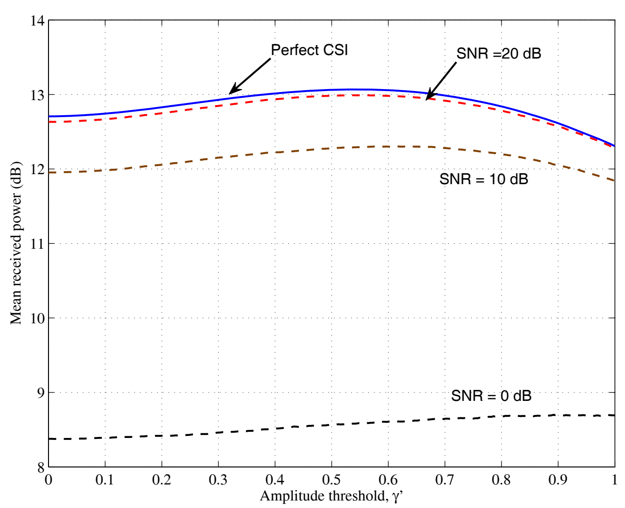

Fig. 6. Mean received power versus the amplitude threshold, γ', in the presence of imperfect CSI with $K = 100$.

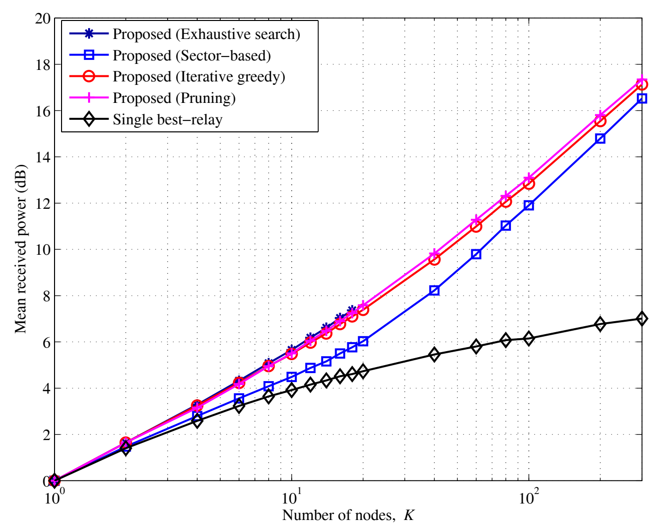

Fig. 8. Mean received power versus the total number of nodes K with imperfect CSI and SNR = 5 dB at the destination.

of the sector-based selection rule becomes less sensitive to the choice of amplitude threshold.

Next, Figure 7 shows the mean received power obtained with the proposed collaborative beamformer with different selection rules and the single-best-relay scheme. $\{\alpha'^*, \gamma'^*\}$ are employed in the proposed collaborative beamforming scheme with the sector-based selection rule. Figure 7 verifies that the proposed collaborative beamformer remains very robust even under noisy channel estimation. Furthermore, Figure 7 indicates that CSI estimates obtained with SNR of 10 dB or larger are sufficiently accurate for the proposed collaborative beamformer to achieve good performance on par with that obtained with perfect CSI.

Finally, we repeat the experiments shown in Figures 3 and 4 but in the presence of noisy channel estimation at SNR of 5 dB. Comparison of Figures 3 and 8 confirms the robustness of the proposed beamformer with imperfect CSI even for smaller K. The proposed collaborative beamformer significantly outperforms the single-best relay scheme even with noisy channel estimates.

Similar to Figure 4, Figure 9 shows the average fraction of selected nodes as a function of the total number of nodes K in the presence of imperfect CSI. Inspection of Figure 9 suggests that the average fraction of participating nodes selected by the sector-based selection rule slightly increases under noisy channel estimation whereas the corresponding impact on the exhaustive search, iterative greedy and pruning selection rules is rather marginal.

C. Performance with heterogeneous relay-destination links

The analytical and simulation results presented so far concentrate on homogenous relay-destination links by modeling $\{h_k\}$ as $\mathcal{CN}\left(0, \sigma_{h_k}^2\right)$ with $\sigma_{h_k}^2 = 1$. Such a model is valid and accurate for wireless ad hoc networks characterized by

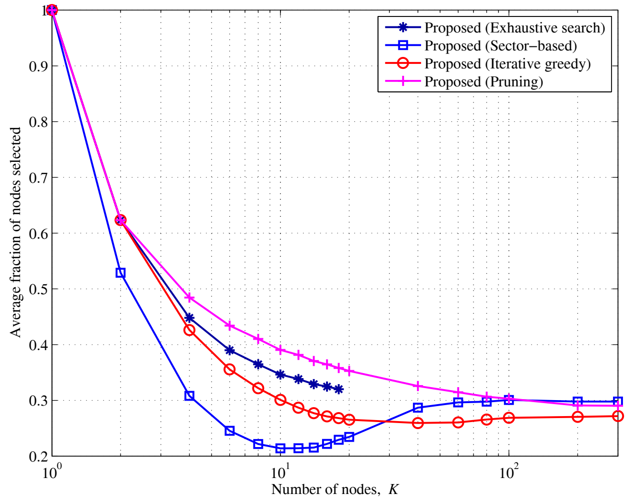

Fig. 9. Average fraction of nodes selected for participation in the collaborative beamformer versus the total number of nodes K with imperfect CSI and SNR = 5 dB at the destination.

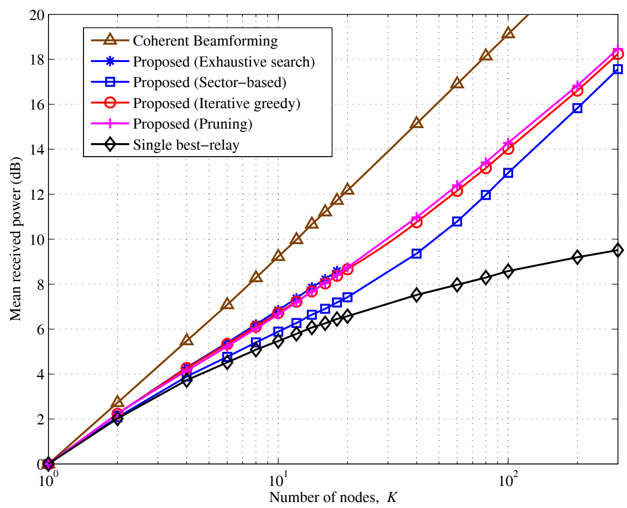

Fig. 10. Mean received power versus the total number of nodes K with heterogeneous relay-destination links.

closely clustered relay nodes, while the effectiveness of the proposed collaborative beamformer in networks composed of heterogeneous relay-destination links has to be confirmed.

In this experiment, rather than fixing $\sigma_{h_k}^2 = 1$, we model $\sigma_{h_k}^2$ as a random variable uniformly distributed over $(0, 2)$. Repeating the experiment depicted in Figure 3, we show the resulting received power as a function of K in Figure 10.

Comparison of Figures 3 and 10 reveals the following interesting observations. First, since the average channel gain is kept at unity, i.e. $E\left\{\sigma_{h_k}^2\right\} = 1$, the performance of the coherent beamformer remains unchanged. In contrast, the single-best-relay scheme exploits the increased maximum channel gain provided by the relay nodes with larger $\sigma_{h_k}^2$, which leads to noticeable performance improvement. More specifically, nearly 1 dB improvement is observed for the single-best-relay scheme at $K = 10$. Finally, Figure 10 confirms that the performance of the proposed collaborative beamformer remains robust even in heterogeneous networks.

VI. Discussion and Conclusions

One of the appeals of the opportunistic collaborative beamforming scheme described in this paper is that each node in the system requires only one bit of feedback in order to commence or halt transmission. This is in contrast to fully-coherent collaborative beamforming schemes that typically require several bits of feedback per node in order to perform local phase precompensation (and perhaps additional bits to exclude nodes with weak channels from transmitting) or several iterations of single-bit feedback to converge to a coherent state. The rate at which the source selection vectors must be sent in an opportunistic collaborative beamformer depends on the channel coherence time as well as any frequency offsets and/or phase noise of the nodes' local oscillators. In systems with channels that exhibit long coherence times, feedback will be required at a rate proportional to the maximum carrier frequency offset among the nodes. Outlier nodes with large carrier offsets could be permanently excluded from the pool of available nodes to reduce the feedback rate requirement. More detailed studies on the feedback rate requirement for opportunistic collaborative beamforming under general channel conditions are of importance.

In addition to a study of the feedback rate requirements, there are several extensions of this work that may also be fruitful directions for further investigation. One unanswered question is whether there exists a polynomial-time algorithm for computing the optimal relay node selection vector. On the one hand, the great similarity between this problem and the classical subset-sum problem hints that the problem may be NP-complete [18]. On the other hand, if the optimal relay node selection problem can be shown to be solvable in polynomial time, more efficient selection algorithms should be devised.

Another potentially valuable extension to the ideas proposed in this paper would be the development of decentralized relay node selection algorithms. Throughout our previous discussions, we have concentrated on centralized selection in which the destination feeds the selection decision back to the relay nodes. It is possible, however, to envision a threshold-based selection rule that could be implemented in a distributed manner. Assuming that each node has only local CSI, obtained perhaps from a pilot signal transmitted from the destination, we can consider a system in which each node sets a timer inversely proportional to its channel gain similar to the procedure described in [2]. Upon its timeout, the node with the strongest channel gain first broadcasts its own channel information (amplitude and phase) to its peer nodes. This is in contrast to [2] in which the best node simply starts sending data to the destination. Exploiting the received information about the strongest channel gain, each node can compare its own channel amplitude and phase against some pre-designed thresholds. In the next time slot, the nodes with channel conditions exceeding the thresholds start transmitting data simultaneously with the best node.

Finally, comprehensive studies on the energy efficiency improvement achieved by the proposed opportunistic collaborative beamforming schemes deserve further investigation. Despite the fact that relay nodes in the proposed schemes are exempted from local channel estimation, pilot signals are required to be transmitted from these nodes to the destination node to facilitate global CSI estimation. Furthermore, the energy required for other coordination aspects of the protocol, e.g. spreading code assignment to the relay nodes, should be also taken into consideration. As a result, it appears more appropriate to evaluate the energy efficiency of the proposed schemes from a holistic prospective by taking into account the energy consumption for both global CSI estimation and data transmission. However, such a holistic approach is a non-trivial task since the overall system energy efficiency is also determined by other design aspects such as energy allocation between global CSI estimation and data transmission. On the one hand, allocating more available energy to global CSI estimation will clearly result in more accurate channel estimates at the price of less energy available for data transmission. On the other hand, noisy CSI estimates due to low-SNR channel estimation incur performance degradation in terms of mean received signal power as shown in Sec. V. Thus, sophisticated channel estimation techniques such as the successive refinement technique [19], are desirable to reduce energy consumption required for accurate global CSI estimation and subsequently, to improve the system energy efficiency.

The opportunistic collaborative beamforming scheme described in this paper fills a gap between the opportunistic relaying schemes proposed in [2] and [3] and the fully coherent collaborative beamforming schemes described in [4]–[9]. The appeal of the opportunistic relaying schemes proposed in [2] and [3] is simplicity: global CSI does not need to be known by any entity in the network and no feedback is required to select the best relay. This simplicity comes at the cost of inefficient undirected transmission in the relay-destination link. Directed transmission is achieved by the fully coherent collaborative beamforming schemes described in [4]–[9], but at the cost of potentially prohibitive complexity and/or feedback overhead. While the proposed opportunistic collaborative beamforming scheme described in this paper does require the destination to estimate the CSI of all K available relay nodes, the relay nodes incur very little additional cost/complexity since all processing is handled at the destination. Moreover, the feedback requirements of opportunistic collaborative beamforming are relatively low with respect to prior approaches.

The main contributions of this work are the development of an opportunistic collaborative beamformer with one bit of feedback per relay node and a unification of the ideas of collaborative beamforming and opportunistic relay selection. Unlike conventional collaborative beamforming, opportunistic collaborative beamforming is applicable in networks with nodes that may not be able to control their carrier frequency or phase. While optimal node selection for opportunistic collaborative beamforming is exponentially complex in the number of available nodes, we have shown that low-complexity selection rules can provide near-optimum beamforming gain with performance within 6.37 dB of an ideal fully-coherent collaborative beamformer with perfect CSI available to the relay nodes. We have also shown, in contrast to single-best-relay selection, that the received power of opportunistic collaborative beamforming scales linearly with the number of available nodes.

APPENDICES

A. PROOF OF (17)

Denote by Ω_{ub} and κ the node index set of the selected relay nodes and its cardinality, respectively. Thus, we can express the received power upper bound normalized by K as

$$\lim_{K\to\infty}\frac{P_{ub}^{(K)}(\gamma)}{K} = \lim_{K\to\infty}\frac{1}{K}\frac{\left|\boldsymbol{a}^T\boldsymbol{s}\right|^2}{\boldsymbol{s}^T\boldsymbol{s}}, \quad (40)$$

$$= \lim_{K\to\infty}\frac{\kappa^2}{K}\frac{\left|\frac{1}{\kappa}\sum_{k\in\Omega_{ub}} s_k\right|^2}{\boldsymbol{s}^T\boldsymbol{s}}. \quad (41)$$

On recalling that $\kappa = \boldsymbol{s}^T\boldsymbol{s}$, we can rewrite (41) in the following form :

$$\lim_{K\to\infty}\frac{P_{ub}^{(K)}(\gamma)}{K}$$

$$= \lim_{K\to\infty}\frac{\boldsymbol{s}^T\boldsymbol{s}}{K}\left[\mathrm{E}\left\{s_k\,|k\in\Omega_{ub}\right\}\right]^2, \quad (42)$$

$$= \lim_{K\to\infty}\frac{\boldsymbol{s}^T\boldsymbol{s}}{K}\left[\frac{1}{\Pr\left(k\in\Omega_{ub}\right)}\int_{\gamma}^{\infty}2x^2e^{-x^2}\,dx\right]^2, \quad (43)$$

$$= \lim_{K\to\infty}\frac{K}{\boldsymbol{s}^T\boldsymbol{s}}\left[\int_{\gamma}^{\infty}2x^2e^{-x^2}\,dx\right]^2, \quad (44)$$

where we have used the fact that $\Pr\left(k\in\Omega_{ub}\right) = \frac{\boldsymbol{s}^T\boldsymbol{s}}{K}$ in obtaining the last equality. Finally, upon substituting (15) into (44) and invoking integration by parts, we can have (17).

B. PROOF OF (30)

For presentational simplicity, we first define the following quantities :

$$X_k \stackrel{\text{def}}{=} -\xi_k, \quad (45)$$

$$Y_k \stackrel{\text{def}}{=} \hat{h}_k, \quad (46)$$

$$Z_k \stackrel{\text{def}}{=} h_k. \quad (47)$$

When K is large, the average number of relay nodes belonging to the selection region Ω'_{lb} can be computed by

$$\kappa = K\cdot\Pr\left(Y_k\in\Omega'_{lb}\right), \quad (48)$$

$$= \frac{K\alpha'}{\pi}\exp\left\{-\frac{\gamma'^2}{\sigma_{\hat{h}}^2}\right\}, \quad (49)$$

where the last equality is obtained by exploiting the fact that

$$\Pr(\hat{h}_k\in\Omega'_{lb}) = \Pr\left(|\hat{\phi}_k|\le\alpha'\right)\Pr\left(\hat{a}_k\ge\gamma'\right). \quad (50)$$

Next, we proceed to evaluate the normalized received power parameterized by α' and γ' and have

$$\lim_{K\to\infty}\frac{P_{lb}^{'(K)}(\alpha',\gamma')}{K} = \frac{1}{K\kappa}\left|\sum_{k=1}^{\kappa}Z_k\mathbb{I}_{Y_k\in\Omega'_{lb}}\right|^2, \quad (51)$$

where $\mathbb{I}$ is the indicator function. Recall $Z_k = X_k + Y_k$, (51) can be expanded into the following form.

$$\begin{aligned}
&\lim_{K\to\infty} \frac{P_{lb}^{'(K)}(\alpha', \gamma')}{K} \\
&= \frac{1}{K\kappa}\left|\sum_{k=1}^{\kappa} X_k \mathbb{I}_{Y_k\in\Omega'_{lb}} + \sum_{k=1}^{\kappa} Y_k \mathbb{I}_{Y_k\in\Omega'_{lb}}\right|^2, \quad (52)\\
&= \frac{\kappa}{K}\left|\frac{1}{\kappa}\sum_{k=1}^{\kappa} X_k \mathbb{I}_{Y_k\in\Omega'_{lb}} + \frac{1}{\kappa}\sum_{k=1}^{\kappa} Y_k \mathbb{I}_{Y_k\in\Omega'_{lb}}\right|^2, \quad (53)\\
&= \frac{\kappa}{K}\left|\mathrm{E}\left[X_k \,|\, Y_k \in \Omega'_{lb}\right] + \mathrm{E}\left[Y_k \,|\, Y_k \in \Omega'_{lb}\right]\right|^2. \quad (54)
\end{aligned}$$

Capitalizing on the following result

$$\int_{-\alpha'}^{\alpha'}\int_{\gamma'}^{\infty} x e^{jy} \frac{1}{2\pi}\frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}}\, dx\, dy = \frac{\sigma \sin\alpha'}{\sqrt{2\pi}} f\left(\frac{\gamma'}{\sqrt{2}\sigma}\right), \quad (55)$$

where $f(\gamma)$ is as defined in (18), we can immediately obtain the first conditional expectation on the right hand side (R.H.S) of (54) as

$$\begin{aligned}
&\mathrm{E}\left[Y_k \,|\, Y_k \in \Omega'_{lb}\right] \\
&= \frac{1}{\Pr\left[Y_k \in \Omega'_{lb}\right]} \cdot \frac{\sigma_{\hat{h}} \sin\alpha'}{2\sqrt{\pi}} f\left(\frac{\gamma'}{\sigma_{\hat{h}}}\right), \quad (56)\\
&= \frac{K\sigma_{\hat{h}} \sin\alpha'}{2\kappa\sqrt{\pi}} f\left(\frac{\gamma'}{\sigma_{\hat{h}}}\right). \quad (57)
\end{aligned}$$

Next, we evaluate the second conditional expectation on the R.H.S of (54). On recalling that X_k and Y_k are jointly Gaussian random variables, we can first derive

$$\begin{aligned}
&\mathrm{E}\left[X_k \,|\, Y_k = y_k\right] \\
&= \mathrm{E}[X] + \frac{\mathrm{cov}(X_k, Y_k)}{\mathrm{var}(Y_k)}(y_k - \mathrm{E}[Y_k]), \quad (58)\\
&= \frac{\mathrm{cov}(X_k, Y_k)}{\mathrm{var}(Y)} y_k, \quad (59)\\
&= \frac{-\sigma_\xi^2}{\sigma_{\hat{h}}^2} y_k. \quad (60)
\end{aligned}$$

Hence, the second conditional expectation can be computed as

$$\begin{aligned}
&\mathrm{E}\left[X \,|\, Y \in \Omega'_{lb}\right] \\
&= \frac{1}{\Pr\left[Y \in \Omega'_{lb}\right]} \int_{\Omega'_{lb}} \mathrm{E}[X \,|\, Y = y] p_Y(y)\, dy, \quad (61)\\
&= -\frac{\sigma_\xi^2 K \sin\alpha'}{2\sigma_{\hat{h}}\kappa\sqrt{\pi}} f\left(\frac{\gamma'}{\sigma_{\hat{h}}}\right). \quad (62)
\end{aligned}$$

Finally, substituting (57) and (62) into (54) followed by some straightforward mathematical manipulations, we obtain (30).

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